# Ch. 3: Oscillators

## Introduction

- The **signal-generator or oscillator** circuits studied in this lecture are collectively capable of providing signals with frequencies in the range of hertz to hundreds of gigahertz. While some can be fabricated on chip, others utilize discrete components.
- There are two distinctly different approaches for the generation of sinusoids, perhaps the most commonly used of the standard waveforms. The first approach, employs a **positive-feedback loop** consisting of an amplifier and an RC or LC **frequency-selective network**. While the frequency of the generated sine wave is determined by the frequency-selective network, the amplitude is set using a nonlinear mechanism, implemented either with a separate circuit or using the nonlinearities of the amplifying device itself. In spite of this, these circuits, which generate sine waves utilizing resonance phenomena, are known as **linear oscillators**. The name clearly distinguishes them from the circuits that generate sinusoids by way of the second approach. In these circuits, a sine wave is obtained by appropriately shaping a triangular waveform.
- Circuits that generate square, triangular, pulse (etc.) waveforms, called **nonlinear oscillators** or **function generators**, employ circuit building blocks known as **multivibrators**. There are three types of multivibrator: the **bistable** , the **astable**, and the **monostable**.

- The **linear oscillator**, which utilizes some form of resonance, and the **nonlinear oscillator** or function generator, which employs a switching mechanism implemented with a multivibrator circuit.

## 3.1 Basic Principles of Sinusoidal Oscillators

### 3.1.1 The Oscillator Feedback Loop

The basic structure of a sinusoidal oscillator consists of an amplifier and a frequency-selective network connected in a **positive-feedback loop**, such as that shown in block diagram form in Fig. 3.1.

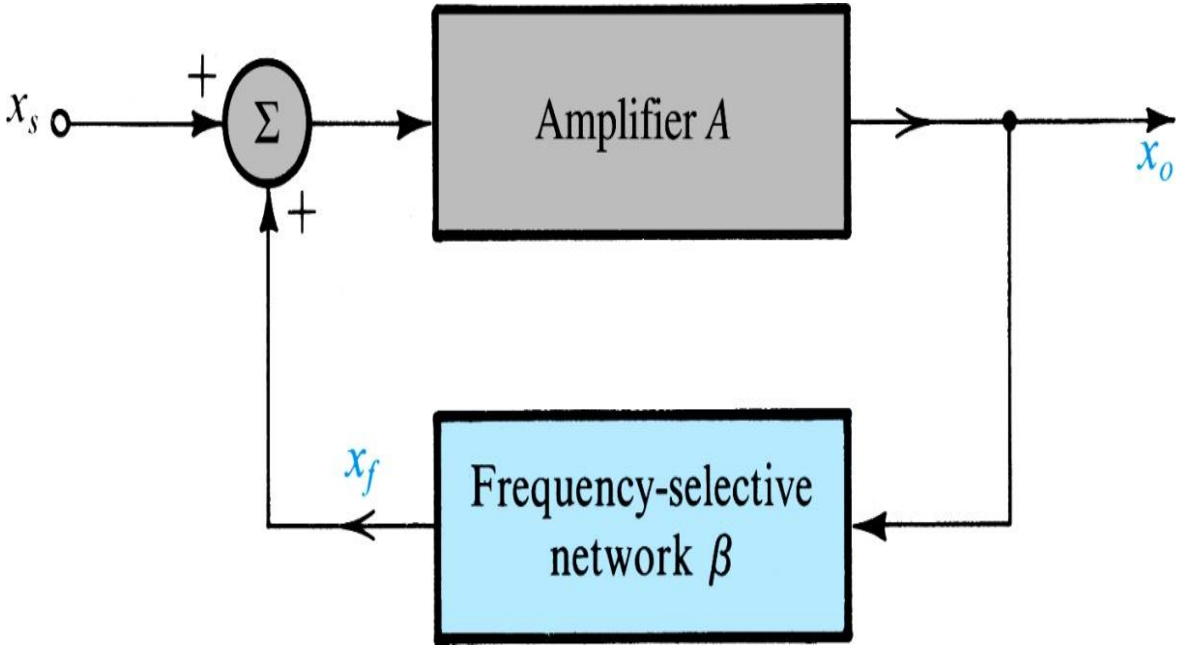


**Figure 3.1** The basic structure of a sinusoidal oscillator. A positive-feedback loop is formed by an amplifier and a frequency-selective network. **In an actual oscillator circuit, no input signal will be present;** here an input signal $x_s$ is employed to help explain the principle of operation.

Thus the gain-with-feedback is given by

$$A_f(s) = \frac{A(s)}{1 - A(S)\beta(S)} \tag{3.1}$$

where we note the negative sign in the denominator. The loop gain *L(s)* is given by

$$L(s) \equiv A(s)\beta(s) \tag{3.2}$$

### 3.1.2 The Oscillation Criterion

If at a specific frequency $f_0$ the **loop gain *Aβ* is equal to unity**, it follows from Eq. (3.1) that $\boldsymbol{A_f}$ **will be infinite**. That is, at this frequency the circuit will have a finite output for zero input signal. Such a circuit is **by definition an oscillator**. Thus the condition for the feedback loop of Fig. 3.1 to provide sinusoidal oscillations of frequency $\omega_0$ is

$$L(j\omega_0) \equiv A(j\omega_0)\beta(j\omega_0) = 1 \tag{3.3}$$

That is, ***at $\boldsymbol{\omega_0}$ the phase of the loop gain should be zero and the magnitude of the loop gain should be unity.*** This is known as the **Barkhausen criterion**. Note that for the circuit to oscillate at one frequency, the oscillation criterion should be satisfied only at one frequency (i.e., $\omega_0$); otherwise the resulting waveform will not be a simple sinusoid. An intuitive feeling for the Barkhausen criterion can be gained by considering once more the feedback loop of Fig. 3.1.

