Chapter Two

Optical Networks

Pulse Propagation in Fibers:

➤ Input/Output signals in Fiber Transmission System

The optical signal (complex) waveform at the input of fiber of length l is $f(t)$. The propagation constant of a particular modal wave carrying the signal is $\beta(\omega)$. Let us find the output signal waveform $g(t)$.

$\Delta\omega$ is the optical signal bandwidth.

$$f(t) = \int_{\omega_c - \Delta\omega}^{\omega_c + \Delta\omega} \tilde{f}(\omega) e^{j\omega t} d\omega$$

$$g(t) = \int_{\omega_c - \Delta\omega}^{\omega_c + \Delta\omega} \tilde{f}(\omega) e^{j\omega t - j\beta(\omega) l} d\omega$$

If $\Delta\omega \ll \omega_c$

$$\beta(\omega) = \beta(\omega_c) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} (\omega - \omega_c) + \frac{1}{2}\left.\frac{d^2\beta}{d\omega^2}\right|_{\omega=\omega_c} (\omega - \omega_c)^2 + \dots$$

Therefore

$$g(t) = \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega t - j\beta(\omega)l} d\omega$$

$$\approx \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega t - j[\beta(\omega_c) + \left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} (\omega-\omega_c)]l} d\omega$$

$$\approx e^{-j\beta(\omega_c)l} \int_{\omega_c-\Delta\omega/2}^{\omega_c+\Delta\omega/2} \tilde{f}(\omega) e^{j\omega(t - l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c})} d\omega$$

$$= e^{-j\beta(\omega_c)l} f(t - l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c}) = e^{-j\beta(\omega_c)l} f(t - \tau_g)$$

$$\tau_g = l\left.\frac{d\beta}{d\omega}\right|_{\omega=\omega_c} = \frac{l}{V_g}$$

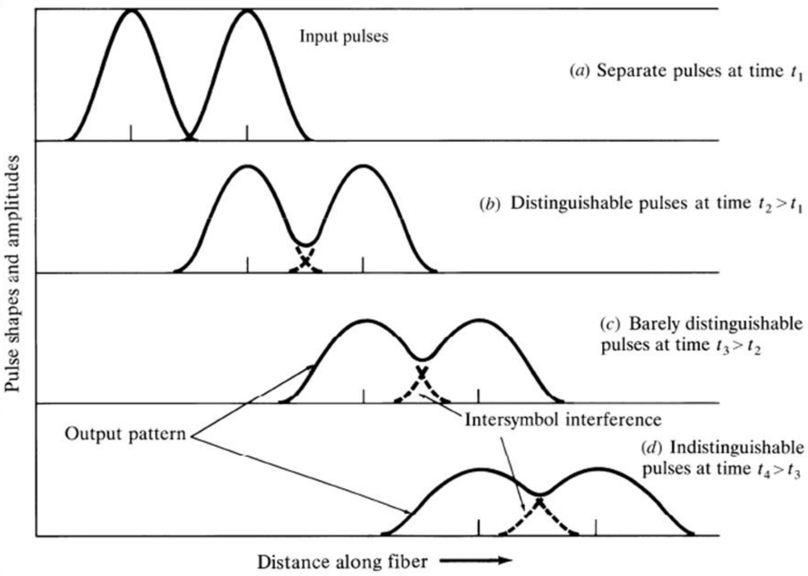
➢ Intramodal Dispersion

As we have seen from Input/output signal relationship in optical fiber, the output is proportional to the delayed version of the input signal, and the delay is inversely proportional to the group velocity of the wave. Since the propagation constant, $\beta(\omega)$, is frequency dependent over band width $\Delta\omega$ sitting at the center frequency ω_c, at each frequency, we have one propagation constant resulting in a specific delay time. As the output signal is collectively represented by group velocity & group delay this phenomenon is called **intramodal dispersion or Group Velocity Dispersion (GVD). This phenomenon arises due to a finite bandwidth of the optical source, dependency of refractive index on the wavelength and the modal dependency of the group velocity**.

In the case of optical pulse propagation down the fiber, GVD causes pulse broadening, leading to Inter Symbol Interference (ISI).

Dispersion & ISI

A measure of information capacity of an optical fiber for digital transmission is usually specified by the **bandwidth-distance product (BW×distance) in GHz.km.** For multi-mode step index fiber this quantity is about 20 MHz·km, for graded index fiber is about 2.5 GHz·km & for single mode fibers are higher than 10 GHz.km.



How to characterize dispersion?

Group delay per unit length can be defined as:

$$\frac{\tau_g}{L} = \frac{d\beta}{d\omega} = \frac{1}{c}\frac{d\beta}{dk} = -\frac{\lambda^2}{2\pi c}\frac{d\beta}{d\lambda}$$

If the spectral width of the optical source is not too wide, then the delay difference per unit wavelength along the propagation path is approximately $\frac{d\tau_g}{d\lambda}$. For spectral components which are $\delta\lambda$ apart, symmetrical around center wavelength, the total delay difference $\delta\tau$ over a distance L is:

$$\delta\tau = \left|\frac{d\tau_g}{d\lambda}\right|\delta\lambda = -\frac{L}{2\pi c}\left(2\lambda\frac{d\beta}{d\lambda} + \lambda^2\frac{d^2\beta}{d\lambda^2}\right)\delta\lambda$$

$$= \left|\frac{d\tau}{d\omega}\right|\delta\omega = \frac{d}{d\omega}\left(\frac{L}{V_g}\right)\delta\omega = L\left(\frac{d^2\beta}{d\omega^2}\right)\delta\omega$$

