Active Filters

Introduction

Filters are circuits that are capable of passing signals with certain selected frequencies while rejecting signals with other frequencies. This property is called **selectivity**.

Filters are usually categorized by the manner in which the output voltage varies with frequency of the input voltage. The categories of active filters are **low-pass, high-pass, band-pass, and band-stop**.

The oldest technology for realizing filters makes use of **inductors and capacitors**, and the resulting circuits are called **passive LC filters**. Such filters work well at high frequencies; however, in **low-frequency applications (dc to 100 kHz) the required inductors are large** and physically bulky, and their characteristics are quite non-ideal. Furthermore, such inductors are impossible to fabricate in monolithic form and are incompatible with any of the modern techniques for assembling electronic systems.

Active-RC filters utilize op amps together with resistors and capacitors and are fabricated using discrete, hybrid thick-film or hybrid thin-film circuit technologies. However, for large-volume production, such technologies do not yield the economies achieved by monolithic (IC) fabrication.

At the present time, there are two popular approaches for realizing fully integrated filters: **the trans-conductance-C approach**, which is particularly suited for high-frequency applications, and **the switched-capacitor approach**, which is used for audio-frequency applications.

2.1 Filter Transmission, Types, and Specification
2.1.1 Filter Transmission

The filters we are about to study are linear circuits that can be represented by the general two-port network shown in Fig. 2.1.

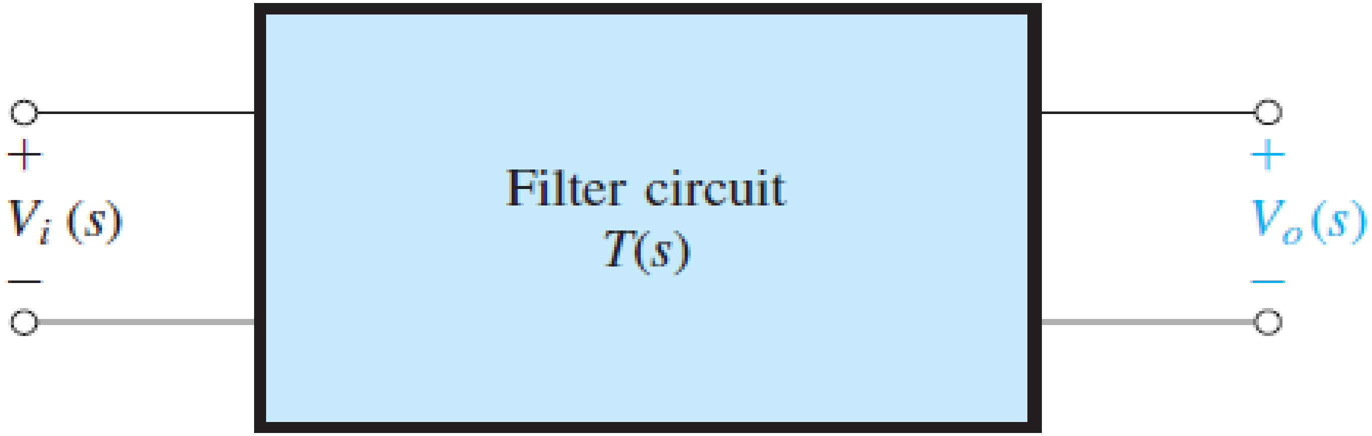


Figure 2.1 The filters studied in this chapter are linear circuits represented by the general two-port network shown. The filter transfer function $T(s) \equiv V_o(s)/V_i(s)$.

The filter **transfer function** $T(s)$ is the ratio of the output voltage $Vo(s)$ to the input voltage $Vi(s)$,

$$T(s) = Vo(s)/Vi(s) \qquad (2.1)$$

The filter **transmission** is found by evaluating $T(s)$ for physical frequencies, $\boldsymbol{s = j\omega}$, and can be expressed in terms of its magnitude and phase as

$$T(j\omega) = |T(j\omega)|e^{j\varphi(\omega)} \qquad (2.2)$$

The magnitude of transmission is often expressed in decibels in terms of the **gain function**

$$G(\omega) \equiv 20\log|T(j\omega)|, \quad \text{dB} \qquad (2.3)$$

or, alternatively, in terms of the **attenuation function**

$$A(\omega) \equiv -20\log|T(j\omega)|, \quad \text{dB} \qquad (2.4)$$

A filter shapes the frequency spectrum of the input signal, $|V_i(j\omega)|$, according to the magnitude of the transfer function $|T(j\omega)|$, thus providing an output $V_o(j\omega)$ with a spectrum

$$|V_o(j\omega)| = |T(j\omega)||V_i(j\omega)| \tag{2.5}$$

2.1.2 Filter Types

Figure 2.2 depicts the ideal transmission characteristics of the four major filter types: **low-pass** (LP) in Fig. 2.2(a), **high-pass** (HP) in Fig. 2.2(b), **band-pass** (BP) in Fig. 2.2(c), and **band-stop** (BS) or **band-reject** in Fig. 2.2(d).

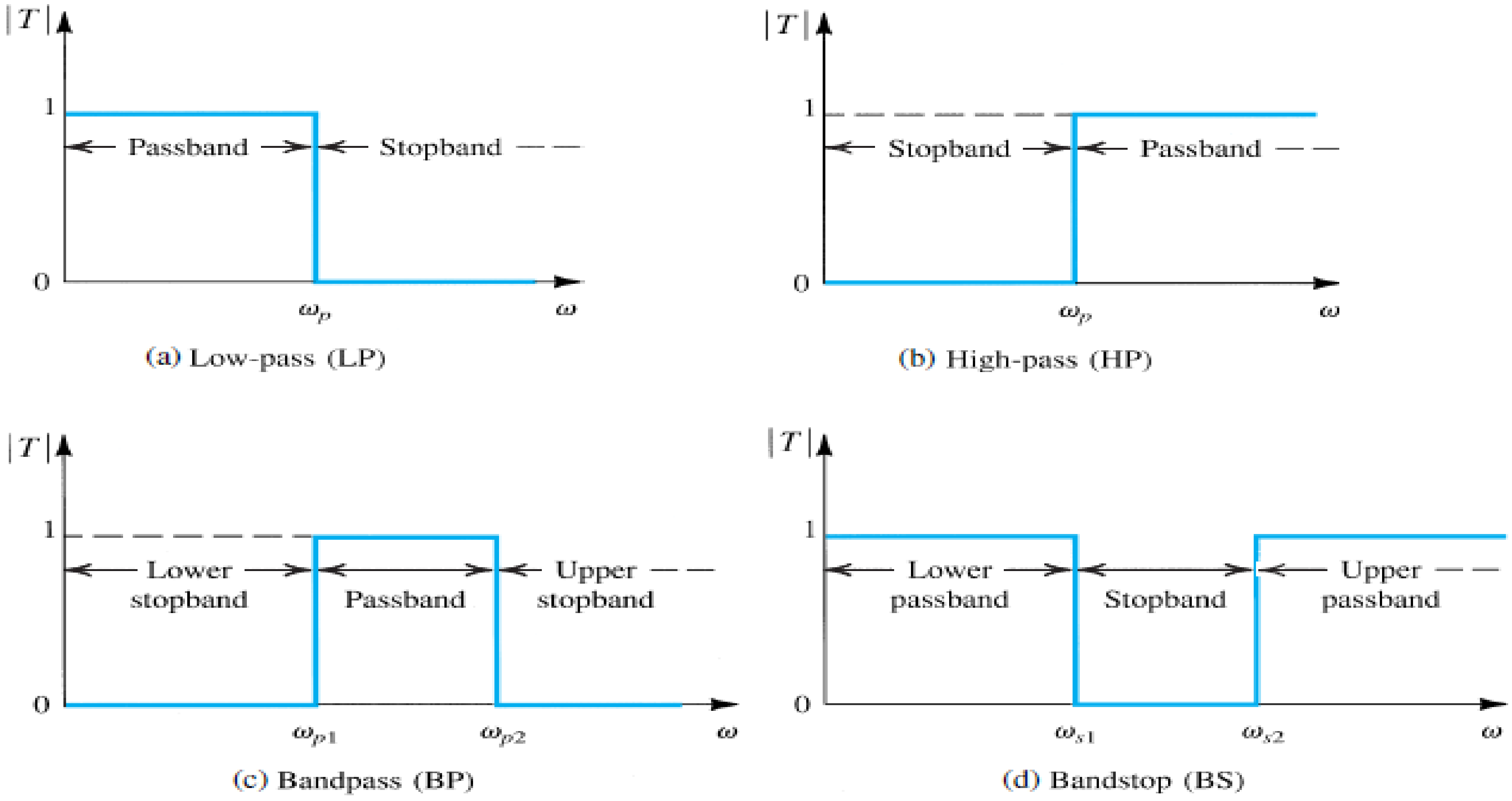


Figure 2.2 Ideal transmission characteristics of the four major filter types: **(a)** low-pass (LP), **(b)** high-pass (HP), **(c)** bandpass (BP), and **(d)** bandstop (BS)**.**

2.1.3 Filter Specification

Figure 2.3 shows realistic specifications for the transmission characteristics of a low-pass filter. Observe that since a physical circuit cannot provide constant transmission at all passband frequencies, the specifications allow for deviation of the passband transmission from the ideal 0 dB, but place an upper bound, ***A*max (dB),** on this deviation. Depending on the application, ***A*max typically ranges from 0.05 dB to 3 dB.** Also, since a physical circuit cannot provide zero transmission at all stopband frequencies, the specifications in Fig. 2.3 allow for some transmission over the stopband. However, the specifications require the stopband signals to **be attenuated by at least *A*min (dB)** relative to the passband signals. Depending on the filter application, ***A*min can range from 20 dB to 100 dB.**

The **ratio** $\frac{\omega_s}{\omega_p}$ is usually used as a measure of **the sharpness of the low-pass filter** response and is called the **selectivity factor**

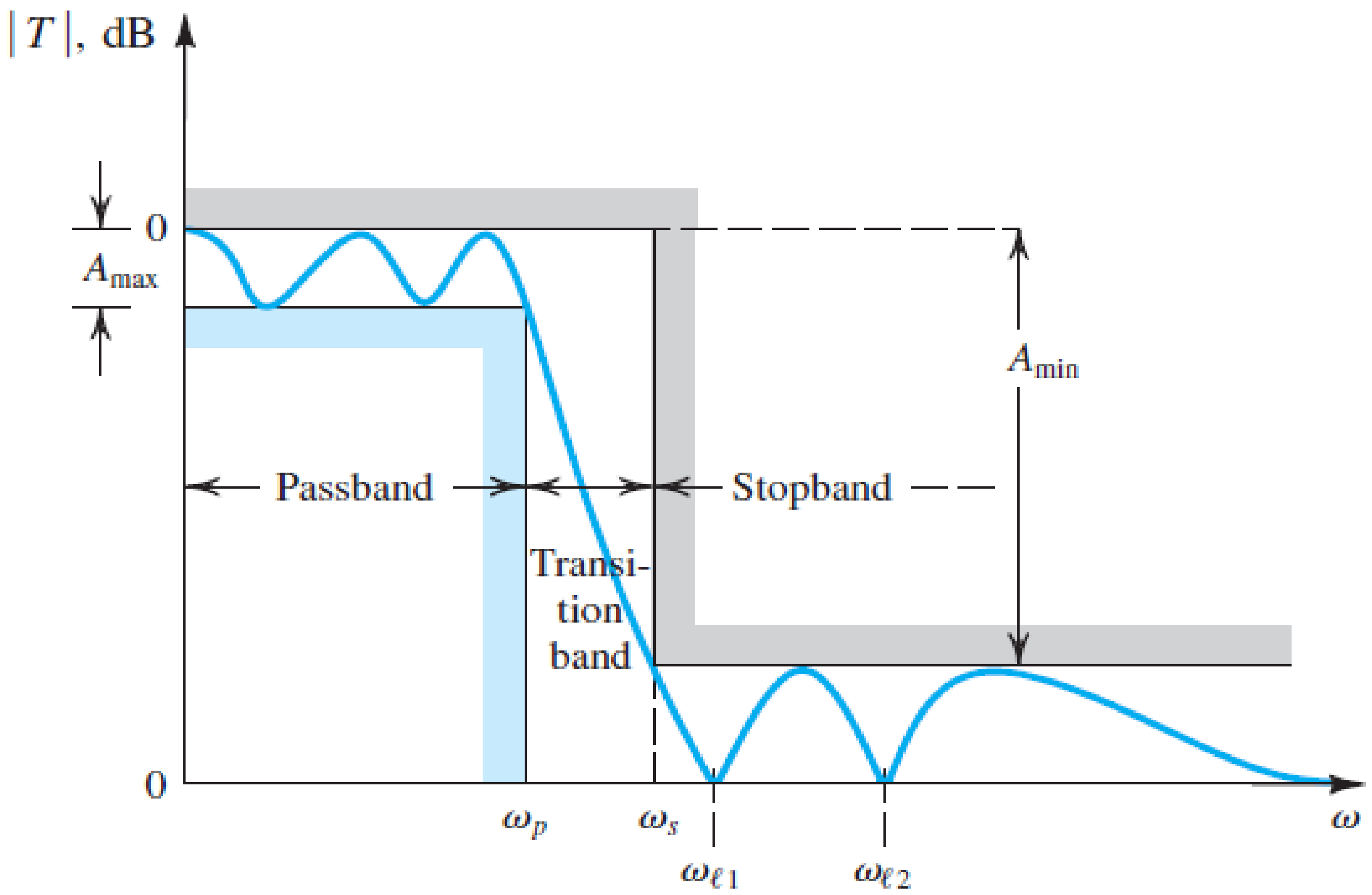


Figure 2.3 Specification of the transmission characteristics of a low-pass filter. The magnitude response of a filter that just meets specifications is also shown.

To summarize, the transmission of a low-pass filter is specified by four parameters:

1. The passband edge ω_p

2. The maximum allowed variation in passband transmission *A*max

3. The stopband edge ω_s

4. The minimum required stopband attenuation *A*min

The more tightly one specifies a filter—that is, **lower *A*max, higher *A*min, and/or a selectivity ratio *ωs*/*ωp* closer to unity**—the closer the response of the resulting filter will be to the ideal.

However, the resulting filter circuit will be of higher order and thus more complex and expensive.

2.2 The Filter Transfer Function

The filter transfer function $T(s)$ can be written as the ratio of two polynomials as

$$T(s) = \frac{a_M S^M + a_{M-1} S^{M-1} + \cdots + a_0}{S^N + b_{N-1} S^{N-1} + \cdots + b_0} \qquad (2.6)$$

The degree of the denominator, N, is the **filter order**. For the filter circuit to be stable, the degree of the numerator must be less than or equal to that of the denominator: $\boldsymbol{M \le N}$. The numerator and denominator coefficients, a_0, a_1, . . . , a_M and b_0, b_1, . . . , b_{N-1}, are real numbers.

