Attenuation / loss in optical fiber is also expressed in terms of decibels per kilometer (*dB/km*). Fiber attenuation can be described by the general relation:

$$\frac{dP}{dz} = -\alpha\, P$$

Where α is the power attenuation coefficient per unit length *z*.

The power light that travels along fibre can be decrease exponentially with distance *L*:

$$P_{out} = P_{in} e^{-\alpha L}$$

Therefore, the attenuation is conveniently expressed in terms of *dB/km* as:

$$\alpha\ (dB/km) = \frac{10}{L}\ \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

Where α is the known as the attenuation coefficient per unit length, (*dB/km*).

Example: Suppose -1.75 *dB/km* is the power attenuation coefficient of communication system using 20 *km* of optical fiber. Calculate the loss efficiency in *dB* ($Loss_{dB}$), and the output power if the input power is 300 *mW*.

$$Loss_{dB} = -1.75 \frac{dB}{km} \times 20\ km = -35\ dB$$

$$Loss_{dB} = 10\ \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

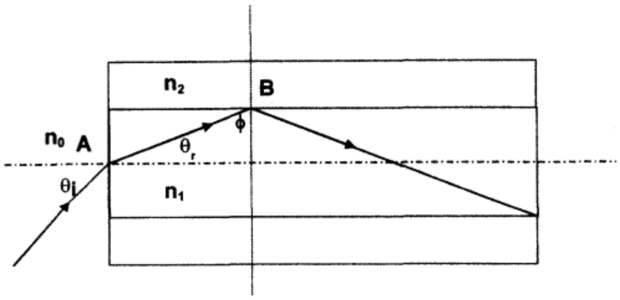
$$-35\ dB = 10\ \log_{10}\left(\frac{P_{out}}{600\ mW}\right)$$

$$P_{out} = 600\ mW\ \times\ 10^{-\frac{35}{10}}$$

$$\therefore \quad P_{out} = 0.1897\ mW$$

Example: Consider a step index fiber with parameters n_1 = 1.475, n_2 = 1.460, n_0 = 1, and core radius a = 25 μm.

(a) Calculate the maximum incident angle and numerical aperture (*NA*) of the fiber.
(b) Under the maximum incident angle, how many total reflections happen for a 1 km long fiber?
(b) If the input power is 100 mW and the output power is 90 mW of this 1 km long fiber, what is the total loss for each reflection (in *dB*)?



(a) $$NA = sin\ \theta_i = \sqrt{{n_{co}}^2 - {n_{cl}}^2}$$

$$NA = \sqrt{(1.475)^2 - (1.460)^2} = 0.21$$

(b) From the relationship $n_0 sin\theta_{\mathrm{i}} = n_1 sin\theta_r$, internal refractive angle, θ_r, can be calculated as:

$$\theta_r = sin^{-1}\left(\frac{n_0 sin\theta_{\mathrm{i}}}{n_1}\right) = sin^{-1}\left(\frac{1\ \times 0.21}{1.475}\right) = 8.19^o$$

Then the propagation length for each reflection is $2a/tan\theta_r$. When L is the length of fiber, the total number of reflections can be calculated as:

$$N = \frac{L}{2a/tan\theta_r} = \frac{10^3 m}{2.25 \times 10^{-6} m} \times tan(8.19^o) = 2.88 \times 10^6 \text{ times}$$

(c) The total loss (in dB) is:

$$Loss_{dB} = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10\log_{10}\left(\frac{90\ mW}{100\ mW}\right) = -0.458$$

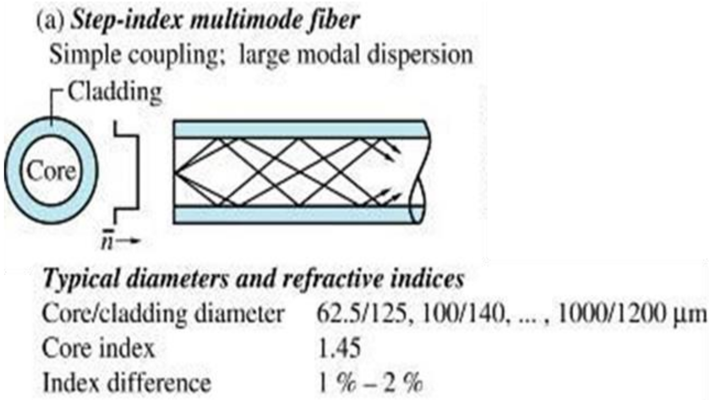
$$\text{The total loss for each reflection } (1\ km) = -0.458 \times 2.88 \times 10^6 \ = -1.32 \times 10^6$$