For this loop to ***produce*** and ***sustain*** an output *xo* with no input applied (*xs*=0), the feedback signal $x_f = \beta x_0$
should be sufficiently large that when multiplied by $A$ it produces $x_0$, that is, $Ax_f = x_0$
that is, $A\beta x_0 = x_0$ which results in $A\beta = 1$

An alternative approach to the study of oscillator circuits consists of examining the circuit poles, which are the roots of the **characteristic equation** (**1-L(s)=0**). For the circuit to produce **sustained oscillations** at a frequency $\omega_0$ the characteristic equation has to have roots at $s = \pm j\omega_0$. Thus $1-A(s)\beta(s)$ should have a factor of the form $s^2 + \omega_0^2$.

### 3.1.3 Nonlinear Amplitude Control

The oscillation condition, the Barkhausen criterion guarantees sustained oscillations in a mathematical sense. It is well known, however, that the parameters of any physical system cannot be maintained constant for any length of time. In other words, suppose we work hard to make $|A\beta| = 1$ at $\omega=\omega_0$, and then the temperature changes and $|A\beta|$ becomes slightly less than unity. Obviously, oscillations will cease in this case. Conversely, if $|A\beta|$ exceeds unity, oscillations will grow in amplitude. We therefore need a mechanism for forcing $|A\beta|$ to remain equal to unity ***at the desired value of output amplitude***. This task is accomplished by providing a nonlinear circuit for gain control.

Basically, the function of the **gain-control mechanism** is as follows: **First**, to ensure that oscillations will start, one designs the circuit such that **$|A\beta|$ is slightly greater than unity**. This corresponds to designing the circuit so that the **poles are in the right half of the *s* plane**. Thus as the power supply is turned on, oscillations will grow in amplitude. When the amplitude reaches the desired level, the nonlinear network comes into action and causes the loop gain to be reduced to exactly unity. In other words, the poles will be "pulled back" to the $j\omega$ axis. This action will cause the circuit to sustain oscillations at this desired amplitude. If, for some reason, the loop gain is reduced below unity, the amplitude of the sine wave will diminish. This will be detected by the nonlinear network, which will cause the loop gain to increase to exactly unity.

One mechanism for amplitude control utilizes an element whose resistance can be controlled by the amplitude of the output sinusoid. By placing this element in the feedback circuit so that its resistance determines the loop gain, the circuit can be designed to ensure that the loop gain reaches unity at the desired output amplitude. Diodes, or JFETs operated in the triode region, are commonly employed to implement the controlled-resistance element.

### 3.1.4 A Popular Limiter Circuit for Amplitude Control

The limiter circuit is shown in Fig. 3.2(a), and its transfer characteristic is depicted in Fig. 3.2(b).