The polynomials in the numerator and denominator can be factored, and $T(s)$ can be expressed in the form

$$T(s) = \frac{a_M(S-Z_1)(S-Z_2)\ldots(S-Z_M)}{(S-P_1)(S-P_2)\ldots(S-P_N)} \quad (2.7)$$

The numerator roots, $z_1, z_2, \ldots, z_M$, are the **transfer function zeros**, or **transmission zeros**; and the denominator roots, $p_1, p_2, \ldots, p_N$, are the **transfer function poles**, or the **natural modes**. Each transmission zero or pole can be either a real or a complex number. Complex zeros and poles, however, must occur in conjugate pairs. Thus, if $-1+j2$ happens to be a zero, then $-1 - j2$ also must be a zero.

Continuing with the example in Fig. 2.3, we observe that the transmission decreases toward zero as ω approaches ∞. Thus the filter must have one or more transmission zeros at $s=\infty$. In general, the number of transmission zeros at $s=\infty$ is the difference between the degree of the numerator polynomial, M, and the degree of the denominator polynomial, N, of the transfer function in Eq. (2.6). This is because as s approaches ∞, $T(s)$ approaches $\boldsymbol{a_M/s^{N-M}}$ and thus is said to have $N - M$ zeros at $s=\infty$.

For a filter circuit to be stable, **all its poles must lie in the left half of the *s* plane**, and thus $p_1, p_2, \ldots, p_N$ must all have negative real parts. Figure 2.4 shows typical pole and zero locations for the low-pass filter whose transmission function is depicted in Fig. 2.3. We have assumed that this filter is of fifth order (*N* = 5). It has two pairs of complex-conjugate poles and one real-axis pole, for a total of five poles. All the poles lie in the vicinity of the passband, which is what gives the filter its high transmission at passband frequencies. The five transmission zeros are at $s = \pm j\omega_{l1}, s = \pm j\omega_{l2}$, and $s = \infty$. Thus, the transfer function for this filter is of the form

$$T(s) = \frac{a_4(s^2+\omega_{l1}^2)(s^2+\omega_{l2}^2)}{s^5+b_4s^4+b_3s^3+b_2s^2+b_1S+b_0} \qquad (2.8)$$

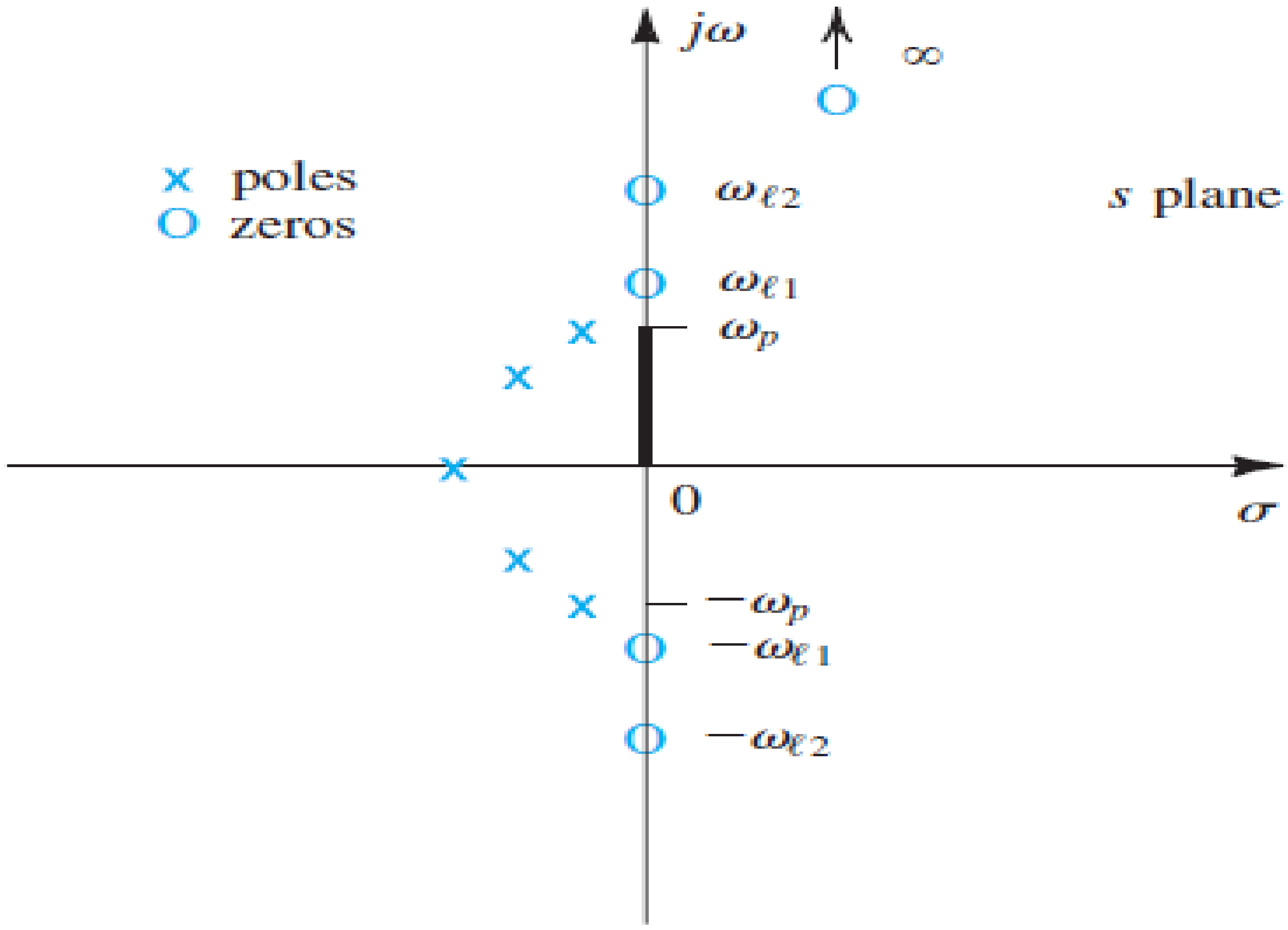


Figure 2.4 Pole–zero pattern for the low-pass filter whose transmission is sketched in Fig. 2.3. This is a fifth-order filter ($N = 5$).

2.3 Butterworth and Chebyshev Filters

2.3.1 The Butterworth Filter

Figure 2.5 shows a sketch of the magnitude response of a Butterworth filter. This filter exhibits a monotonically decreasing transmission with all the transmission zeros at $\omega=\infty$, making it an all-pole filter. The magnitude function for an Nth-order Butterworth filter with a passband edge ω_p is given by

$$|T(j\omega)|=\frac{1}{\sqrt{1+\epsilon^2(\frac{\omega}{\omega_p})^{2N}}} \quad (2.11)$$

At $\omega = \omega_p$,

$$\left|T(j\omega_p)\right|=\frac{1}{\sqrt{1+\epsilon^2}} \quad (2.12)$$

Thus, the parameter ϵ determines the maximum variation in passband transmission, Amax, according to

$$A_{max} = 20 \log \sqrt{1+\epsilon^2} \qquad (2.13)$$

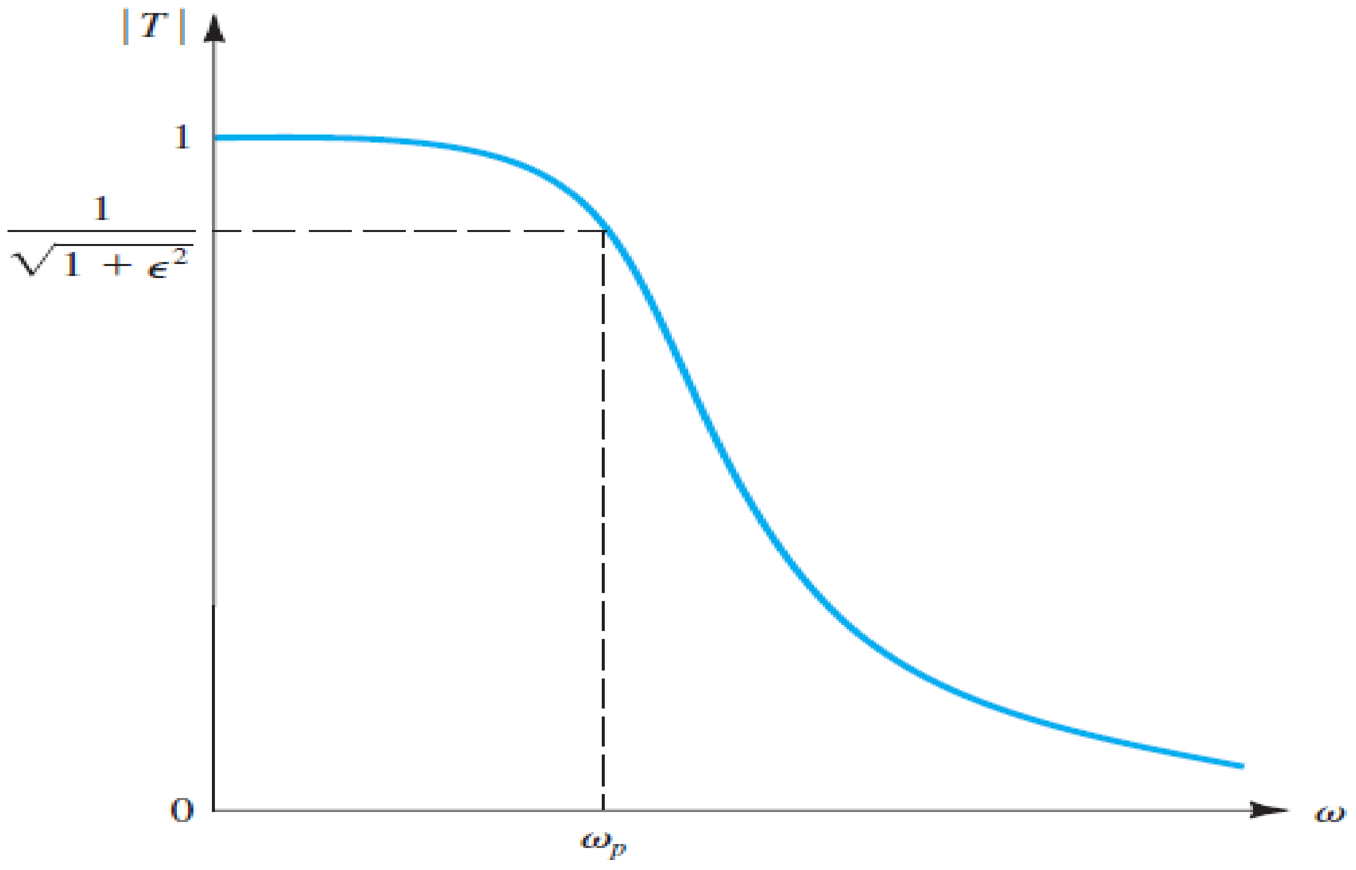


Figure 2.5 The magnitude response of a Butterworth filter.

Conversely, given *A*max, the value of can be determined from

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} \qquad (2.14)$$

At the edge of the stopband, $\omega = \omega_s$, the attenuation of the Butterworth filter can be obtained by substituting $\omega = \omega_s$ in Eq. (2.11). The result is given by

$$A(\omega_s) = -20 \log\left[\frac{1}{\sqrt{1+\epsilon^2 (\frac{\omega_s}{\omega_p})^{2N}}}\right] \qquad (2.15)$$

$$= 10 \log\left[1 + \epsilon^2 (\frac{\omega_s}{\omega_p})^{2N}\right]$$

This equation can be used to determine the filter order required, which is the lowest integer value of *N* that yields $\boldsymbol{A(\omega_s) \geq Amin}$.

Note that the degree of passband flatness increases as the order *N* is increased.

The natural modes of an Nth-order Butterworth filter can be determined from the graphical construction shown in Fig. 2.6(a). Observe that the natural modes lie on a circle of radius $\omega_p(1/\epsilon)^{1/N}$ and are spaced by equal angles of π/N, with the first mode at an angle $\pi/2N$ from the $+j\omega$ axis. Since the natural modes all have equal radial distance from the origin, they all have the same frequency $\omega_0 = \omega_p(1/\epsilon)^{1/N}$. See Fig. 2.6(b), (c), and (d) for the natural modes of Butterworth filters of order N = 2, 3, and 4, respectively. Once the N natural modes $p_1, p_2, \ldots, p_N$ have been found, the transfer function can be written as

$$T(s) = \frac{K\omega_0^N}{(s-p_1)(s-p_2)\ldots(s-p_N)} \qquad (2.16)$$

where K is a constant equal to the required dc gain of the filter.

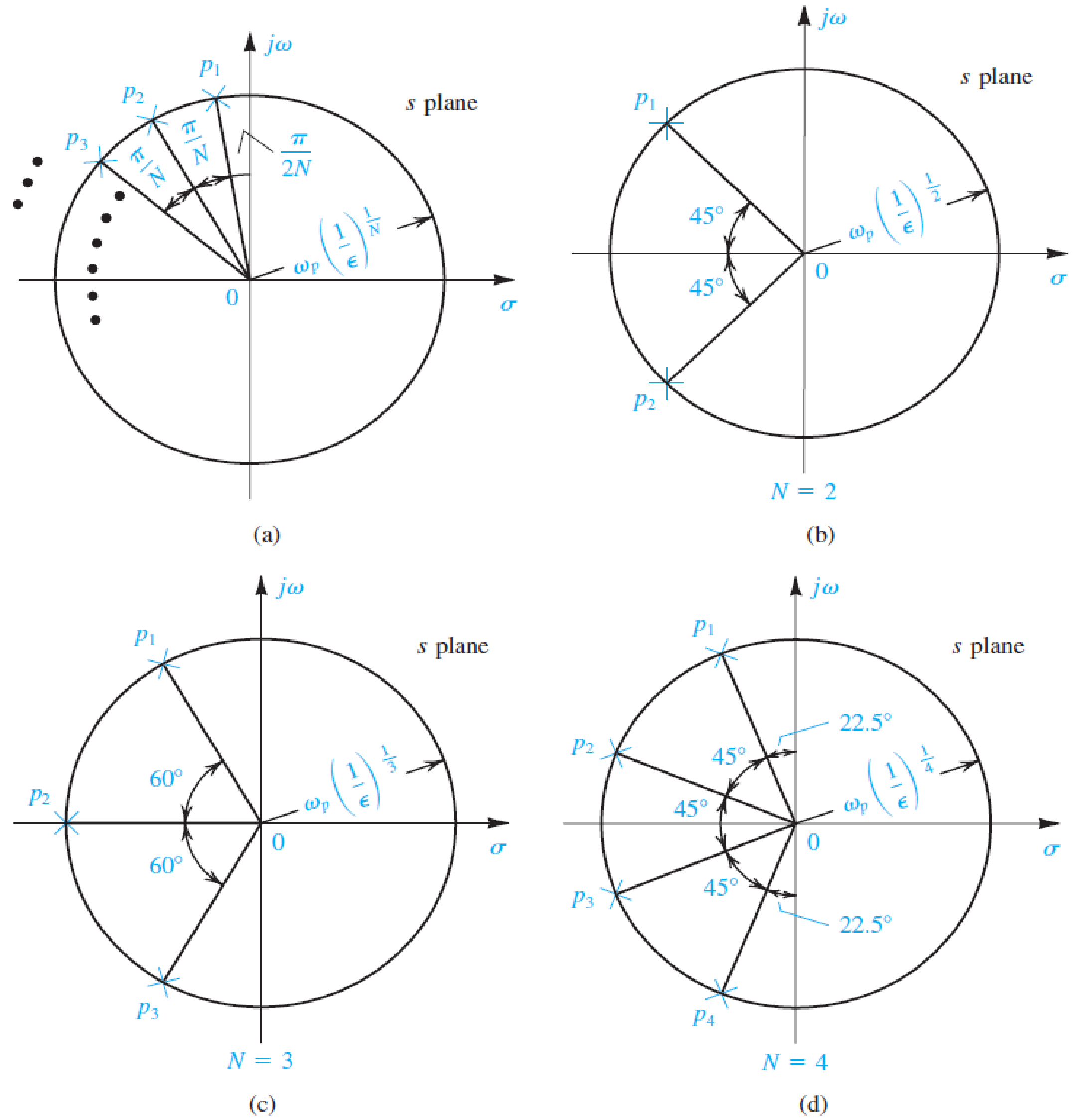


Figure 2.6 Graphical construction for determining the poles of a Butterworth filter of order N. **(a)** the general case; **(b)** N = 2; **(c)** N = 3; **(d)** N = 4.