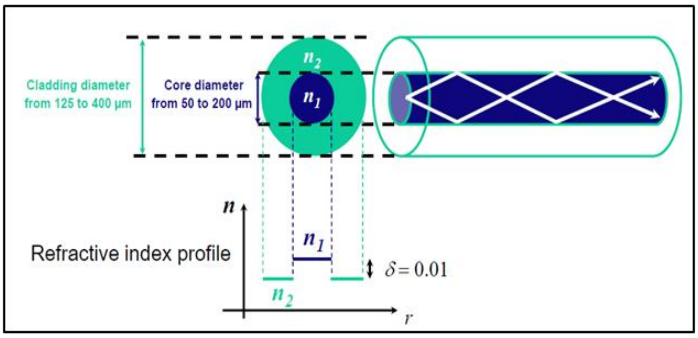
H.W.: Derive the approximation of the Numerical Aperture ($NA \cong n_1\sqrt{2\Delta}$) for $\Delta \ll 1$. What is the difference in approximate and exact expression for the value of NA if n_1 = 1.49 and n_2 = 1.48?

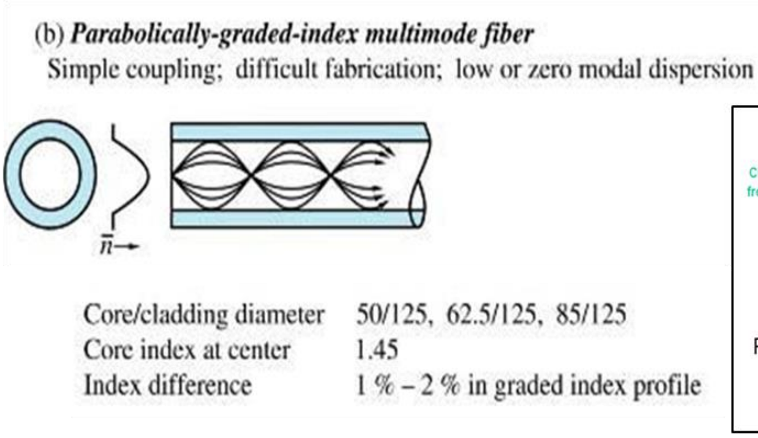
Types of Fiber Optic:

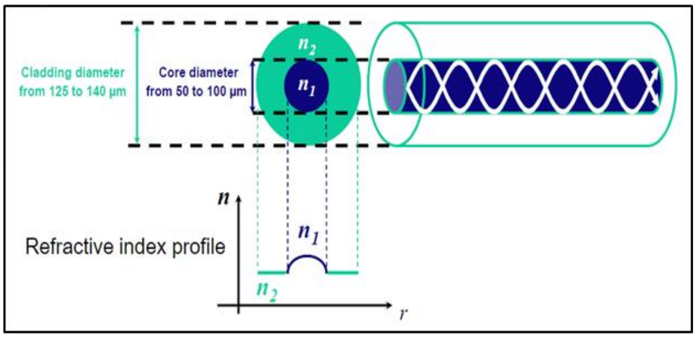
In communication systems, there are three basic types of fiber optic:

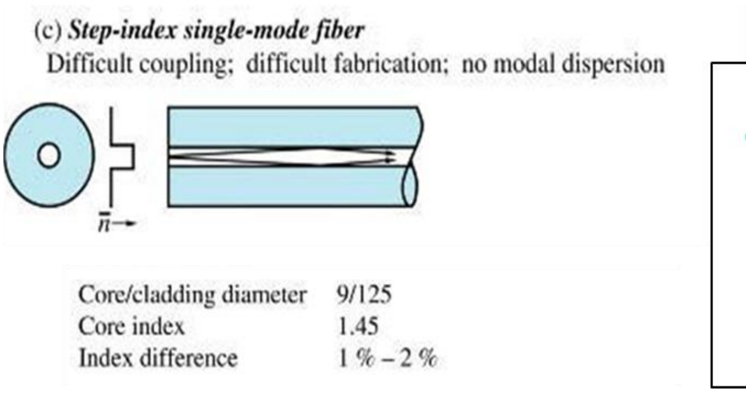
- ➢ Step-index multimode fiber.
- ➢ Parabolically-graded-index multimode fiber.
- ➢ Step-index single mode fiber.