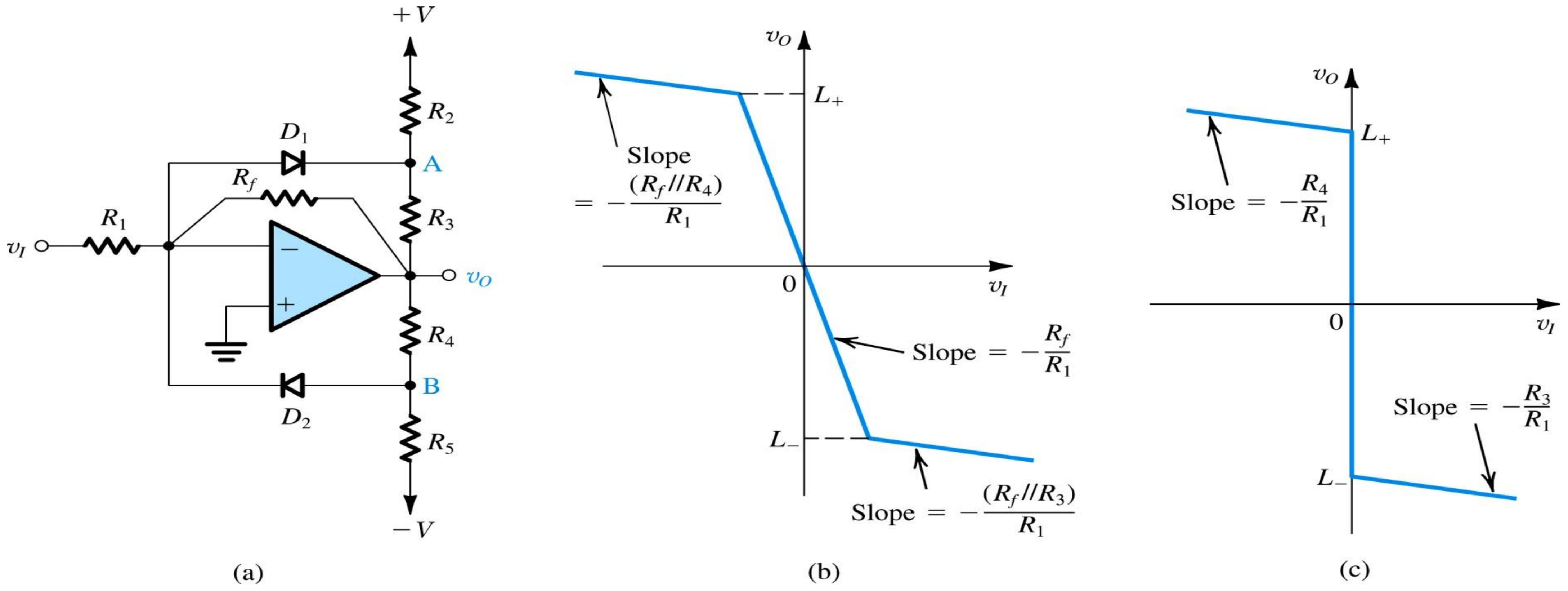


**Figure 3.2 (a)** A popular limiter circuit. **(b)** Transfer characteristic of the limiter circuit; **(c)** When $R_f$ is removed, the limiter turns into a comparator with the characteristic shown.

# Limiter Circuit operation

- For small amplitude ($D_1$ off, $D_2$ off)

  $\rightarrow$ incremental gain (slope) = $-R_f/R_1$

- For large negative swing ($D_1$ on, $D_2$ off)

  $\rightarrow$ incremental gain (slope) = $-(R_f \| R_4)/R_1$

- For large positive swing ($D_1$ off, $D_2$ on)

  $\rightarrow$ incremental gain (slope) = $-(R_f \| R_3)/R_1$

$$v_A = \frac{R_3}{R_2 + R_3}V + \frac{R_2}{R_2 + R_3}v_O \qquad v_B = -\frac{R_4}{R_4 + R_5}V + \frac{R_5}{R_4 + R_5}v_O$$

$$L_+ = \frac{R_4}{R_5}V + \frac{R_4 + R_5}{R_5}V_D \qquad L_- = -\frac{R_3}{R_2}V - \frac{R_2 + R_3}{R_2}V_D$$

## 3.2 Op Amp–RC Oscillator Circuits

In this section we shall study some practical oscillator circuits utilizing op amps and RC networks. These circuits are usually assembled on printed-circuit boards; their frequency of operation extends from very low frequencies to at most 1 MHz.

### 3.2.1 The Wien-Bridge Oscillator

Figure 4.3 shows a Wien-bridge oscillator without the nonlinear gain-control network. The circuit consists of an op amp connected in the noninverting configuration, with a closed-loop gain of 1+*R*2/*R*1. In the feedback path of this positive-gain amplifier, an RC network is connected. The loop gain can be easily obtained by multiplying the transfer function *Va*(*s*)/*Vo*(*s*) of the feedback network by the amplifier gain,