To find a Butterworth transfer function that meets transmission specifications of the form in Fig. 2.3 we perform the following procedure:

1. Determine ϵ from Eq. (2.14).
2. Use Eq. (2.15) to determine the required filter order as the lowest integer value of N that results in $A(\omega_s) \geq A$min.
3. Use Fig. 2.6(a) to determine the N natural modes.
4. Use Eq. (2.16) to determine $T(s)$

Example 2.1: Determine the order of the Butterworth low-pass filter that have the specifications: *fp*=10 kHz, *A*max= 1 dB, *fs*= 15 kHz, *A*min= 25 dB, dc gain = 1.

Solution:

$\epsilon = \sqrt{10^{A_{max}/10} - 1}$=0.5088

To determine the order of the filter, apply the following condition

$A(\omega_s) \geq A\text{min}$

$$10 \log \left[1 + \epsilon^2 (\frac{\omega_s}{\omega_p})^{2N}\right] \geq A\text{min}$$

By solving the above equation yields N≥ 8.76

Therefore, N should be approximated to 9.

2.3.2 The Chebyshev Filter

Figure 2.7 shows representative transmission functions for Chebyshev filters of even and odd orders. The Chebyshev filter exhibits an equiripple response in the passband and a monotonically decreasing transmission in the stopband. While the odd-order filter has $|T(0)| = 1$, the even-order filter exhibits its maximum magnitude deviation at $\omega = 0$. In both cases the total number of passband maxima and minima equals the order of the filter, N.

All the transmission zeros of the Chebyshev filter are at $\omega = \infty$, making it an all-pole filter.

The magnitude of the transfer function of an Nth-order Chebyshev filter with a passband edge (ripple bandwidth) ω_p is given by

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 \cos^2\left[N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right]}} \quad for \quad \omega \leq \omega_p \qquad (2.18)$$

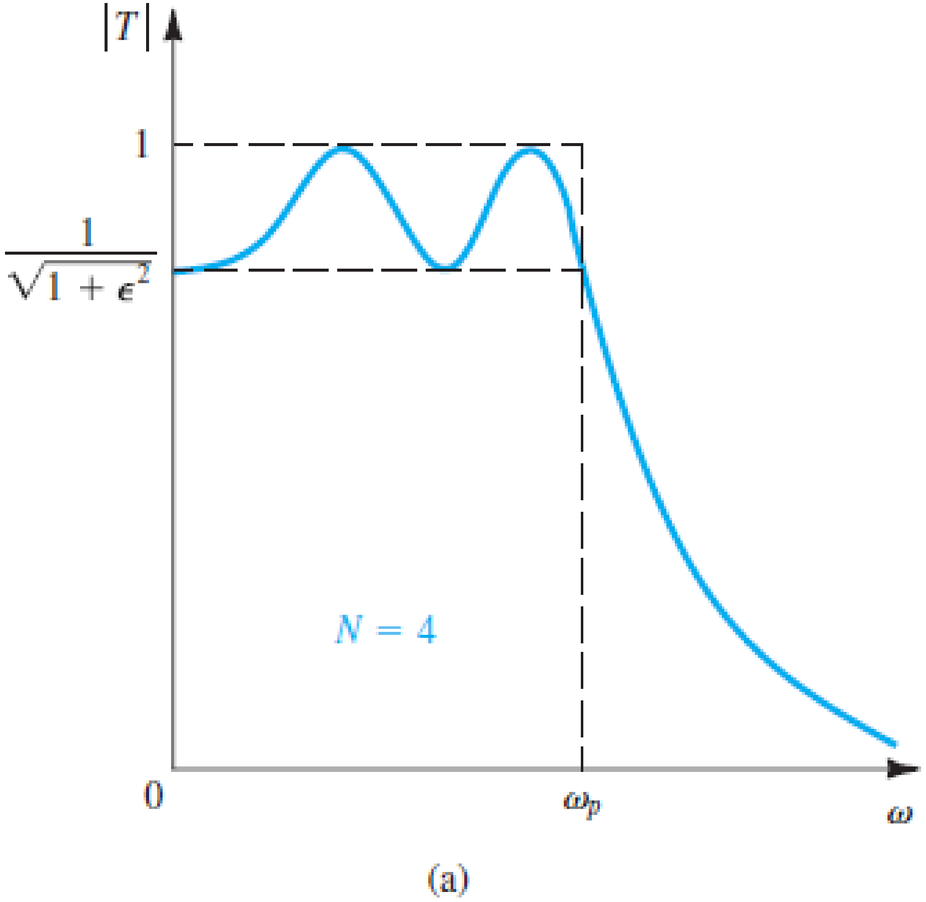


Figure 2.7 Sketches of the transmission characteristics of representative **(a)** even-order and **(b)** odd-order Chebyshev filters.

and

$$|T(j\omega)| = \frac{1}{\sqrt{1+\epsilon^2 cosh^2\left[Ncosh^{-1}(\frac{\omega}{\omega_p})\right]}} \quad for \quad \omega \geq \omega_p \qquad (2.19)$$

At the passband edge, $\omega = \omega_p$, the magnitude function is given by

$$\left|T(j\omega_p)\right| = \frac{1}{\sqrt{1+\epsilon^2}}$$

Thus, the parameter ϵ determines the passband ripple according to

$$A_{max} = 10 \log \sqrt{1+\epsilon^2} \qquad (2.20)$$

Conversely, given A_{max}, the value of ϵ is determined from

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} \qquad (2.21)$$

The attenuation achieved by the Chebyshev filter at the stopband edge ($\omega = \omega_s$) is found using Eq. (2.19) as

$$A(\omega_s) = 10 \log \left[1 + \epsilon^2 cosh^2(Ncosh^{-1}(\frac{\omega_s}{\omega_p}))\right] \qquad (2.22)$$

With the aid of a calculator, this equation can be used to determine the order *N* required to obtain a specified *A*min by finding the lowest integer value of *N* that yields
$\boldsymbol{A(\omega_s) \geq A}$**min.**
As in the case of the Butterworth filter, increasing the order *N* of the Chebyshev filter causes its magnitude function to approach the ideal brick-wall low-pass response.
The poles of the Chebyshev filter are given by

$$P_k = -\omega_p \sin\left(\frac{2k-1}{N}\frac{\pi}{2}\right) sinh\left(\frac{1}{N}\ sinh^{-1}\frac{1}{\epsilon}\right)$$
$$+j\omega_p \cos\left(\frac{2k-1}{N}\frac{\pi}{2}\right) cosh\left(\frac{1}{N}\ sinh^{-1}\frac{1}{\epsilon}\right) \qquad k=1, 2, \ldots, N \qquad (2.23)$$

Finally, the transfer function of the Chebyshev filter can be written as

$$T(s) = \frac{K\omega_p^N}{\epsilon 2^{N-1}(s-p_1)(s-p_2)\ldots.(s-p_N)} \qquad (2.24)$$

where *K* is the dc gain that the filter is required to have.

To summarize, given low-pass transmission specifications of the type shown in Fig. 2.3, the transfer function of a Chebyshev filter that meets these specifications can be found as follows:

1. Determine ϵ from Eq. (2.21).
2. Use Eq. (2.22) to determine the order required.
3. Determine the poles using Eq. (2.23).
4. Determine the transfer function using Eq. (2.24).

The Chebyshev filter provides a more efficient approximation than the Butterworth filter. Thus, for the same order and the same A_{max}, the Chebyshev filter provides greater stopband attenuation than the Butterworth filter. Alternatively, to meet identical specifications, one requires a lower order for the Chebyshev than for the Butterworth filter.

Example 2.2: Find the Chebyshev transfer function that meets the same low-pass filter specifications given in Example 2.1: namely, f_p= 10 kHz, A_{max}= 1 dB, f_s= 15 kHz, A_{min} = 25 dB, dc gain = 1.

Solution

Substituting A_{max}= 1 dB into Eq. (2.21) yields = 0.5088. By trying various values for N in Eq. (2.22) we find that N = 4 yields $A(\omega s)$= 21.6 dB and N =5 provides 29.9 dB. We thus select N =5.

Recall that we required a ninth-order Butterworth filter to meet the same specifications in Example 2.1.

The poles are obtained by substituting in Eq. (2.23) as

P1, p5= ω_p(-0.0895$\pm j$0.9901)

P2, p4=ω_p(-0.2342$\pm j$0.6119)

P3=ω_p(-0.2895)

The transfer function is obtained by substituting these values in Eq. (2.24) as

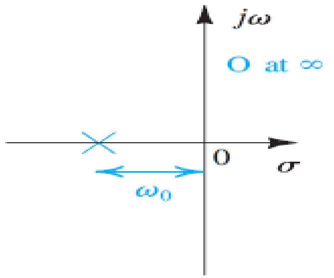
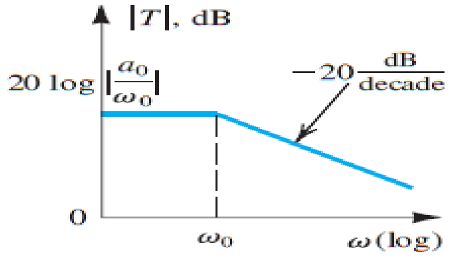
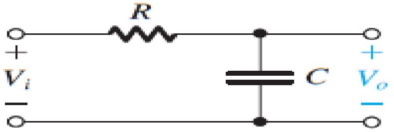
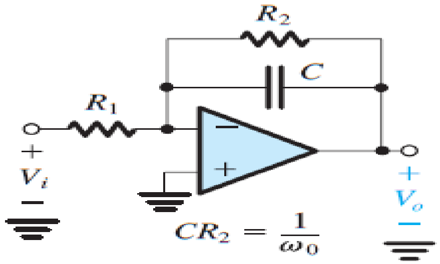
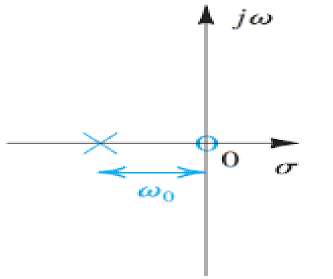
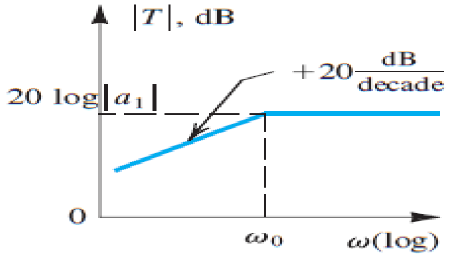
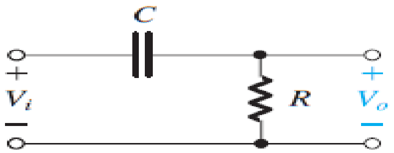
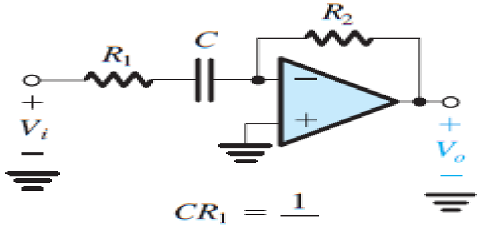
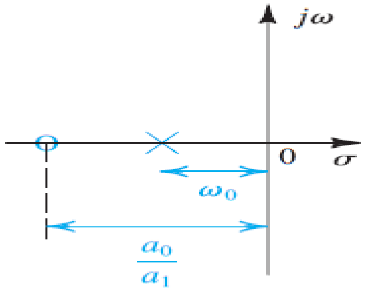
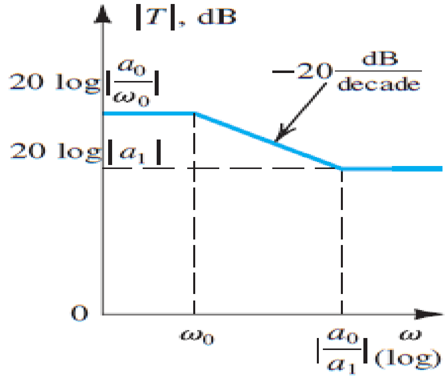
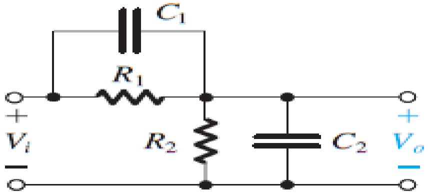
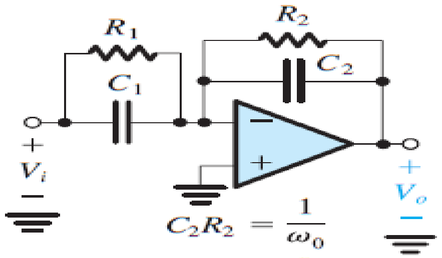
$$T(s) = \frac{\omega_p^5}{8.1408\left(s+0.2895\,\omega_p\right)\left(s^2+s0.4684\,\omega_p+0.4293\,\omega_p^2\right)}$$

$$\times \frac{1}{s^2+s\,0.1789\,\omega_p+0.9883\,\omega_p^2}$$

where $\omega_p = 2\pi \times 10^4$ rad/s.