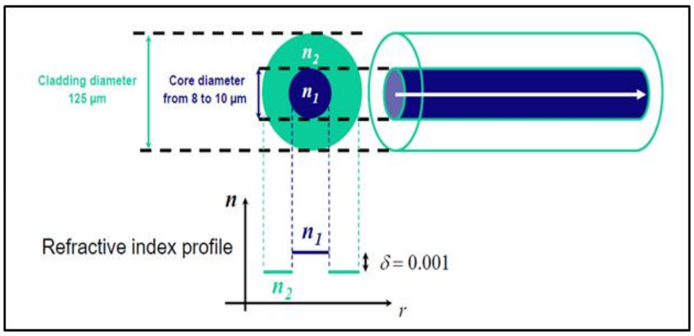












- ❖ Each mode corresponds to a particular ray angle.
- ❖ Hence it is clear that the rays for the different modes will progress at different speeds.
- ❖ This leads to unsatisfactory performance of a telecommunication system.

In general:

- *Multimode Fiber*
    - Several signals can be transmitted
    - Several frequencies used to modulate the signal
- *Single Mode Fiber*
    - only one signal can be transmitted
    - use of single frequency

Theory of Light Propagation in Optical Fiber:

- Geometrical optics can't describe rigorously light propagation in fibers
- Must be handled by electromagnetic theory (wave propagation)
- Starting point: Maxwell's equations

$$\nabla \times E = -\frac{\partial B}{\partial T} \quad (1)$$

$$\nabla \times H = J + \frac{\partial D}{\partial T} \quad (2)$$

$$\nabla \cdot D = \rho_f \quad (3)$$

$$\nabla \cdot B = 0 \quad (4)$$

with

$B = \mu_0 H + M$: Magnetic flux density

$D = \varepsilon_0 E + P$: Electric flux density

$J = 0$: Current density

$\rho_f = 0$: Charge density

Light Propagation = linear propagation + non-linear propagation.

$$P(r,t) = P_L(r,t) + P_{NL}(r,t)$$

$P_L(r,t) = \varepsilon_0 \int_{-\infty}^{+\infty} \chi^{(1)}(t-t_1) E(r,t_1) dt_1$: Linear Polarization

$P_{NL}(r,T)$: Nonlinear Polarization

$\chi^{(1)}$: linear susceptibility

We consider only linear propagation: $P_{NL}(r,T)$ negligible.

$$\nabla\times\nabla\times E(r,t)+\frac{1}{c^2}\frac{\partial^2 E(r,t)}{\partial t^2}=-\mu_0\frac{\partial^2 P_L(r,t)}{\partial t^2}$$

We now introduce the Fourier transform: $\tilde{E}(r,\omega)=\int_{-\infty}^{+\infty}E(r,t)e^{i\omega t}dt$

$$\frac{\partial^k E(r,t)}{\partial t^k}\Leftrightarrow(i\omega)^k\,\tilde{E}(r,\omega)$$

And we get: $\nabla\times\nabla\times\tilde{E}(r,\omega)-\frac{\omega^2}{c^2}\tilde{E}(r,\omega)=+\mu_0\varepsilon_0\chi^{(1)}(\omega)\omega^2\tilde{E}(r,\omega)$

which can be rewritten as

$$\nabla\times\nabla\times\tilde{E}(r,\omega)-\frac{\omega^2}{c^2}\left[1+c^2\mu_0\varepsilon_0\chi^{(1)}(\omega)\right]\tilde{E}(r,\omega)=0$$

i.e. $\nabla\times\nabla\times\tilde{E}(r,\omega)-\frac{\omega^2}{c^2}\varepsilon(\omega)\tilde{E}(r,\omega)=0$

$$\varepsilon(\omega)=\left[n+i\frac{\alpha c}{2\omega}\right]^2 \text{ with } n=1+\frac{1}{2}\Re\left[\chi^{(1)}(\omega)\right]$$

$$\text{and } \alpha=\frac{\omega}{cn(\omega)}\Im\left[\chi^{(1)}(\omega)\right]$$

n: refractive index
α: absorption

$$\nabla\times\nabla\times\tilde{E}(r,\omega)=\nabla\left(\nabla\cdot\tilde{E}(r,\omega)\right)-\nabla^2\tilde{E}(r,\omega)=-\nabla^2\tilde{E}(r,\omega)$$

$$\left(\nabla\cdot\tilde{E}(r,\omega)\propto\nabla\cdot\tilde{D}(r,\omega)=0\right)$$

$$\nabla^2\tilde{E}(r,\omega)+n^2\frac{\omega^2}{c^2}\tilde{E}(r,\omega)=0 \;:\; \text{Helmoltz Equation!}$$

Each components of $E(x,y,z,t)=U(x,y,z)e^{j\omega t}$ must satisfy the Helmoltz equation

$$\nabla^2 U+n^2k_0^{\,2}U=0 \text{ with } \begin{cases} n=n_1 \text{ for } r\le a\\ n=n_2 \text{ for } r>a\\ k_0=2\pi/\lambda\end{cases}$$

Note: $\lambda=\omega/c$