$$L(s)=\left[1+\frac{R_2}{R_1}\right]\frac{Z_p}{Z_p+Z_s}$$

$$=\frac{1+{}^{R_2}/_{R_1}}{1+Z_sY_p}$$

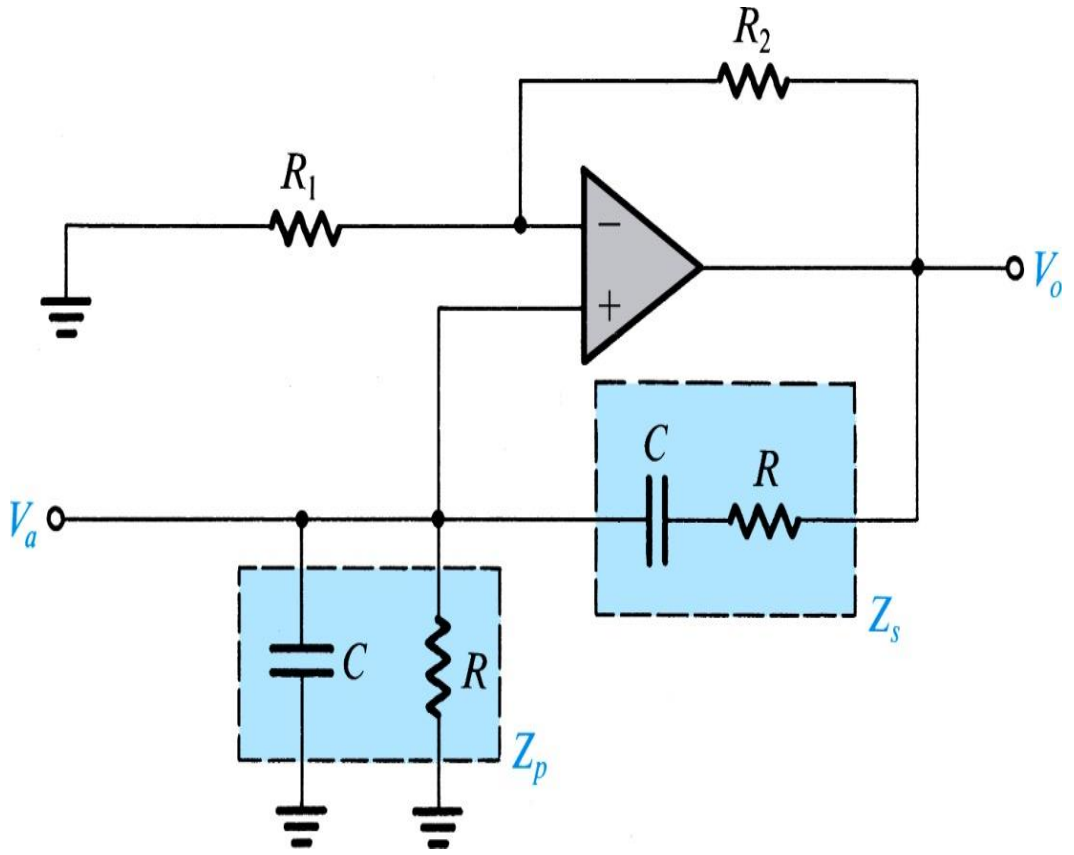


**Figure 3.3** A Wien-bridge oscillator without amplitude stabilization.

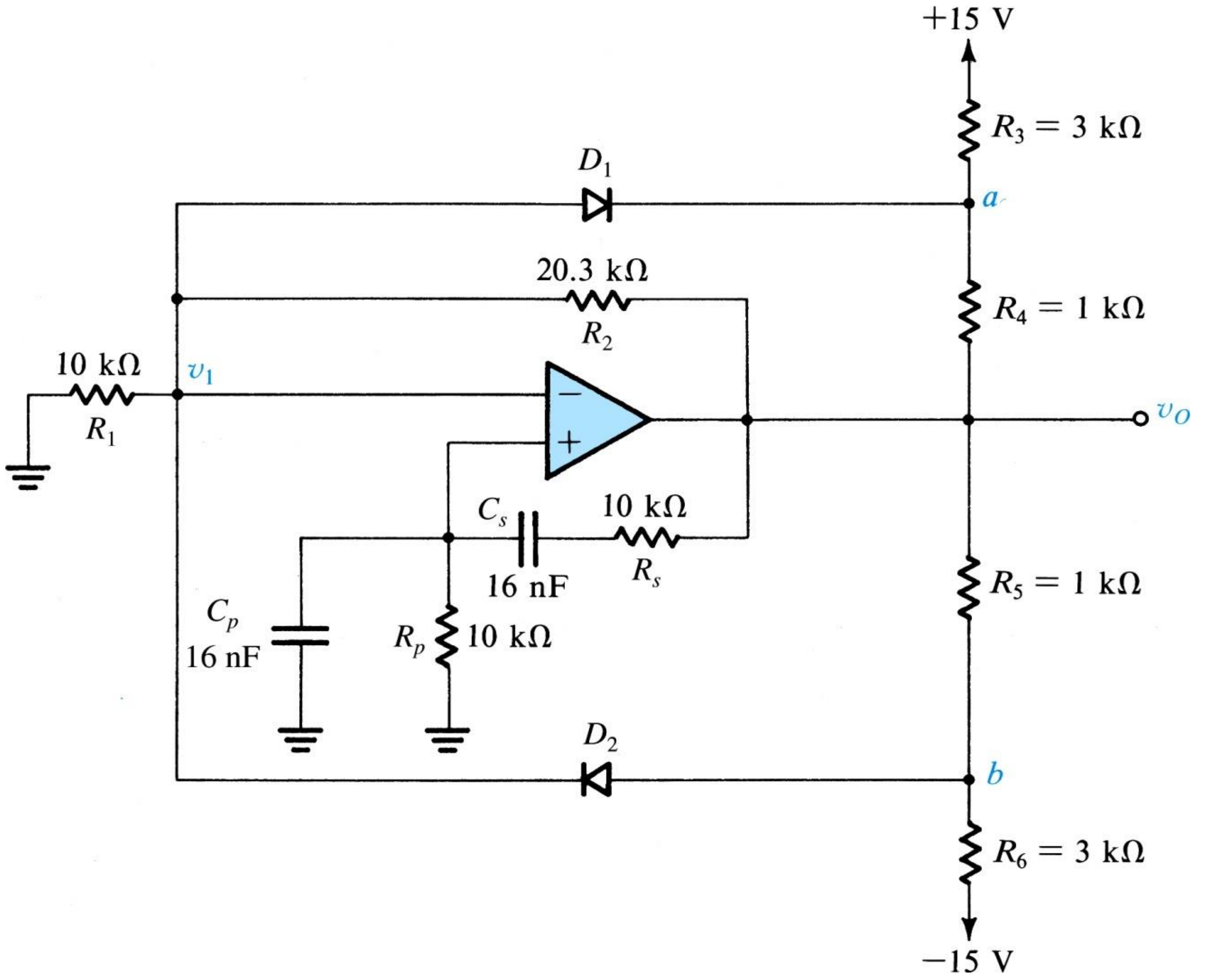


**Figure 3.4** A Wien-bridge oscillator with a limiter used for amplitude control.

Thus, $$L(S) = \frac{1 + {}^{R_2}/_{R_1}}{3 + sCR + {}^{1}/_{sCR}} \qquad (3.4)$$

Substituting $s$=$j\omega$ results in

$$L(j\omega) = \frac{1 + {}^{R_2}/_{R_1}}{3 + j(\omega CR - {}^{1}/_{\omega}\, CR)} \qquad (3.5)$$

The loop gain will be a real number (i.e., the phase will be zero) at one frequency given by

$$\omega_0 \mathrm{CR} = \frac{1}{\omega_0 CR}$$

That is, $$\omega_0 = \frac{1}{CR} \qquad (3.6)$$

Oscillations will start at this frequency if the loop gain is at least unity. This can be achieved by selecting

$$\frac{R_2}{R_1} = 2 \qquad (3.7)$$

### 3.2.2 The Phase-Shift Oscillator

The basic structure of the phase-shift oscillator is shown in Fig. 3.5. It consists of a negative-gain amplifier ($-K$) with a three-section (third-order) RC ladder network in the feedback. The circuit will oscillate at the frequency for which the phase shift of the RC network is 180°.