2.4 First-Order and Second-Order Filter Functions

2.4.1 First-Order Filters

Filter Type and $T(s)$	s-Plane Singularities	Bode Plot for $\|T\|$	Passive Realization	Op Amp–RC Realization
(a) Low pass (LP) $T(s) = \frac{a_0}{s + \omega_0}$			$CR = \frac{1}{\omega_0}$ DC gain = 1	$CR_2 = \frac{1}{\omega_0}$ DC gain $= -\frac{R_2}{R_1}$
(b) High pass (HP) $T(s) = \frac{a_1 s}{s + \omega_0}$			$CR = \frac{1}{\omega_0}$ High-frequency gain = 1	$CR_1 = \frac{1}{\omega_0}$ High-frequency gain $= -\frac{R_2}{R_1}$
(c) General $T(s) = \frac{a_1 s + a_0}{s + \omega_0}$			$(C_1 + C_2)(R_1 // R_2) = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= \frac{R_2}{R_1 + R_2}$ HF gain $= \frac{C_1}{C_1 + C_2}$	$C_2 R_2 = \frac{1}{\omega_0}$ $C_1 R_1 = \frac{a_1}{a_0}$ DC gain $= -\frac{R_2}{R_1}$ HF gain $= -\frac{C_1}{C_2}$

An important special case of the first-order filter function is the **all-pass filter** shown in Fig. 2.8

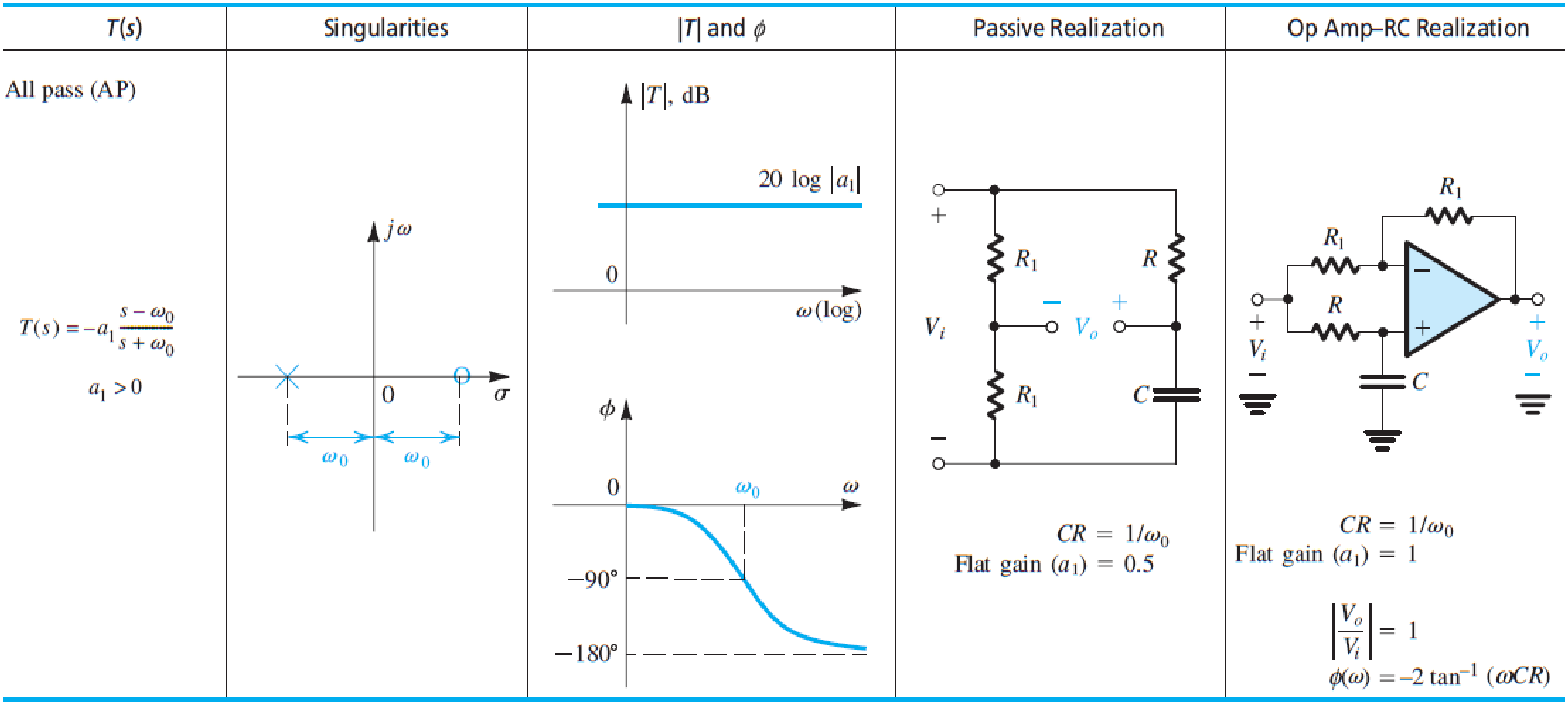


Fig.2.8 First order all pass filter

2.4.2 Second-Order Filter Functions

The general second-order (or **biquadratic**) filter transfer function is usually expressed in the standard form

$$T(s) = \frac{a_2 s^2 + a_1 S + a_0}{s^2 + \left(\frac{\omega_0}{Q}\right)s + \omega_0^2} \tag{2.27}$$

where ω_0 and Q determine the natural modes (poles) according to

$$P_1, P_2 = -\frac{\omega_0}{2Q} \pm j\omega_0 \sqrt{1 - \left({}^{1}/_{4Q^2}\right)} \tag{2.28}$$

Figure 2.9 shows the location of the pair of complex-conjugate poles in the *s* plane. Observe that the radial distance of the natural modes (from the origin) is equal to ω_0, which is known as the **pole frequency**. **The parameter *Q* determines the distance of the poles from the *jω* axis: the higher the value of *Q*, the closer the poles are to the *jω* axis, and the more selective the filter response becomes.**

An infinite value for Q locates the poles on the $j\omega$ axis and can yield sustained oscillations in the circuit realization. A negative value of Q implies that the poles are in the right half of the s plane, which certainly produces oscillations. The parameter Q is called the **pole quality factor**, or simply **pole *Q***.

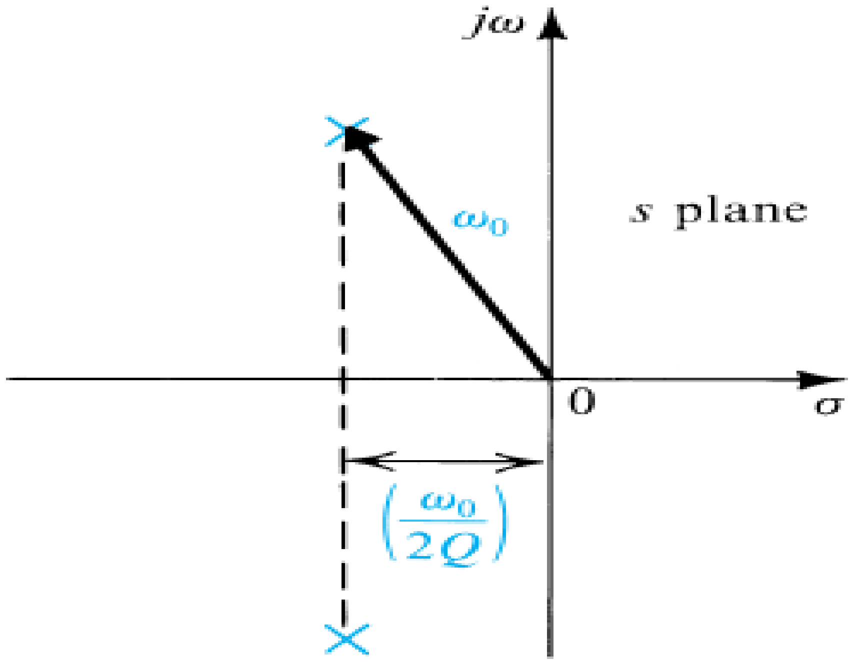


Figure 2.9 Definition of the parameters ω_0 and Q of a pair of complex-conjugate poles.

The transmission zeros of the second-order filter are determined by the numerator coefficients, $a0$, $a1$, and $a2$. It follows that the numerator coefficients determine the type of second-order filter function (i.e., LP, HP, etc.). Several special cases of interest are illustrated in Fig. 2.10. For each case we give the transfer function, the s-plane locations of the transfer function singularities, and the magnitude response. All special second-order filters have a pair of complex-conjugate natural modes characterized by a frequency $\omega 0$ and a quality factor Q.

In the low-pass (LP) case, shown in Fig. 2.10(a), the two transmission zeros are at $s=\infty$. The magnitude response can exhibit a peak with the details indicated. It can be shown that the peak occurs only for $Q > 1/\sqrt{2}$. The response obtained for $Q = 1/\sqrt{2}$ is the Butterworth, or maximally flat, response.

The high-pass (HP) function shown in Fig. 2.10(b) has both transmission zeros at $s = 0$ (dc). The magnitude response shows a peak for $Q > 1/\sqrt{2}$, with the details of the response as indicated. Observe the duality between the LP and HP responses.

Next consider the bandpass (BP) filter function shown in Fig. 2.10(c). Here, one transmission zero is at $s = 0$ (dc), and the other is at $s = \infty$. The magnitude response peaks at $\omega = \omega 0$. Thus the **center frequency** of the bandpass filter is equal to the pole frequency $\omega 0$.

The selectivity of the second-order bandpass filter is usually measured by its *3-dB bandwidth*. This is the difference between the two frequencies $\omega 1$ and $\omega 2$ at which the magnitude response is 3 dB below its maximum value (at $\omega 0$). It can be shown that

$$\omega_1, \omega_2 = \frac{\omega_0}{2Q} \pm \omega_0 \sqrt{1 + \left({}^{1}/_{4Q^2}\right)} \qquad 2.29$$

Thus,

$$BW \equiv \omega 2 - \omega 1 = \omega 0 / Q \qquad (2.30)$$

Observe that as Q increases, the bandwidth decreases and the bandpass filter becomes more selective.

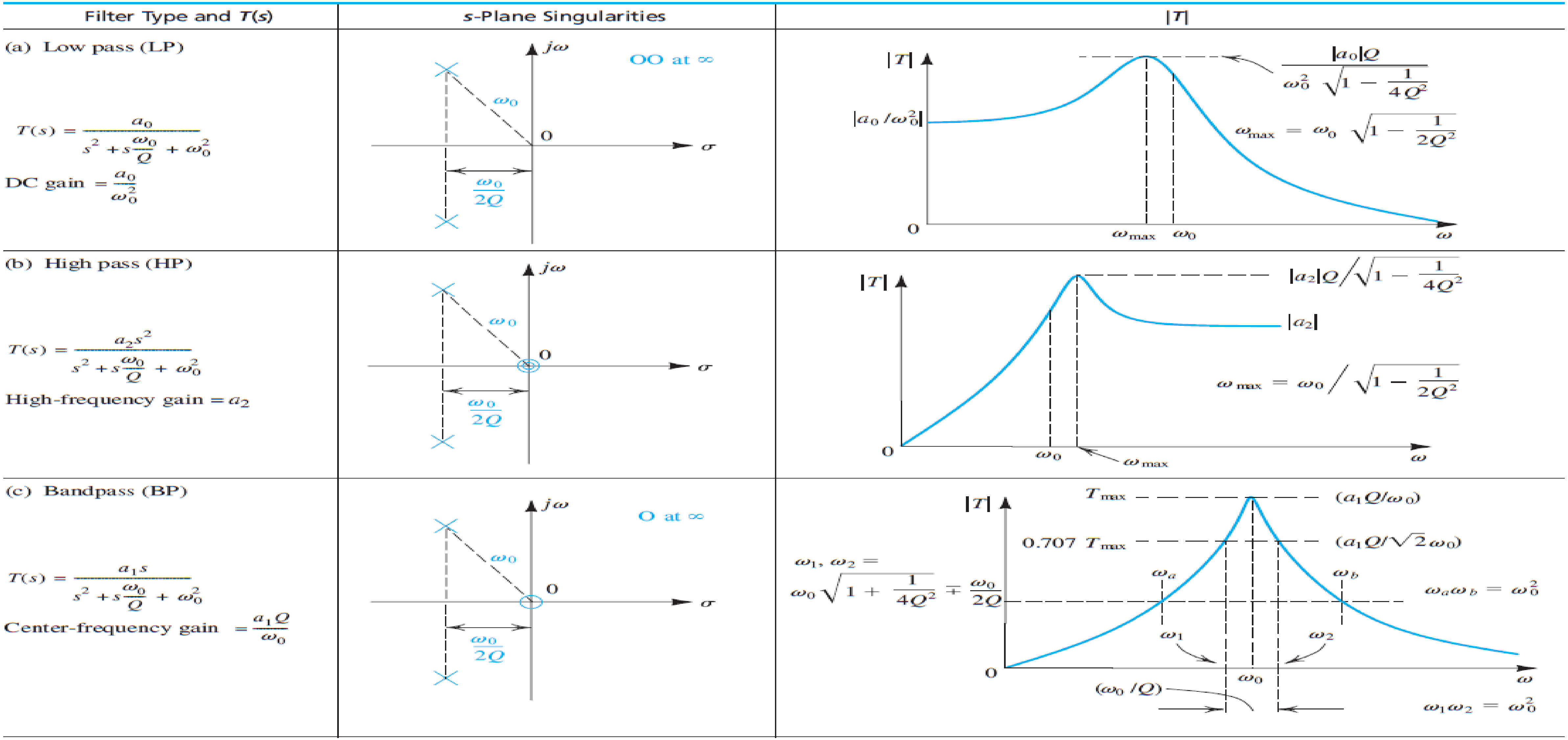
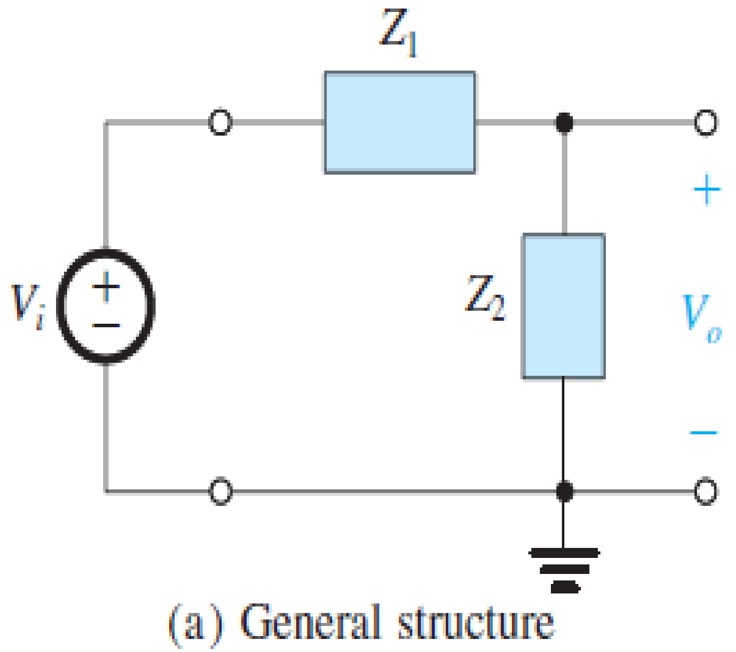


Fig.2.10 Second order filtering functions

2.5 The Second-Order RLC Resonator

Figure 2.11-a shows the general structure of the realization of the second order filter using RLC resonator.