$\beta_2 \equiv \dfrac{d^2\beta}{d\omega^2}$ is called **GVD parameter,** and shows how much a light pulse broadens as it travels along an optical fiber. The more common parameter is called **Dispersion,** and can be defined as the delay difference per unit length per unit wavelength as follows:

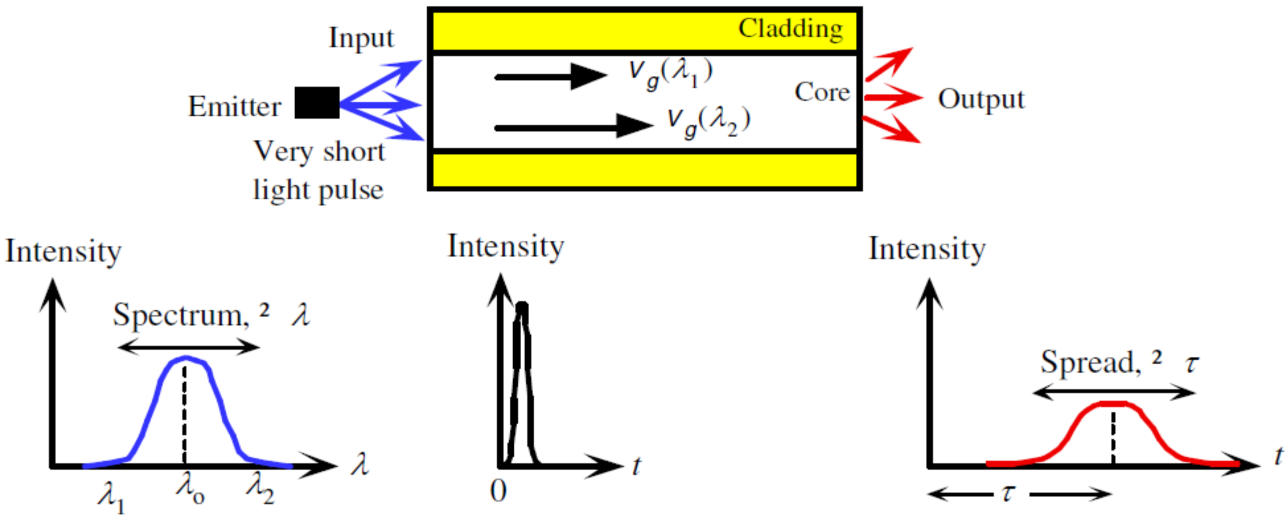
$$D = \frac{1}{L}\frac{d\tau_g}{d\lambda} = \frac{d}{d\lambda}\left(\frac{1}{V_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2$$

In the case of optical pulse, if the spectral width of the optical source is characterized by its rms value of the Gaussian pulse σ_λ [nm], the pulse spreading over the length of L, σ_g [ps] can be well approximated by:

$$\sigma_g \approx \left|\frac{d\tau_g}{d\lambda}\right|\sigma_\lambda = DL\sigma_\lambda$$

D has a typical unit of [ps/(nm.km)].

Material Dispersion:



All excitation sources are inherently non-monochromatic and emit within a spectrum, ^{2} λ, of wavelengths. Waves in the guide with different free space wavelengths travel at different group velocities due to the wavelength dependence of n_1. The waves arrive at the end of the fiber at different times and hence result in a broadened output pulse.

The refractive index of the material varies as a function of wavelength, $n(\lambda)$

Material-induced dispersion for a plane wave propagation in homogeneous medium of refractive index n:

$$\tau_{mat} = L\frac{d\beta}{d\omega} = -\frac{\lambda^2}{2\pi c}L\frac{d\beta}{d\lambda} = -\frac{\lambda^2}{2\pi c}L\frac{d}{d\lambda}\left[\frac{2\pi}{\lambda}n(\lambda)\right]$$
$$= \frac{L}{c}\left(n - \lambda\frac{dn}{d\lambda}\right)$$

The pulse spread due to material dispersion is therefore:

$$\sigma_g \approx \left|\frac{d\tau_{mat}}{d\lambda}\right|\sigma_\lambda = \frac{L\sigma_\lambda}{c}\left|\lambda\frac{d^2n}{d\lambda^2}\right| = L\sigma_\lambda\left|D_{mat}(\lambda)\right|$$

$D_{mat}(\lambda)$ is material dispersion

Waveguide Dispersion

Waveguide dispersion is due to the dependency of the group velocity of the fundamental mode as well as other modes on the V number, (see Fig 2-18 of the textbook). In order to calculate waveguide dispersion, we consider that n is not dependent on wavelength. Defining the normalized propagation constant b as:

$$b = \frac{\beta^2/k^2 - n_2^{\ 2}}{n_1^{\ 2} - n_2^{\ 2}} \approx \frac{\beta/k - n_2}{n_1 - n_2}$$

solving for propagation constant:

$$\beta \approx n_2k(1 + b\Delta); \Delta = (n_1 - n_2)/n_1$$

Using V number:

$$V = ka(n_1^{\ 2} - n_2^{\ 2})^{1/2} \approx kan_1\sqrt{2\Delta}$$