Only at this frequency will the total phase shift around the loop be 0° or 360°. Here we should note that the reason for using a three-section RC network is that three is the minimum number of sections (i.e., lowest order) that is capable of producing a 180° phase shift at a finite frequency.

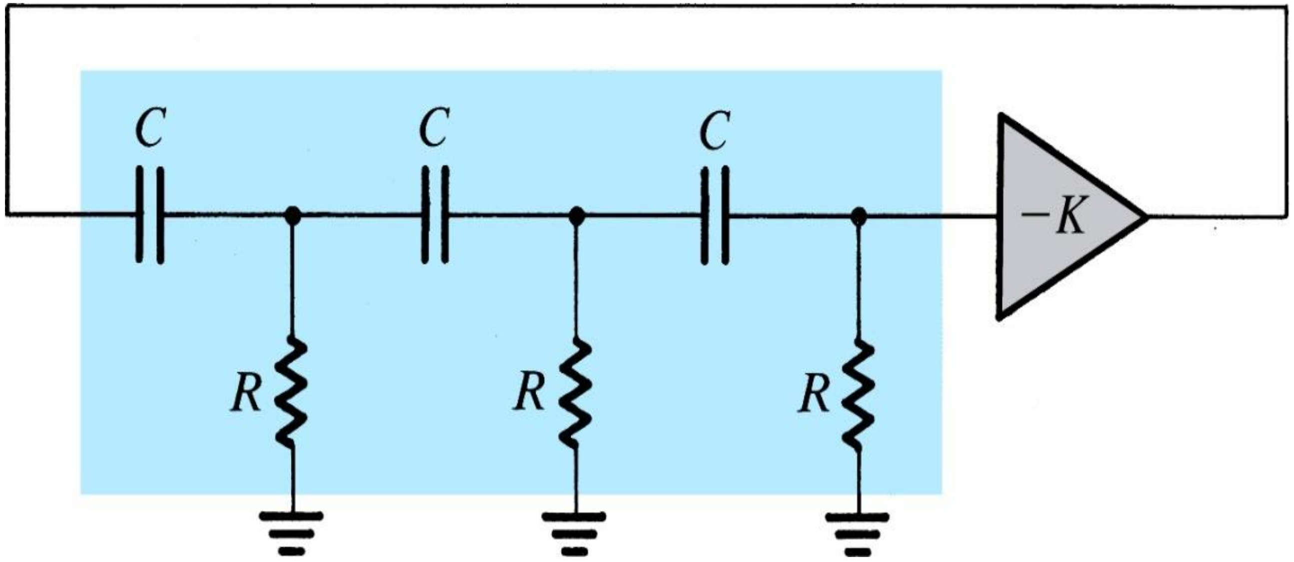


**Figure 3.5** A phase-shift oscillator.

## 3.3 LC and Crystal Oscillators

Oscillators utilizing transistors (FETs or BJTs), with LC circuits or crystals as the frequency-selective feedback elements, are used in the frequency range of 100 kHz to hundreds of gigahertz. They exhibit higher *Q* than the RC types. However, LC oscillators are difficult to tune over wide ranges, and crystal oscillators operate at a single frequency.

### 3.3.1 The Colpitts and Hartely Oscillators

Figure 4.6 shows two commonly used configurations of LC oscillators. They are known as the **Colpitts oscillator** and the **Hartley oscillator**. Both utilize a parallel LC circuit connected between collector and base (or between drain and gate if a FET is used) with a fraction of the tuned-circuit voltage fed to the emitter (the source in a FET).

This feedback is achieved by way of a capacitive divider in the Colpitts oscillator and by way of an inductive divider in the Hartley circuit. **Observe that in both circuits the voltage *Veb* gives rise to a current *Ic* in the direction shown, which in turn results in a positive voltage across the LC circuit. Thus, we do have a positive-feedback loop.**

If the frequency of operation is sufficiently low that we can neglect the transistor capacitances, the frequency of oscillation will be determined by the resonance frequency of the parallel-tuned circuit (also known as a *tank circuit* because it behaves as a reservoir for energy storage).