Note that the output will be zero either when $Z_2(s)$ behaves as a short circuit or when $Z_1(s)$ behaves as an open circuit. If there is a value of s at which both Z_1 and Z_2 are zero, then V_o/V_i will be finite and no transmission zero is obtained. Similarly, if there is a value of s at which both Z_1 and Z_2 are infinite, then $\boldsymbol{V_o/V_i}$ will be finite and no transmission zero is realized.



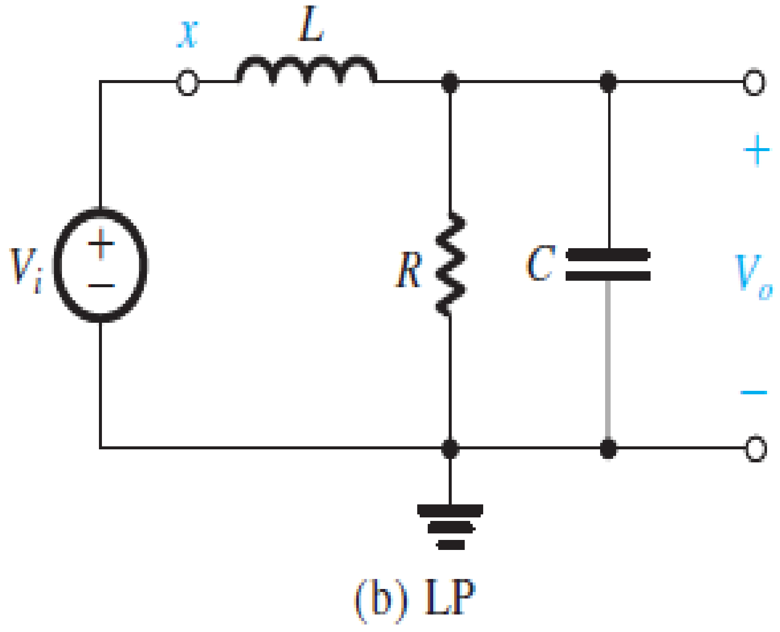
$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Fig.2.11 (a) Realization of the second order filter using RLC Resonator (General structure)

2.5.1 Realization of the Low-Pass Function

Figure 2.11-b shows the second order low pass filter using RLC resonator.

This circuit has two transmission zeros at $s=\infty$, as a second-order LP is supposed to. The transfer function can be written either by inspection or by using the voltage divider rule. Following the latter approach, we obtain



$$T(s) \equiv \frac{V_o}{V_i} = \frac{Z_2}{Z_1 + Z_2} = \frac{Y_1}{Y_1 + Y_2} = \frac{1/sL}{(1/sL) + sC + (1/R)}$$

$$= \frac{1/LC}{s^2 + s(1/CR) + (1/LC)}$$

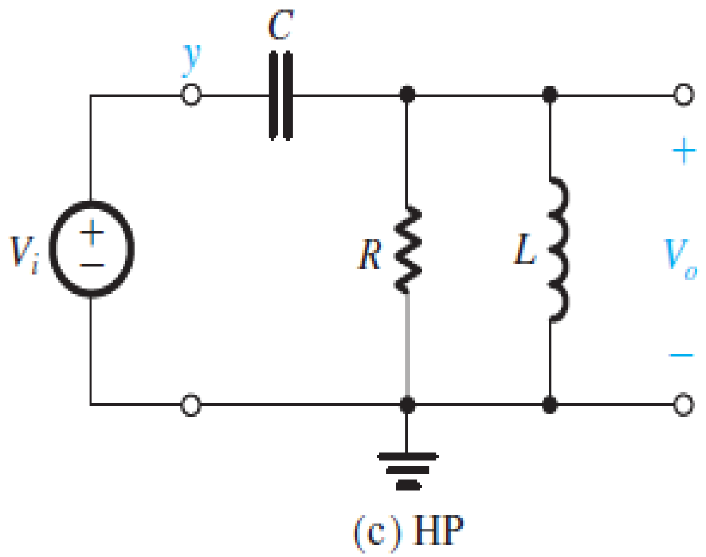
$$\omega_0 = 1/\sqrt{LC}$$

$$Q = \omega_0 CR$$

Fig.2.11 (b) Second order LP filter using RLC Resonator

2.5.2 Realization of the High-Pass Function

Figure 2.11-c shows the second order high pass filter using RLC resonator.

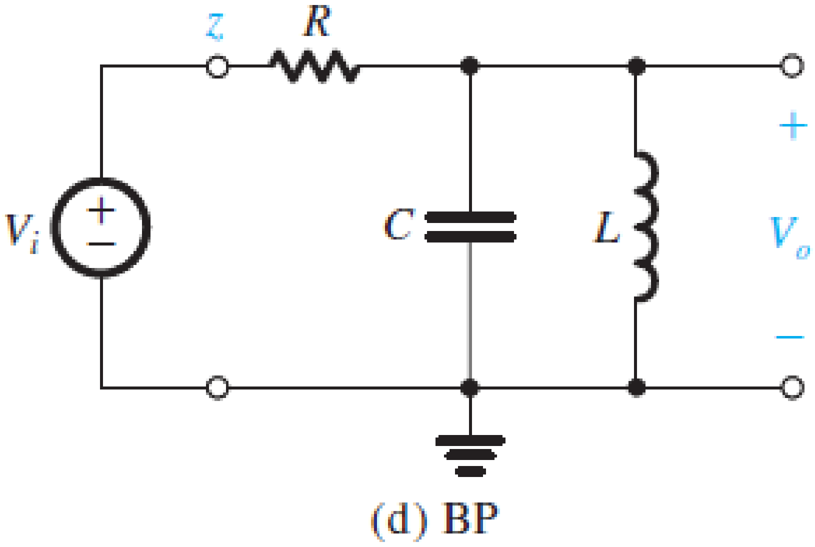


$$T(s) \equiv \frac{V_o}{V_i} = \frac{a_2 s^2}{s^2 + s(\omega_0/Q) + \omega_0^2}$$

Fig.2.11 (c) Second order HP filter using RLC Resonator

2.5.3 Realization of the Bandpass Function

Figure 2.11-d shows the second order band pass filter using RLC resonator.



$$T(s) = \frac{Y_R}{Y_R + Y_L + Y_C} = \frac{1/R}{(1/R) + (1/sL) + sC}$$

$$= \frac{s(1/CR)}{s^2 + s(1/CR) + (1/LC)}$$

Fig.2.11 (c) Second order BP filter using RLC Resonator

2.6 Second-Order Active Filters Based on Inductor Replacement

In this section, we study a family of op amp–RC circuits that realize the various second-order filter functions. The circuits are based on an op amp–RC resonator obtained by replacing the inductor L in the LCR resonator with an op amp–RC circuit that has an inductive input impedance.

2.6.1 The Antoniou Inductance-Simulation Circuit

Figure 2.12 (a) shows the Antoniou inductance simulation circuit.

If the circuit is fed at its input (node 1) with a voltage source $V1$ and the input current is denoted $I1$, then for ideal op amps the input impedance can be shown to be

$$Z_{in} \equiv V_1/I_1 = sC_4R_1R_3R_5/R_2$$

which is that of an inductance L given by

$$L = C_4R_1R_3R_5/R_2$$

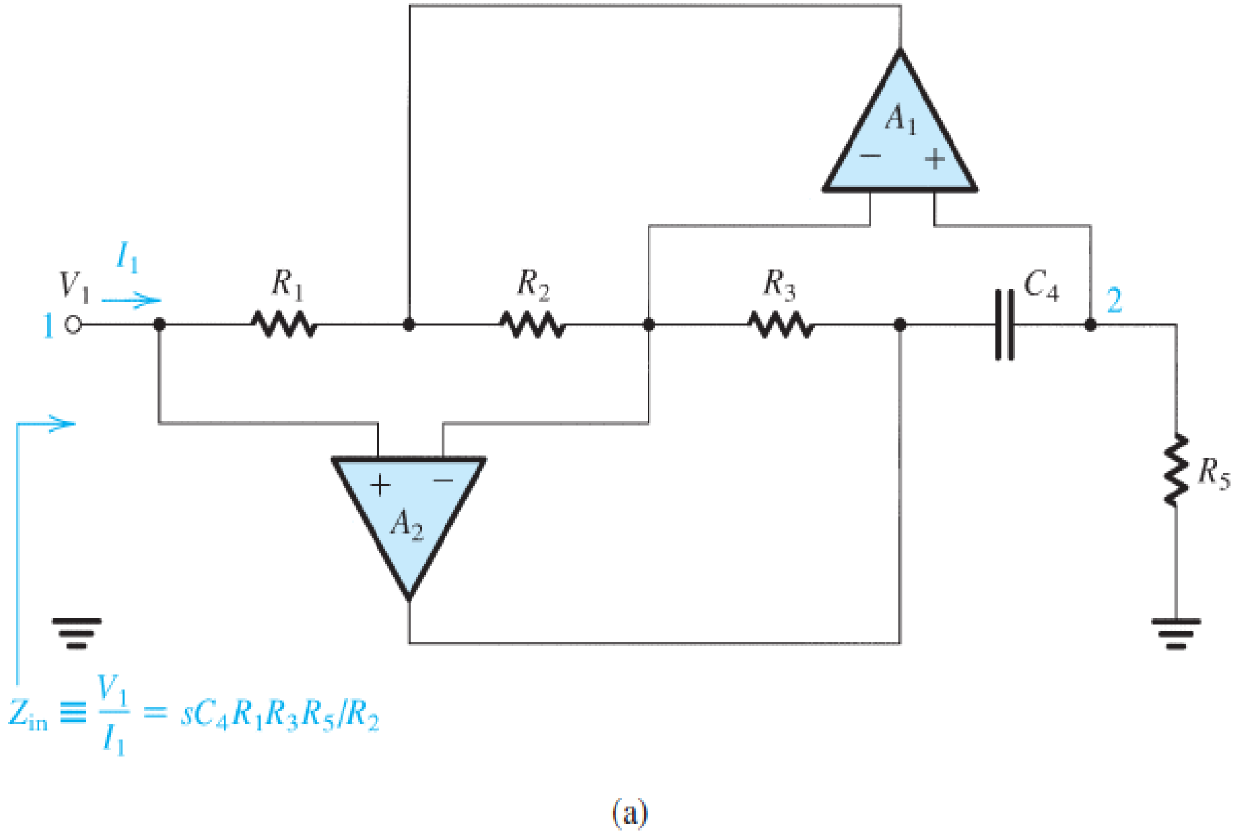
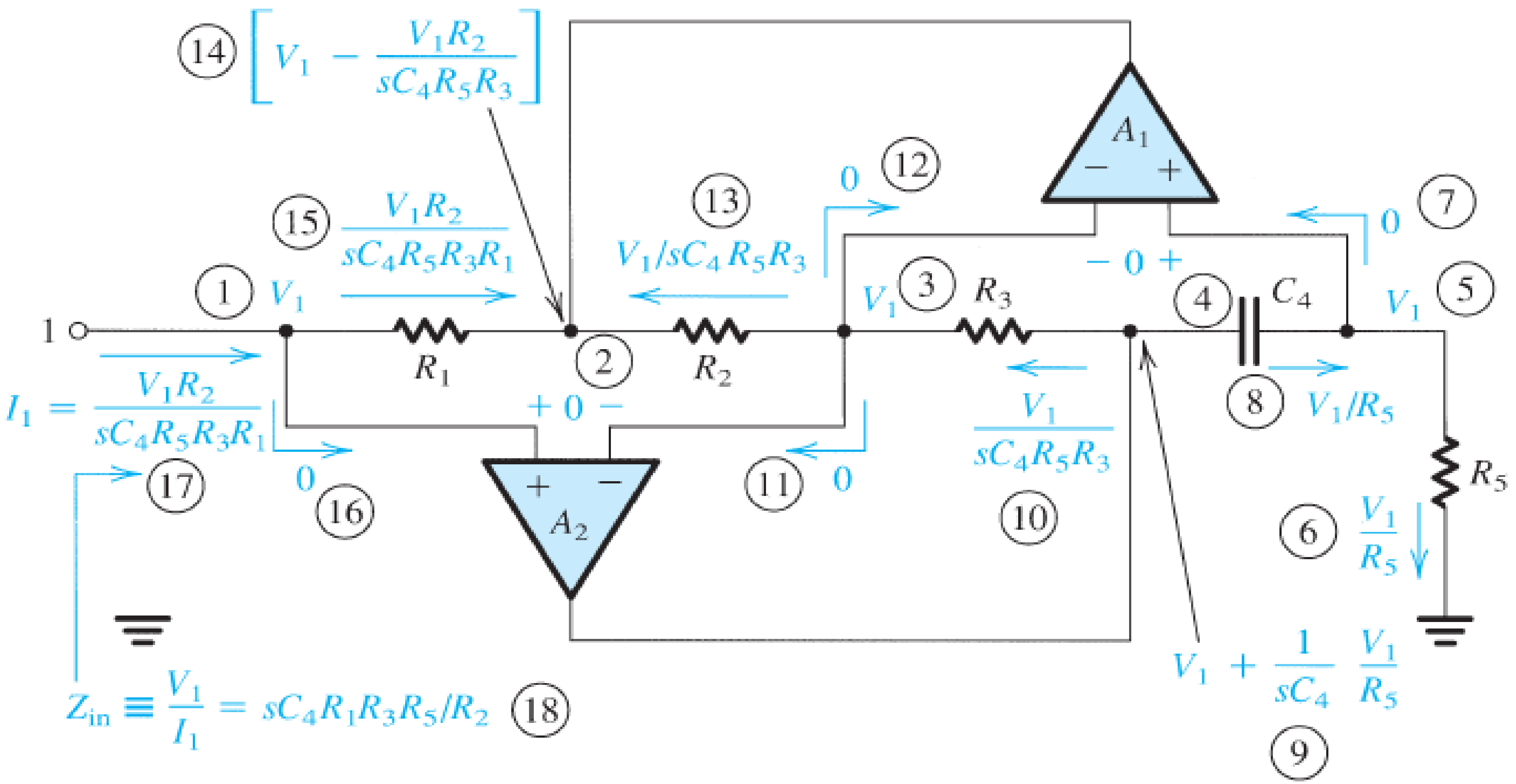


Fig. 2.12 (a) The Antoniou inductance simulation circuit

Note that the design of this circuit is usually based on selecting R1=R2=R3=R5=R and C4=C, which leads to **L = CR^2.** Convenient values are then selected for C and R to yield the desired inductance value L.



