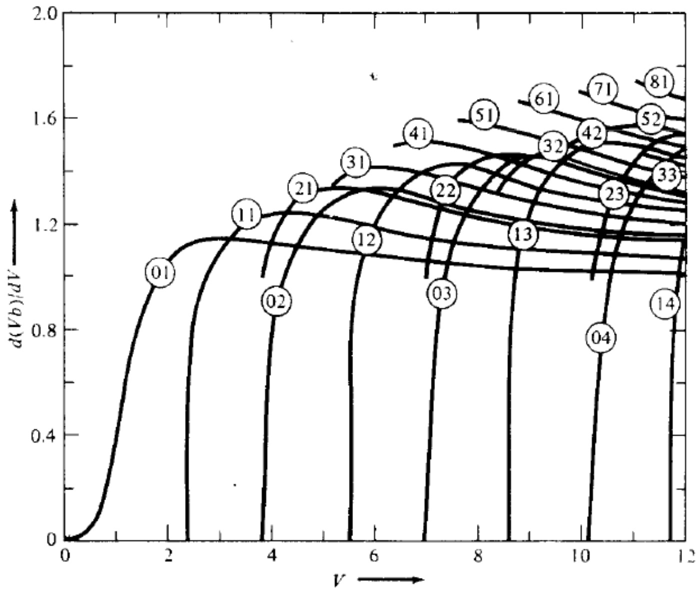
Delay time due to waveguide dispersion can then be expressed as:

$$\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(kb)}{dk}\right] \cong \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(Vb)}{dV}\right]$$



Waveguide dispersion in single mode fibers

For single mode fibers, waveguide dispersion is in the same order of material dispersion. The pulse spread can be well approximated as:

$$\sigma_{wg} \approx \left|\frac{d\tau_{wg}}{d\lambda}\right|\sigma_{\lambda} = L\sigma_{\lambda}\left|D_{wg}(\lambda)\right| = -\frac{n_2 L\Delta\sigma_{\lambda}}{c\lambda}V\frac{d^2(Vb)}{dV^2}$$

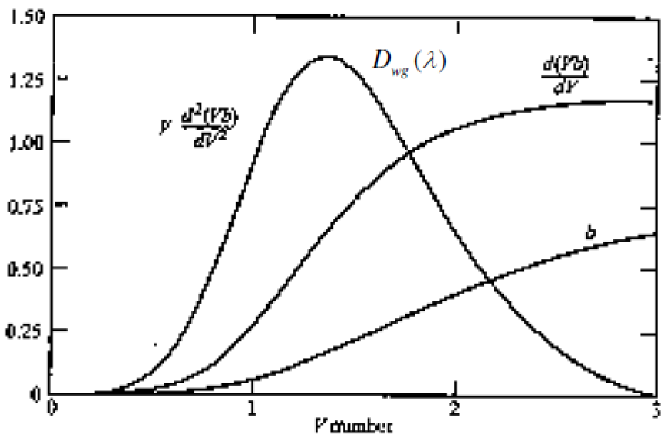


FIGURE 3-15
The waveguide parameter b and its derivatives $d(Vb)/dV$ and $V\,d^2(Vb)/dV^2$ plotted as a function of the V number for the HE_{11} mode.

Modal Dispersion

In multi-mode fiber, the pulse spread due to the modal dispersion can be approximated by ray tracing as:

$$\sigma_{modal} \approx \tau_{\max} - \tau_{\min}$$

where τ_{max}, τ_{min} denote the maximum, minimum group delays, respectively. Since