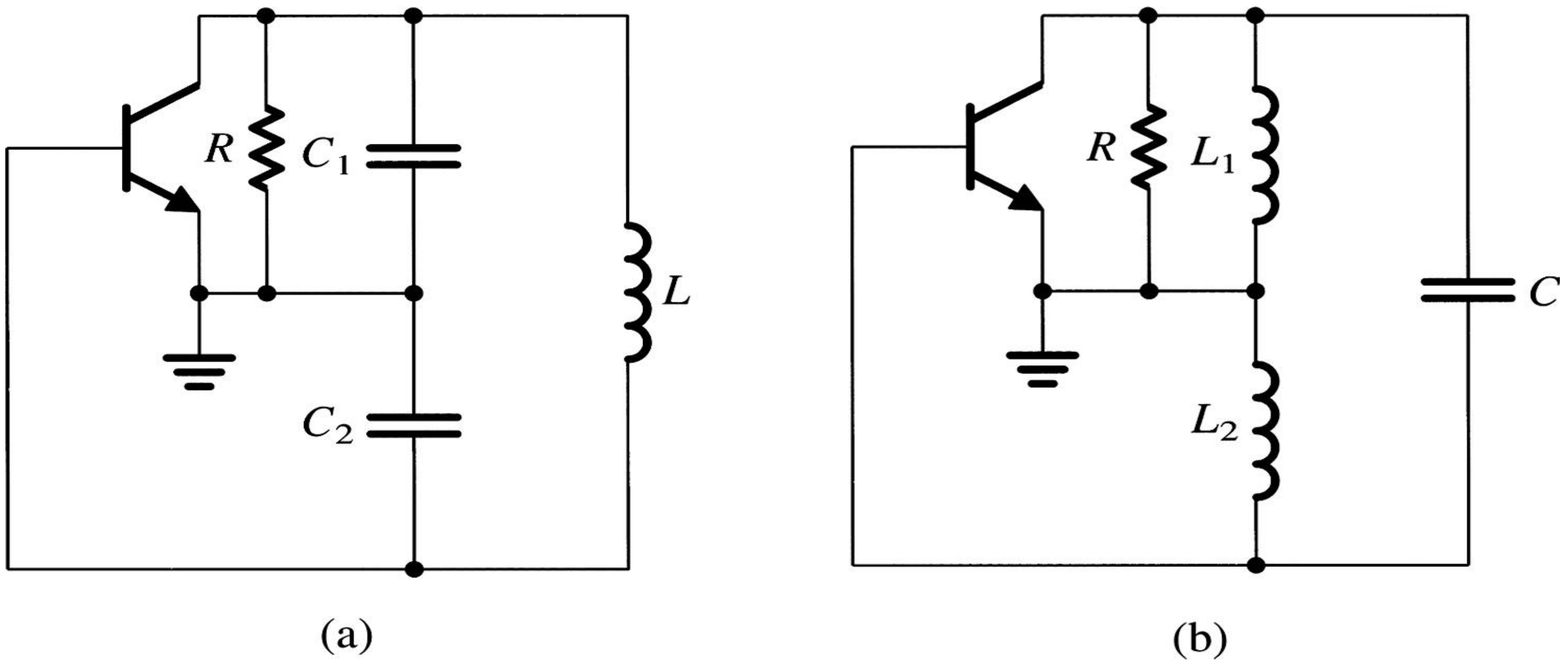


**Figure 3.6** Two commonly used configurations of LC-tuned oscillators: **(a)** Colpitts and **(b)** Hartley.

Thus for the **Colpitts oscillator** we have

$$\omega_0 = 1\bigg/\sqrt{L(\frac{C_1 C_2}{C_1 + C_2}} \qquad (3.8)$$

and for the **Hartley oscillator** we have

$$\omega_0 = 1/\sqrt{(L_1 + L_2)C} \qquad (3.9)$$

**The ratio *L*1/*L*2 or *C*1/*C*2 determines the feedback factor and thus must be adjusted in conjunction with the transistor gain to ensure that oscillations will start.**

To determine the oscillation condition for the Colpitts oscillator in Fig. 3.7(a), we replace the transistor with its equivalent circuit, as shown in Fig. 3.7(b). To simplify the analysis, we have neglected the transistor capacitance $C\mu$ ($Cgd$ for a FET). Capacitance $C_\pi$ ($Cgs$ for a FET), although not shown, can be considered to be a part of $C2$. The input resistance $r_\pi$ (infinite for a FET) has also been neglected, assuming that at the frequency of oscillation $r_\pi \gg (1/\omega C_2)$. Finally, as mentioned earlier, the resistance $R$ includes $r_o$ of the transistor.

From Fig. (3.7)b, a node equation at the transistor collector (node C) yields

$$sC_2 V_\pi + g_m V_\pi + \left(\frac{1}{R} + sC_1\right)(1 + s^2 LC_2)V_\pi = 0$$