(b)

Fig.2.12 (b) Analysis of circuit in (a) using ideal OP-amps

2.6 Second-Order Active Filters Based on the Two-Integrator-Loop Topology

2.6.1 Derivation of the Two-Integrator-Loop Biquad

To derive the two-integrator-loop biquadratic circuit, or **biquad** as it is commonly known, consider the second-order high-pass transfer function

$$\frac{V_{hp}}{V_i} = \frac{Ks^2}{s^2 + s(\omega_0/Q) + \omega_0^2} \qquad (1)$$

where K is the high-frequency gain. Cross-multiplying the above equation and dividing both sides of the resulting equation by $s2$ (to get all the terms involving s in the form $1/s$, which is the transfer function of an integrator) gives

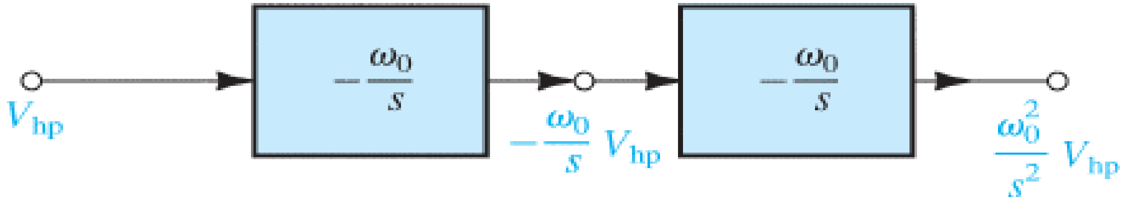
$$V_{hp} + \frac{1}{Q}\left(\frac{\omega_0}{s} V_{hp}\right) + \left(\frac{\omega_0^2}{s^2} V_{hp}\right) = KV_i \qquad (2)$$

In this equation we observe that the signal $(\omega_0/s)V_{hp}$ can be obtained by passing V_{hp} through an integrator with a time constant equal to $1/\omega_0$.

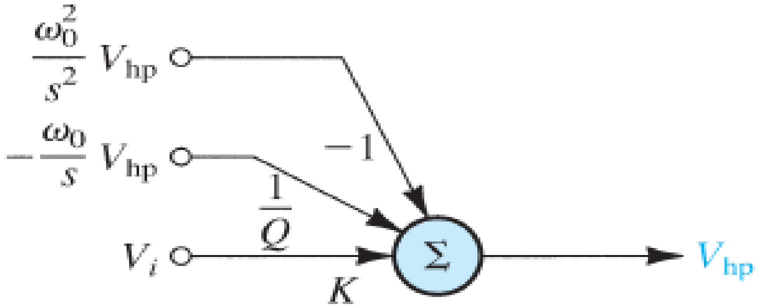
we rearrange Eq. (2), expressing V_{hp} in terms of its single- and double-integrated versions and of Vi as

$$V_{hp} = KV_i - \frac{1}{Q}\frac{\omega_0}{s}V_{hp} - \frac{\omega_0^2}{s^2}V_{hp}$$

which suggests that *V*hp can be obtained by using the weighted summer of Fig. 2.13(b). Now it should be easy to see that a complete block diagram realization can be obtained by combining the integrator blocks of Fig. 2.13(a) with the summer block of Fig. 2.13(b), as shown in Fig. 2.13(c).



(a)



(b)

In the realization of Fig. 2.13(c), V_{hp}, obtained at the output of the summer, realizes the high-pass transfer function $T_{hp} \equiv V_{hp}/V_i$ of Eq. (1). The signal at the output of the first integrator is $-(\omega_0/s)V_{hp}$, which is a bandpass function,

$$\frac{(-\omega_0/s)V_{hp}}{V_i} = -\frac{K\omega_0 s}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{bp}(s)$$

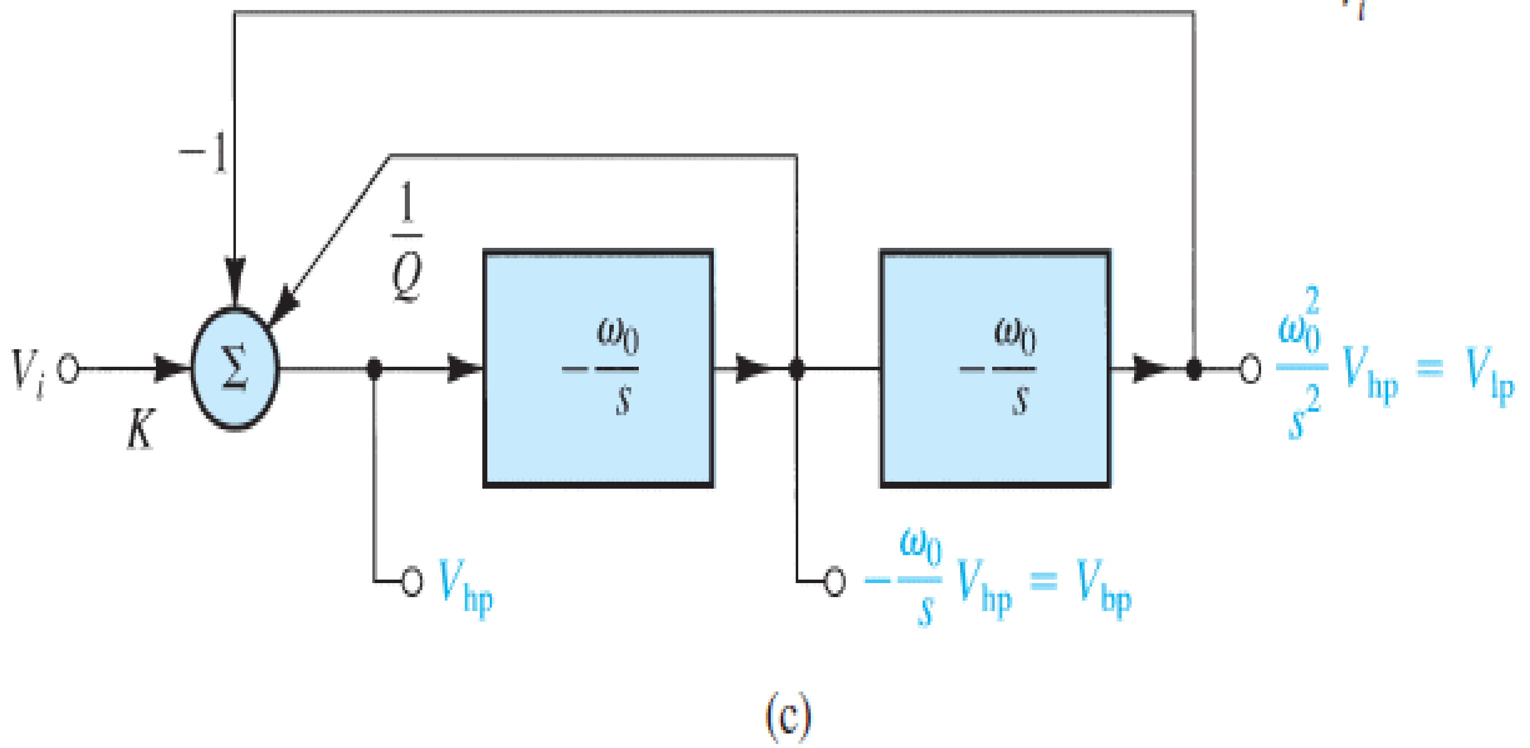


Fig.2.13 Derivation of a block diagram realization of the two integrator loop biquad

Therefore the signal at the output of the first integrator is labeled V_{bp}. Note that the center-frequency gain of the bandpass filter realized is equal to –*KQ*.
We can also show that the transfer function realized at the output of the second integrator is the low-pass function,

$$\frac{(\omega_0^2/s^2)V_{hp}}{V_i} = \frac{K\omega_0^2}{s^2 + s(\omega_0/Q) + \omega_0^2} = T_{lp}(s)$$

Thus the output of the second integrator is labeled *V*lp. Note that the dc gain of the low-pass filter realized is equal to *K*.
We conclude that the two-integrator-loop biquad shown in block diagram form in Fig. 2.13(c) realizes the three basic second-order filtering functions, LP, BP, and HP, *simultaneously*. This versatility has made the circuit very popular and has given it the name ***universal active filter***.

2.6.2 Circuit Implementation

To obtain an op-amp circuit implementation of the two-integrator-loop biquad of Fig. 2.13(c), we replace each integrator with a Miller integrator circuit having $CR = 1/\omega 0$, and we replace the summer block with an op-amp summing circuit that is capable of assigning both positive and negative weights to its inputs. The resulting circuit, is shown in Fig. 2.14.

Given values for ω_0, Q, and K, the design of the circuit is straightforward: We select suitably practical values for the components C and R of the integrators so that $CR=1/\omega_0$. To determine the values of the resistors associated with the summer, we first use *superposition* to express the output of the summer V_{hp} in terms of its inputs, V_i, V_{bp}, and V_{lp} as

$$V_{hp} = V_i \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) + V_{bp}\frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right) - V_{lp}\frac{R_f}{R_1}$$

Substituting $V_{bp} = -(\omega_0/s)V_{hp}$ and $V_{lp} = ({\omega_0}^2/s^2)V_{hp}$ gives

$$V_{hp} = \frac{R_3}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)V_i + \frac{R_2}{R_2 + R_3}\left(1 + \frac{R_f}{R_1}\right)\left(-\frac{\omega_0}{s}V_{hp}\right) - \frac{R_f}{R_1}\left(\frac{\omega_0^2}{s^2}V_{hp}\right) \qquad (3)$$

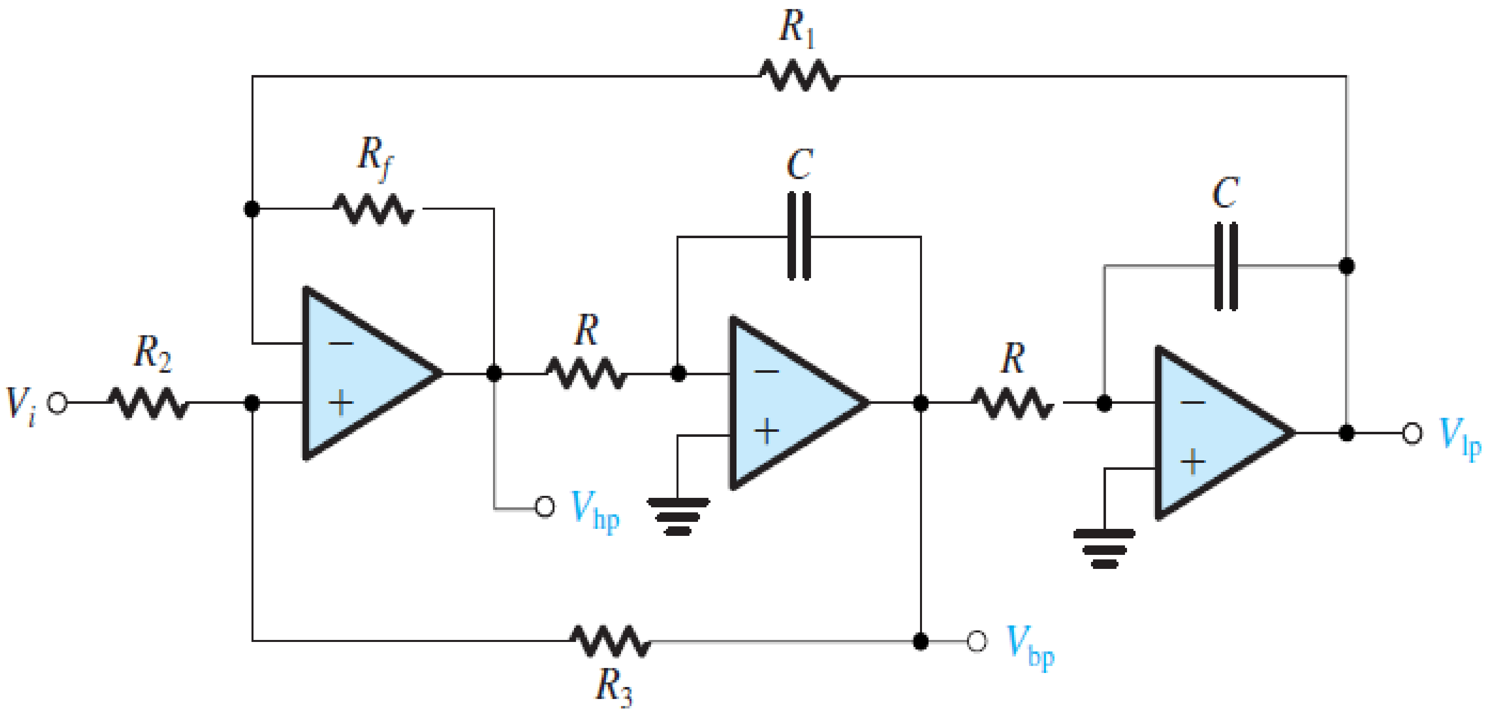
Equating the last right-hand-side terms of Eqs. (3) and (2) gives $R_f/R_1 = 1$

which implies that we can select arbitrary but practically convenient equal values for $R1$ and Rf . Then, equating the second-to-last terms on the right-hand side of Eqs. (3) and (2) and setting $R1= Rf$ yields the ratio $R3/R2$ required to realize a given Q as $R_3/R_2 = 2Q - 1$

Finally, equating the coefficients of Vi in Eqs. (3) and (2) and substituting $Rf= R1$ and for $R3/R2$=2Q-1 results in

$$K = 2 - (1/Q)$$

Fig.2.14 Two integrators loop topology circuit for the three basic filtering functions HP, BP, and LP, which are simultaneously realized



2.7 Single Amplifier Biquad Sections

Low-Pass Filter

A first-order, low-pass filter using a single resistor and capacitor as shown in Fig. 2.15.