$$\tau_{\max} = L/(v_p \sin\phi_c); \tau_{\min} = L/v_p; v_p = c/n_1$$

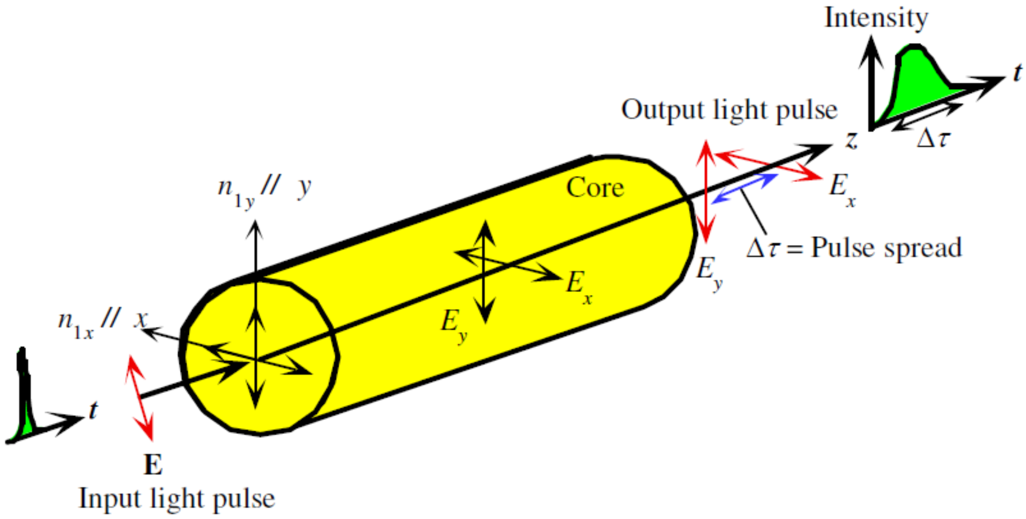
It can be shown that

$$\sigma_{modal} \approx \frac{Ln_1}{c}\left(\frac{n_1}{n_2} - 1\right) = \frac{Ln_1^2}{cn_2}\Delta$$

In general, $\sigma_{modal} < 1/B$ is required, thus bit rate-distance product :

$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$

Polarization Mode dispersion



Suppose that the core refractive index has different values along two orthogonal directions corresponding to electric field oscillation direction (polarizations). We can take x and y axes along these directions. An input light will travel along the fiber with E_x and E_y polarizations having different group velocities and hence arrive at the output at different times

The effects of fiber-birefringence on the polarization states of an optical are another source of pulse broadening. **Polarization mode dispersion** (PMD) is due to slightly different velocity for each polarization mode because of the lack of perfectly symmetric & anisotropicity of the fiber. If the group velocities of two orthogonal polarization modes are v_{gx} and v_{gy} then the differential time delay $\Delta\tau_{PMD}$ between these two polarization over a distance L is

$$\Delta\tau_{PMD} = \left| \frac{L}{v_{gx}} - \frac{L}{v_{gy}} \right|$$

The rms value of the differential group delay can be approximated as:

$$\langle \Delta\tau_{PMD} \rangle \approx D_{PMD}\sqrt{L}$$

Chromatic & Total Dispersion

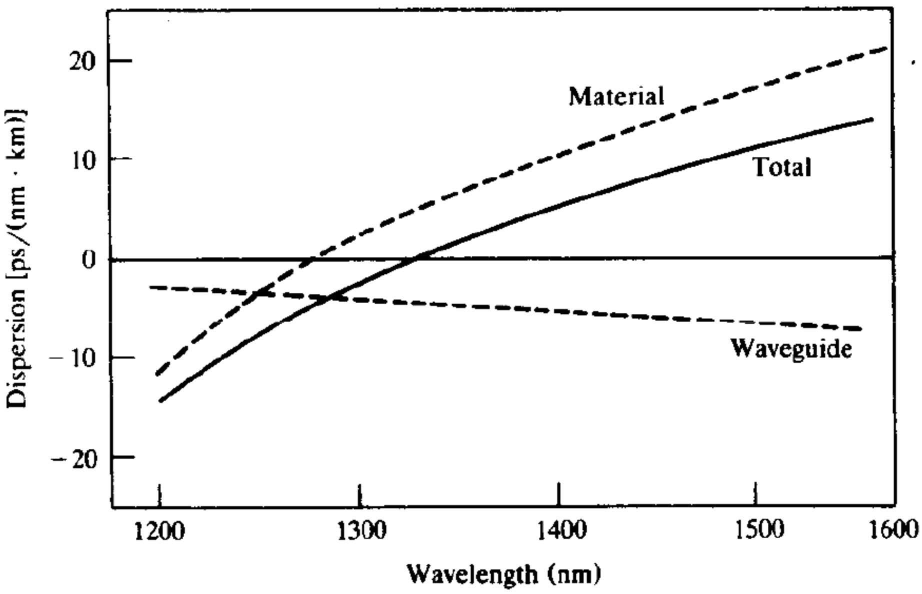
- Chromatic dispersion includes the material & waveguide dispersions.