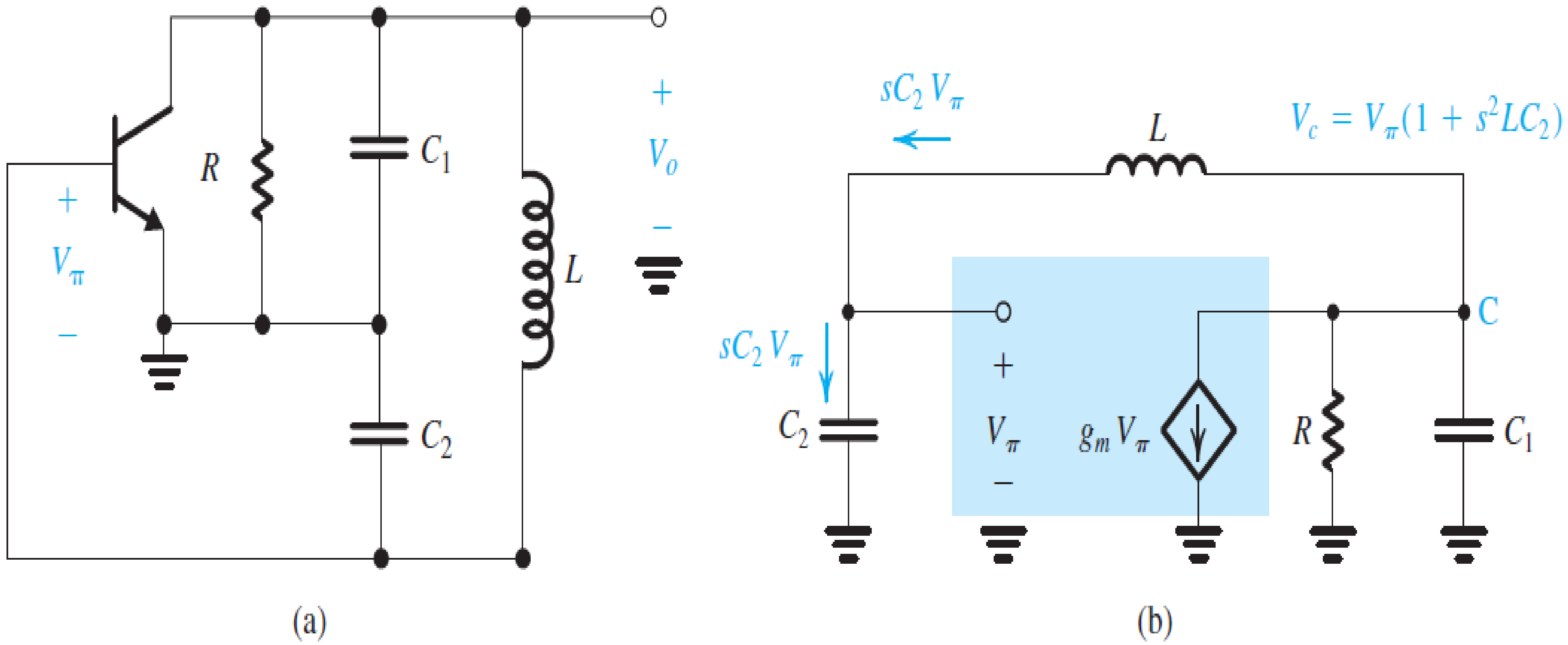


**Figure 3.7** (**a**)A Colpitts oscillator in which the emitter is grounded and the output is taken at the collector. (**b**) Equivalent circuit of the Colpitts oscillator of (**a**). To simplify the analysis, $C_\mu$ and $r_\pi$ are neglected. We can consider $C_\pi$ to be part of $C_2$, and we can include $r_o$ in $R$.

Since $V_\pi \neq 0$ (oscillations have started), it can be eliminated, and the equation can be rearranged in the form

$$s^3LC_1C_2 + s^2(LC_2/R) + s(C_1 + C_2) + \left(g_m + \frac{1}{R}\right) = 0$$

Substituting $s = j\omega$ gives

$$\left(g_m + \frac{1}{R} - \frac{\omega^2 LC_2}{R}\right) + j[\omega(C_1 + C_2) - \omega^3 LC_1C_2] = 0$$

For oscillations to start, both the real and imaginary parts must be zero. Equating the imaginary part to zero gives the frequency of oscillation as

$$\omega_0 = 1\bigg/\sqrt{L\left(\frac{C_1C_2}{C_1 + C_2}\right)}$$

Equating the real part to zero

$$C_2/C_1 = g_mR$$

Of course, for oscillations to start, the loop gain must be made greater than unity, a condition that can be

$$g_mR > C_2/C_1$$

**EXERCISES:** Show that for the Hartley oscillator of Fig. 3.6(b), the frequency of oscillation is given by Eq. (3.9) and that for oscillations to start $g_mR > L_1/L_2$.

### 3.3.2 Crystal Oscillators

A piezoelectric crystal, such as quartz, exhibits electromechanical-resonance characteristics that are very stable (with time and temperature) and highly selective (having very high *Q* factors). The circuit symbol of a crystal is shown in Fig. 3.8(a), and its equivalent-circuit model is given in Fig. 3.8(b). The resonance properties are characterized by a large inductance *L* (as high as hundreds of henrys), a very small series capacitance *Cs* (as small as 0.0005 pF), a series resistance *r* representing a *Q* factor $\omega_0 L/r$ that can be as high as a few hundred thousand, and a parallel capacitance *Cp* (a few picofarads). Capacitor *Cp* represents the electrostatic capacitance between the two parallel plates of the crystal. Note that $C_p \gg C_s$. Since the *Q* factor is very high, we may neglect the resistance *r* and express the crystal impedance as

$$Z(s) = 1 \Big/ \left[ sC_p + \frac{1}{sL + 1/sC_s} \right]$$

which can be manipulated to the form

$$Z(s) = \frac{1}{sC_p} \frac{s^2 + (1/LC_s)}{s^2 + \left[ (C_p + C_s)/LC_sC_p \right]}$$

From the above equation and from Fig. 3.8(b), we see that the crystal has two resonance frequencies: a series resonance at $\omega_s$

$$\omega_s = 1/\sqrt{LC_s}$$

and a parallel resonance at $\omega_p$

$$\omega_p = 1 \Big/ \sqrt{L\left(\frac{C_s C_p}{C_s + C_p}\right)}$$

Thus for $s = j\omega$ we can write

$$Z(j\omega) = -j\frac{1}{\omega C_p}\left(\frac{\omega^2 - \omega_s^2}{\omega^2 - \omega_p^2}\right)$$