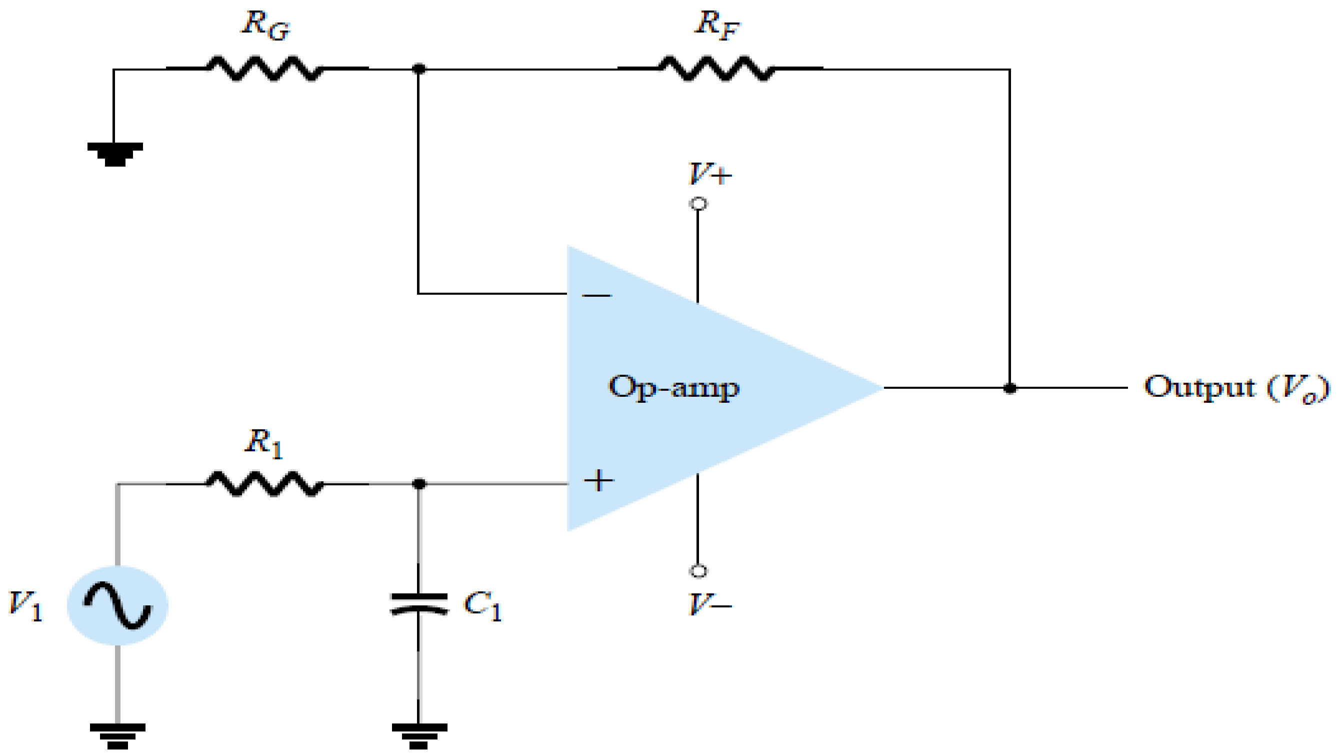


Fig. 2.15 (a) LP 1st order filter

The Sallen-Key Low-Pass Filter

The Sallen-Key is one of the most common configurations for a second-order (two-pole) filter. It is also known as a VCVS (voltage-controlled voltage source) filter. A low-pass version of the Sallen-Key filter is shown in Figure 2.16.

The critical frequency for the Sallen-Key filter is

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

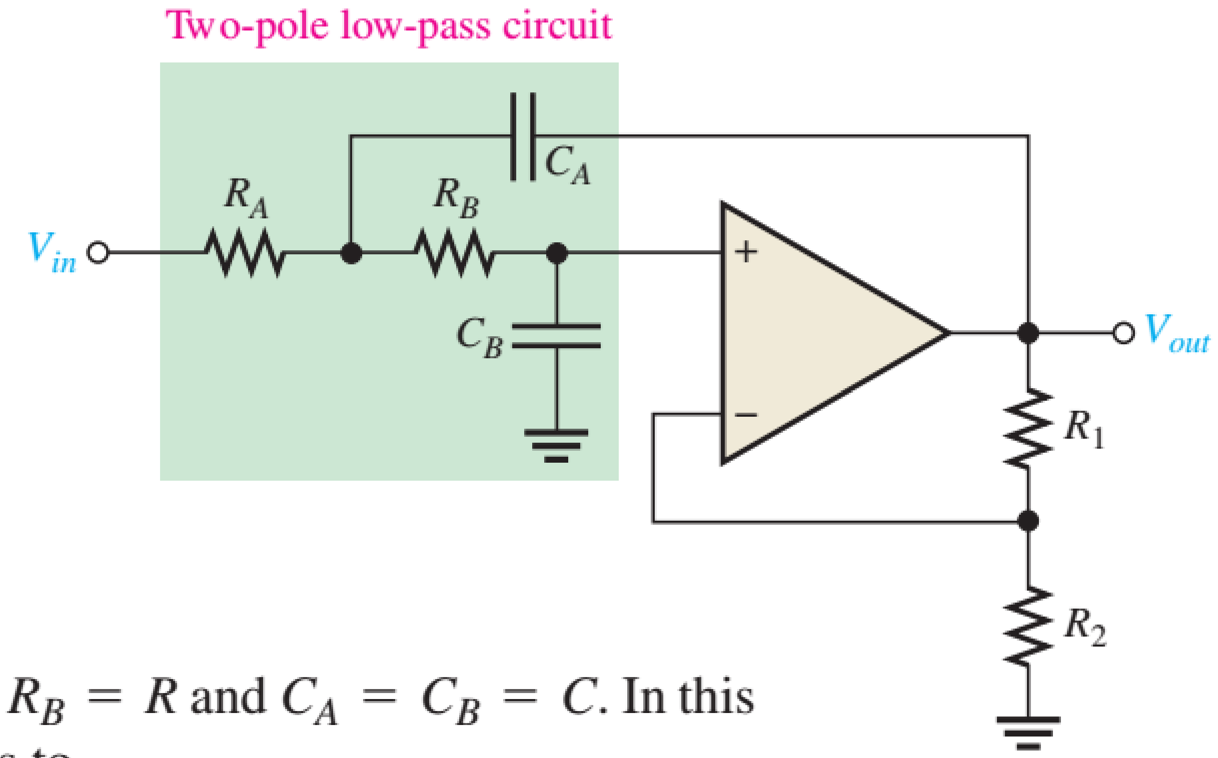


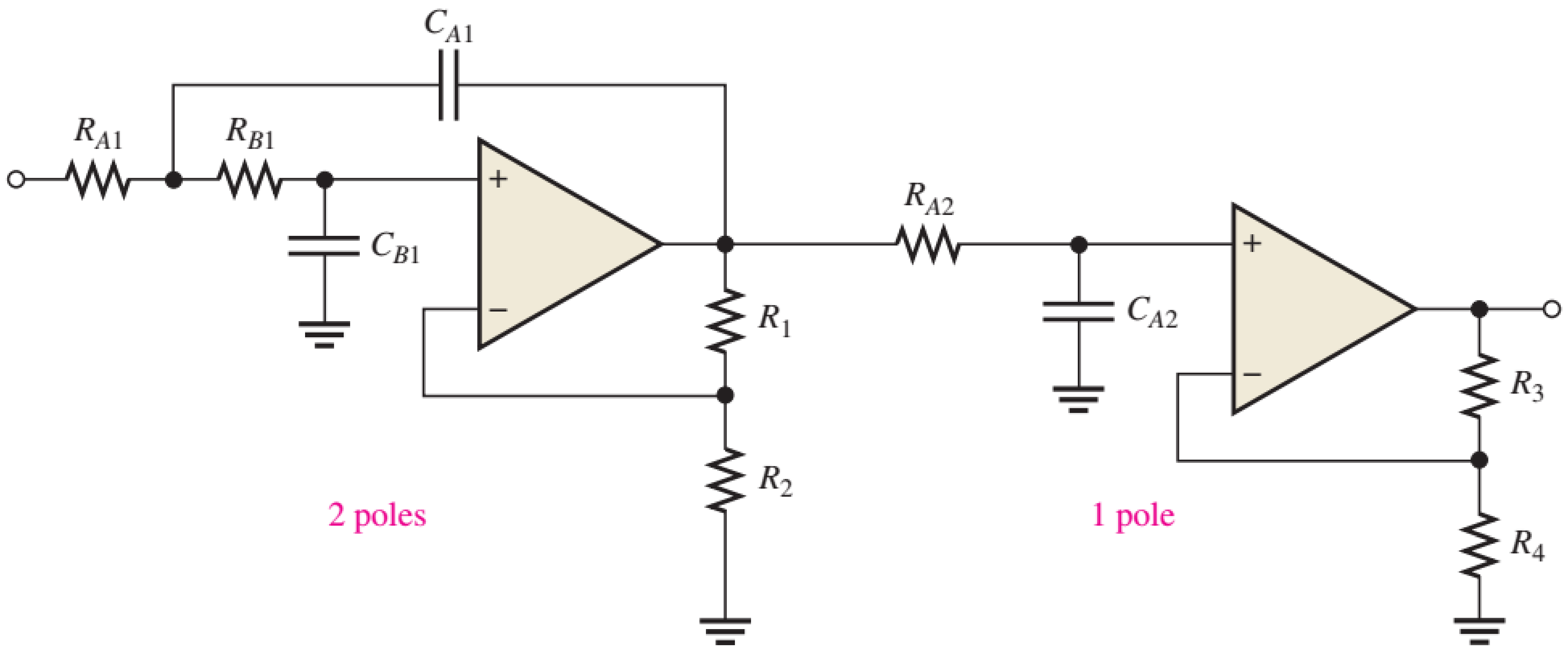
Figure 2.16

The component values can be made equal so that $R_A = R_B = R$ and $C_A = C_B = C$. In this case, the expression for the critical frequency simplifies to

$$f_c = \frac{1}{2\pi RC}$$

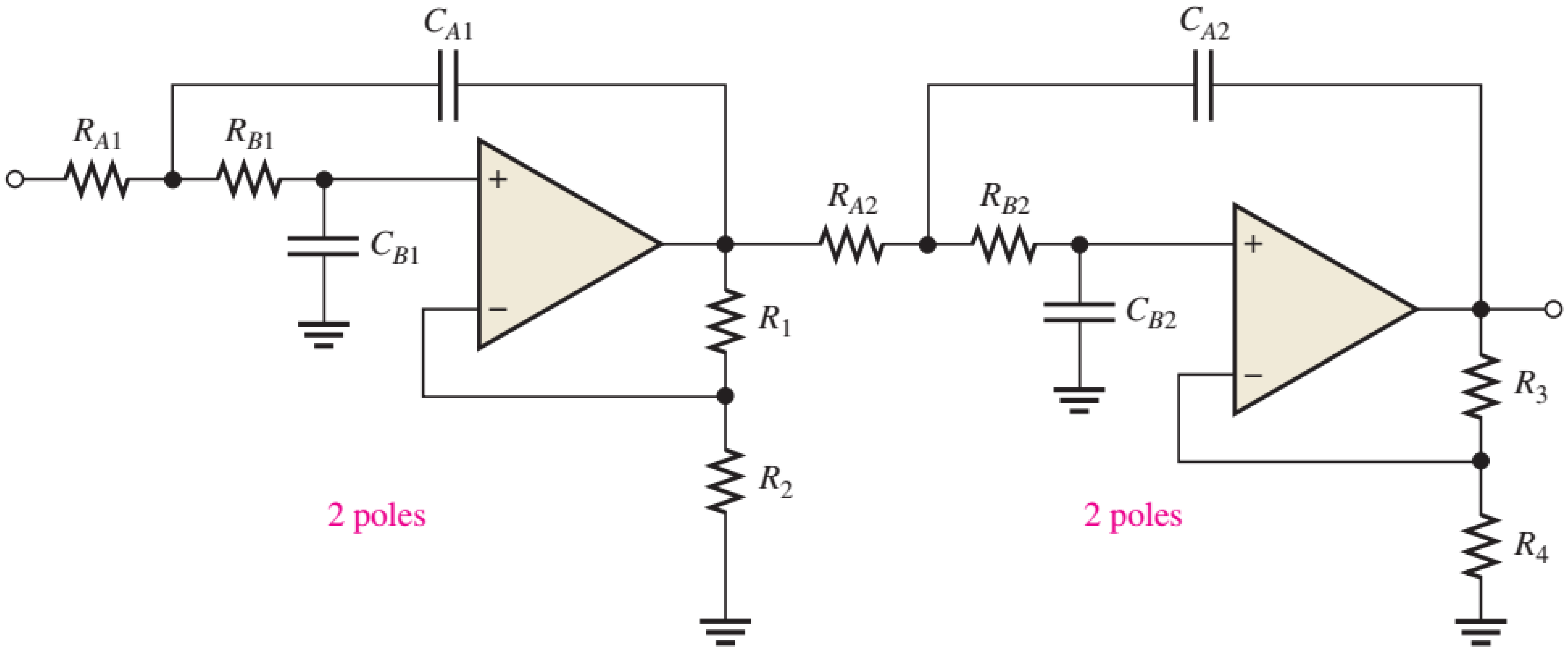
Cascaded Low-Pass Filters

To obtain more order filters, we can cascaded more different ordered filters as the examples shown in Figure 2.17.



(a) Third-order configuration

Figure 2.17



(b) Fourth-order configuration

Figure 2.17

The Sallen-Key High-Pass Filter

A high pass Sallen-Key configuration is shown in Figure 2.18. Notice that the positions of the resistors and capacitors in the frequency-selective circuit are opposite to those in the low-pass configuration. As with the other filters, the response characteristic can be optimized by proper selection of the feedback resistors, and *R*1 *R*2.