$$D_{ch}(\lambda) \approx \left| D_{mat} + D_{wg} \right|$$
$$\sigma_{ch} = D_{ch}(\lambda) L \sigma_{\lambda}$$

- Total dispersion is the sum of chromatic , polarization dispersion and other dispersion types and the total rms pulse spreading can be approximately written as:

$$D_{total} \approx \left| D_{ch} + D_{pol} + \ldots \right|$$
$$\sigma_{total} = D_{total} L \sigma_{\lambda}$$

Total Dispersion, zero Dispersion



Fact 1) Minimum distortion at wavelength about 1300 nm for single mode silica fiber.
Fact 2) Minimum attenuation is at 1550 nm for sinlge mode silica fiber.
Strategy: shifting the zero-dispersion to longer wavelength for minimum attenuation and dispersion.

➢ Absorption and Scattering losses

SCATTERING

- LINEAR ⇒
  - Rayleigh (diameter physical anomalies < $\lambda/10$)
  - Mie (diameter physical anomalies > $\lambda/10$)
- NON-LINEAR ⇒
  - Brillouin (acoustic phonon → SBS)
  - Raman (optical phonon → SRS)

Rayleigh Scattering Coefficient ⇒ Transmission Loss factor

$$\gamma_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_T k_B T_F \quad \Longrightarrow \quad F_{Rayleigh} = e^{-\gamma_R L}$$

n : refractive index of the material,
β_T : isothermal compressibility,
p : photoelastic coefficient,
T_F : Solidification temperature,
k_B : Boltzman constant,
L : the fibre length.

ABSORPTION
- INTRINSIC ⇨ Interaction of free electrons and the λ within the fibre material.
- EXTRINSIC ⇨ Impurities atoms in glass material (metal ions)
- INDUCED ⇨ Curvature (micro and macrobending)

Urbach's rule (empirical relationship)

$$\alpha_{UV}[dB/km] = Ce^{\frac{\lambda_{UV}}{\lambda}} = 1.108\times10^{-3}\, e^{\frac{4.582}{\lambda[\mu m]}}$$

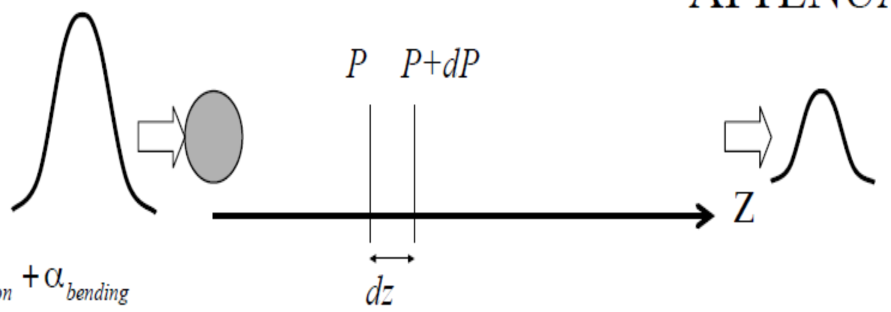
$$\alpha_{IR}[dB/km] = 4\times10^{-11}\, e^{\frac{48.48}{\lambda[\mu m]}}$$

Macrobending (critical radius)

$$R_{MMF} = \frac{3\lambda n_1^2}{4\pi\,(n_1^2 - n_2^2)}$$

$$R_{SMF} = \frac{20\lambda}{\sqrt{n_1^2 - n_2^2}}\left(2.748 - \frac{0.996\lambda}{\lambda_{cutoff}}\right)^{-3}$$

ATTENUATION



$$\alpha = \alpha_{scattering} + \alpha_{absorption} + \alpha_{bending}$$

$$\frac{dP}{dz} = -\alpha L \Rightarrow P(L) = P(0)e^{-\alpha L} \Rightarrow \alpha = \frac{1}{L}\ln\left[\frac{P(0)}{P(L)}\right] \quad [1/\text{km or } 1/\text{m}]$$

$$\frac{P(0)}{P(L)} = e^{\alpha L} \Leftrightarrow 10\log\left[\frac{P(0)}{P(L)}\right] = 10\log\left[e^{\alpha L}\right] = 10\alpha L \log e = 4.343\alpha$$

$$\alpha[dB/km] = \frac{10}{L}\log\left[\frac{P(0)}{P(L)}\right]$$

$$\alpha[dB/km] = 4.343\alpha[1/km]$$