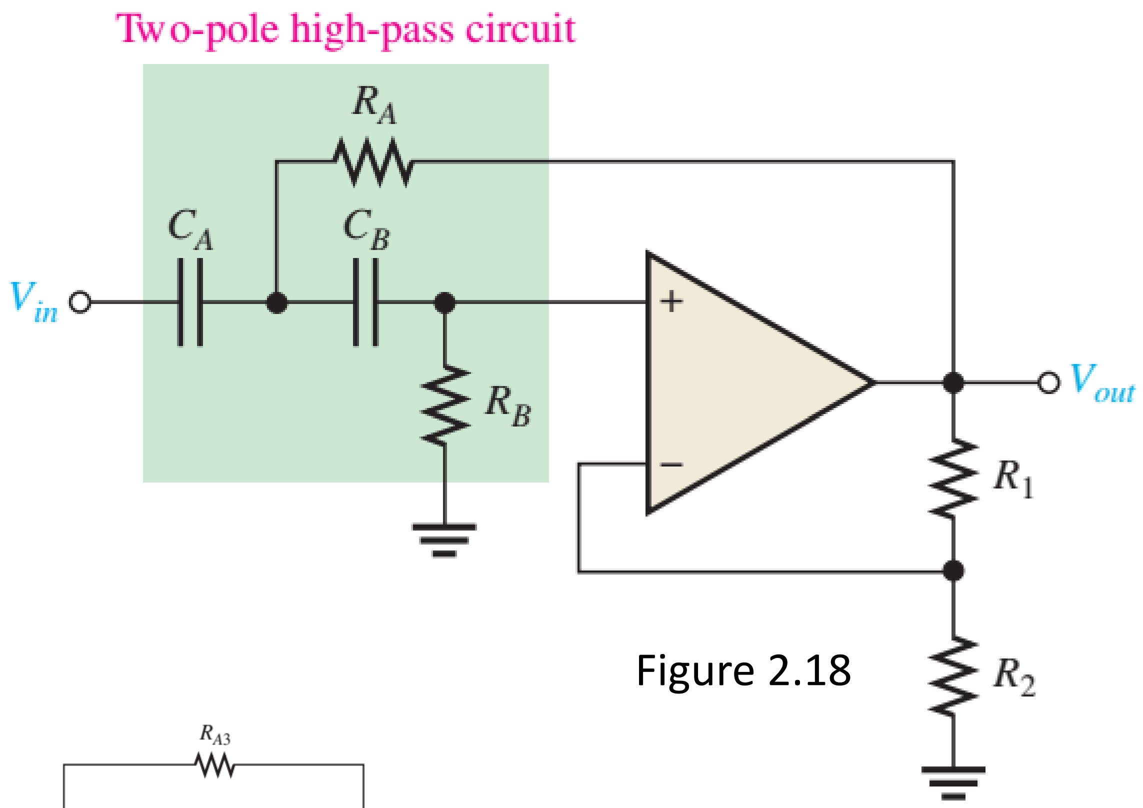
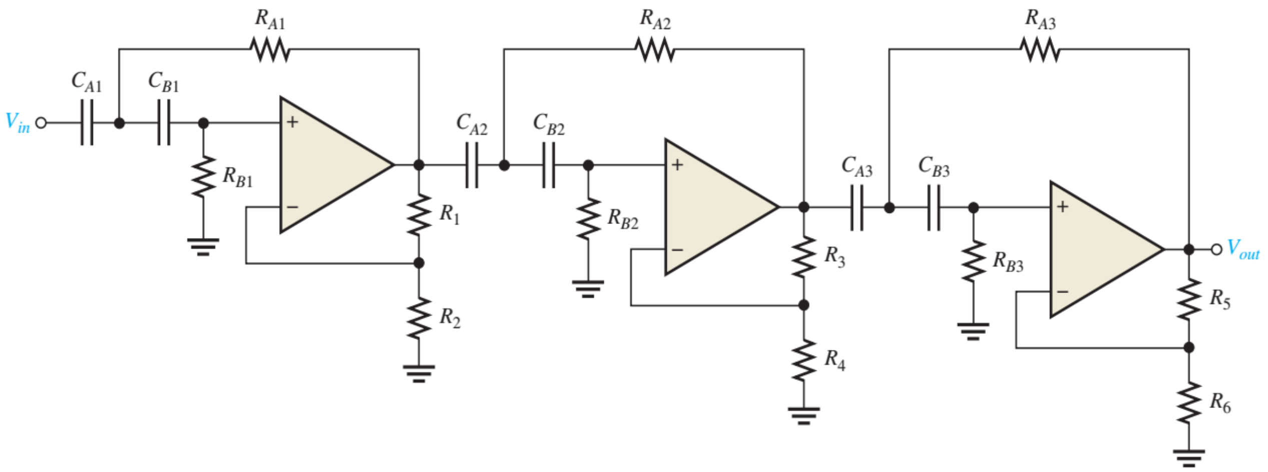


Figure 2.18

Cascading High-Pass Filters



Cascaded Low-Pass and High-Pass Filters

One way to implement a band-pass filter is a cascaded arrangement of a high-pass filter and a low-pass filter, as shown in Figure 2.19. The critical frequency of each filter is chosen so that the response curves overlap sufficiently, as indicated in Figure 2.20. The critical frequency of the high-pass filter must be sufficiently lower than that of the low-pass stage. This filter is generally limited to wide bandwidth applications.

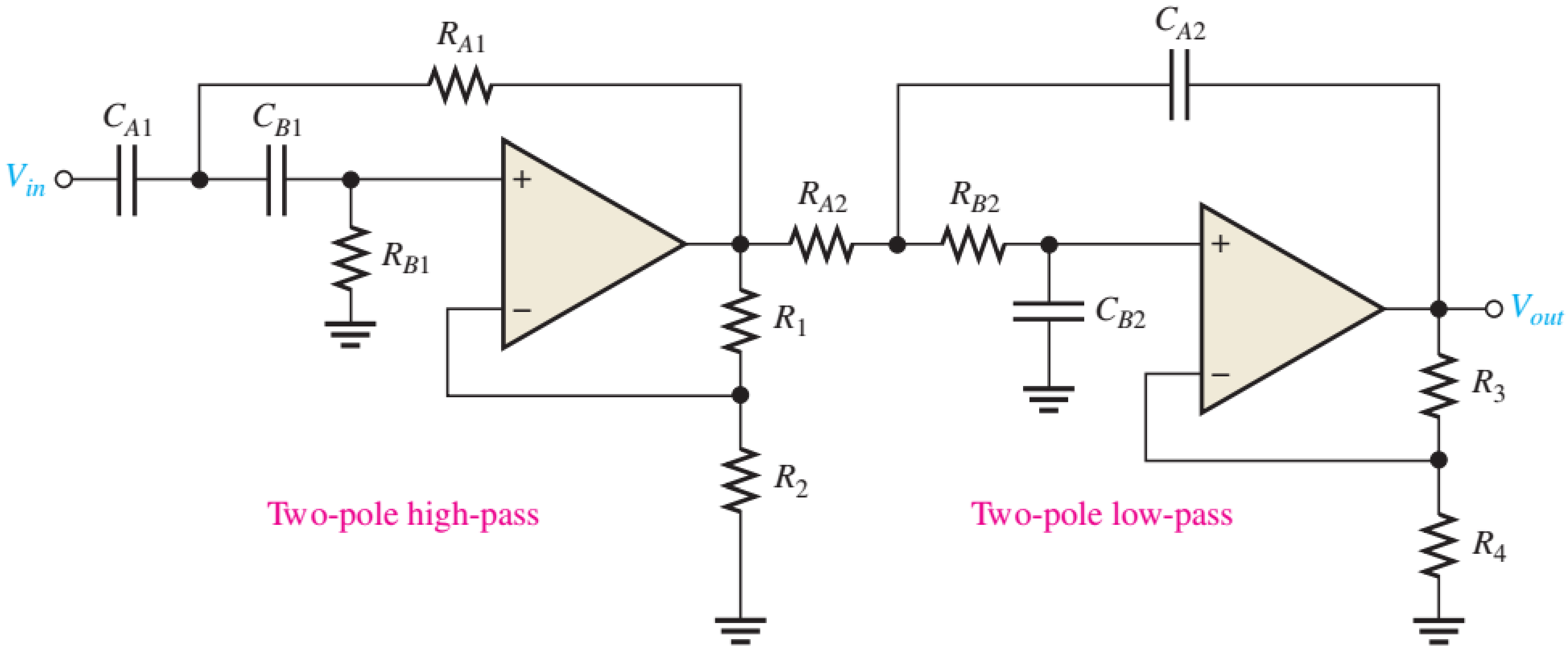


Figure 2.19

The lower frequency *fc1* of the passband is critical frequency of high-pass filter. The upper frequency *fc2* is the critical frequency of the low-pass filter. Ideally, the center frequency *fo* of passband is the geometric mean of *fc1* and *fc2*. The following formulas express the three frequencies of the bandpass filter in Figure 2.20.

$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

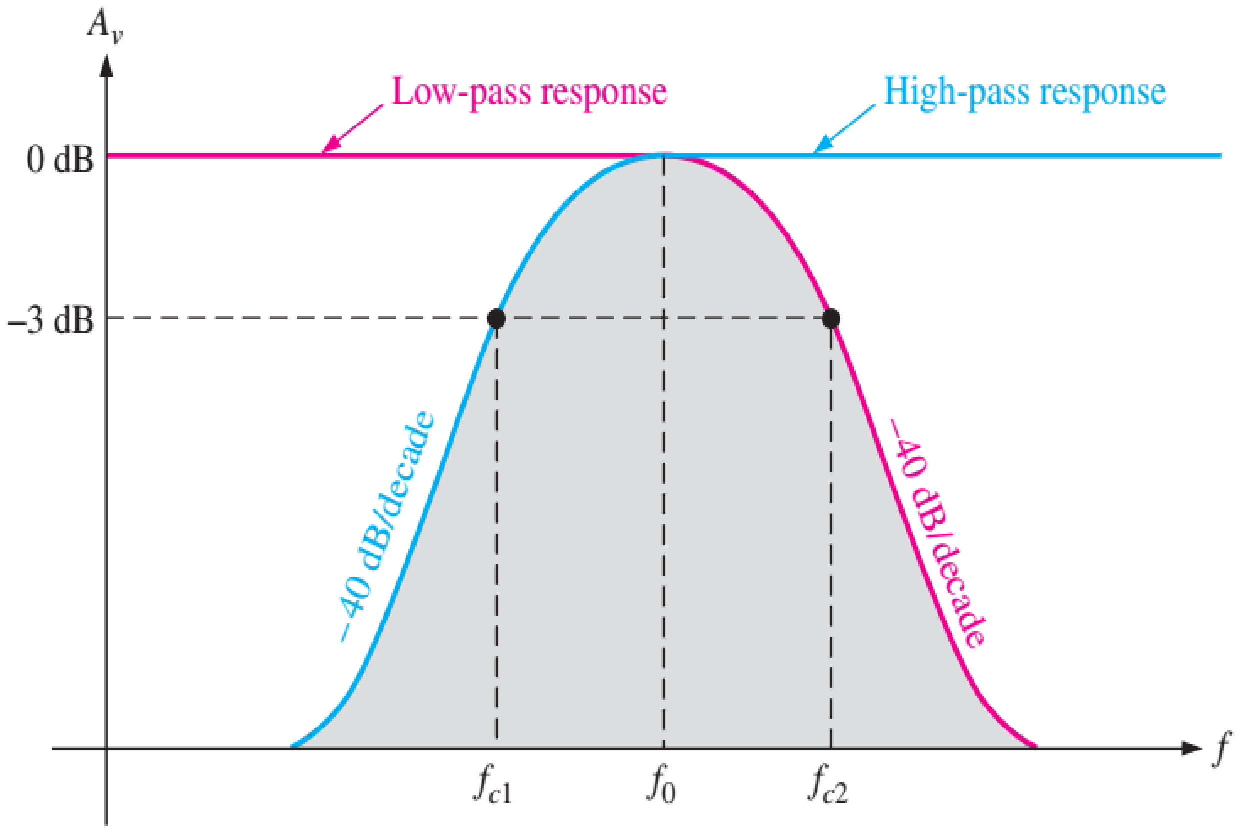


Figure 2.20

Multiple-Feedback Band-Pass Filter

Another type of filter configuration, shown in Figure 2.21, is a multiple-feedback bandpass filter. The two feedback paths are through *R*2 and *C*1. Components *R*1 and *C*1 provide the low-pass response, and *R*2 and *C*2 provide the high-pass response. The maximum gain, *A*, occurs at the center frequency. *Q* values of less than 10 are typical in this type of filter.

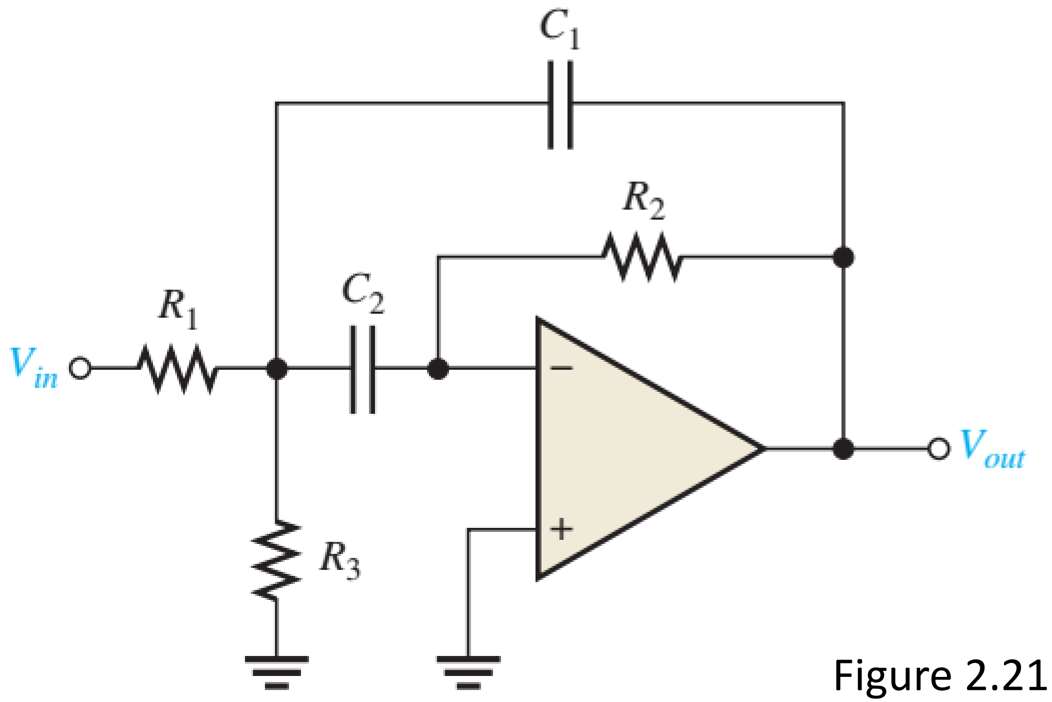


Figure 2.21

An expression for the center frequency is developed as follows, recognizing that R_1 and R_3 appear in parallel as viewed from the C_1 feedback path (with the V_{in} source replaced by a short).

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C_1C_2}}$$

Making $C_1 = C_2 = C$ yields

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C^2}} = \frac{1}{2\pi C\sqrt{(R_1 \| R_3)R_2}}$$

$$= \frac{1}{2\pi C}\sqrt{\frac{1}{R_2(R_1 \| R_3)}} = \frac{1}{2\pi C}\sqrt{\left(\frac{1}{R_2}\right)\left(\frac{1}{R_1R_3/R_1 + R_3}\right)}$$

$$\mathbf{f_0 = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1R_2R_3}}}$$