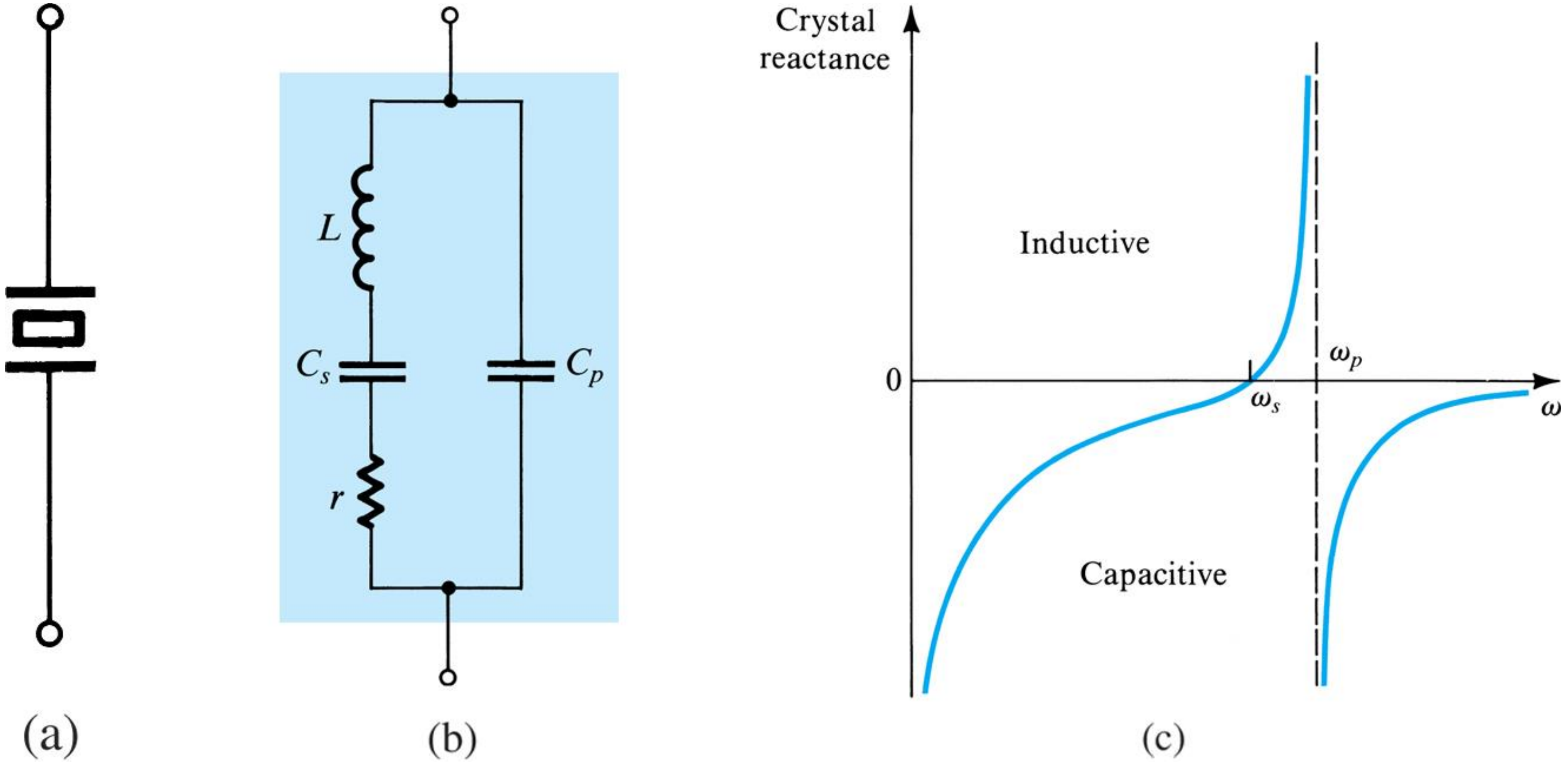


**Figure 3.8:** A piezoelectric crystal. **(a)** Circuit symbol. **(b)** Equivalent circuit. **(c)** Crystal reactance versus frequency [note that, neglecting the small resistance *r*, *Z*crystal= *jX(ω)*].

From $\omega_s$ , $\omega_p$ Eqs., we note that $\omega_p > \omega_s$. However, since $C_p \gg C_s$, the two resonance frequencies are very close. Expressing $Z(j\omega)=jX(\omega)$, the crystal reactance $X(\omega)$ will have the shape shown in Fig. 3.8(c). We observe that the crystal reactance is inductive over the very narrow frequency band between $\omega_s$ and $\omega_p$. For a given crystal, this frequency band is well defined. Thus we may use the crystal to replace the inductor of the Colpitts oscillator [Fig. 3.6(a)]. The resulting circuit will oscillate at the resonance frequency of the crystal inductance $L$ with the series equivalent of $Cs$ and $(Cp+C1C2/(C1+C2))$. Since $Cs$ is much smaller than the three other capacitances, it will be dominant and

$$\omega_0 \simeq 1/\sqrt{LC_s} = \omega_s$$

In addition to the basic Colpitts oscillator, a variety of configurations exist for crystal oscillators. Figure 3.9 shows a popular configuration (called the **Pierce oscillator**) utilizing a CMOS inverter as an amplifier. Resistor $Rf$ determines a dc operating point in the high-gain region of the VTC of the CMOS inverter. Resistor $R1$ together with capacitor $C1$ provides a low-pass filter that discourages the circuit from oscillating at a higher harmonic of the crystal frequency. Note that this circuit also is based on the Colpitts configuration.

The extremely stable resonance characteristics and the very high *Q* factors of quartz crystals result in oscillators with very accurate and stable frequencies. Crystals are available with resonance frequencies in the range of a few kilohertz to hundreds of megahertz. Unfortunately, however, crystal oscillators, being mechanical resonators, are fixed-frequency circuits.

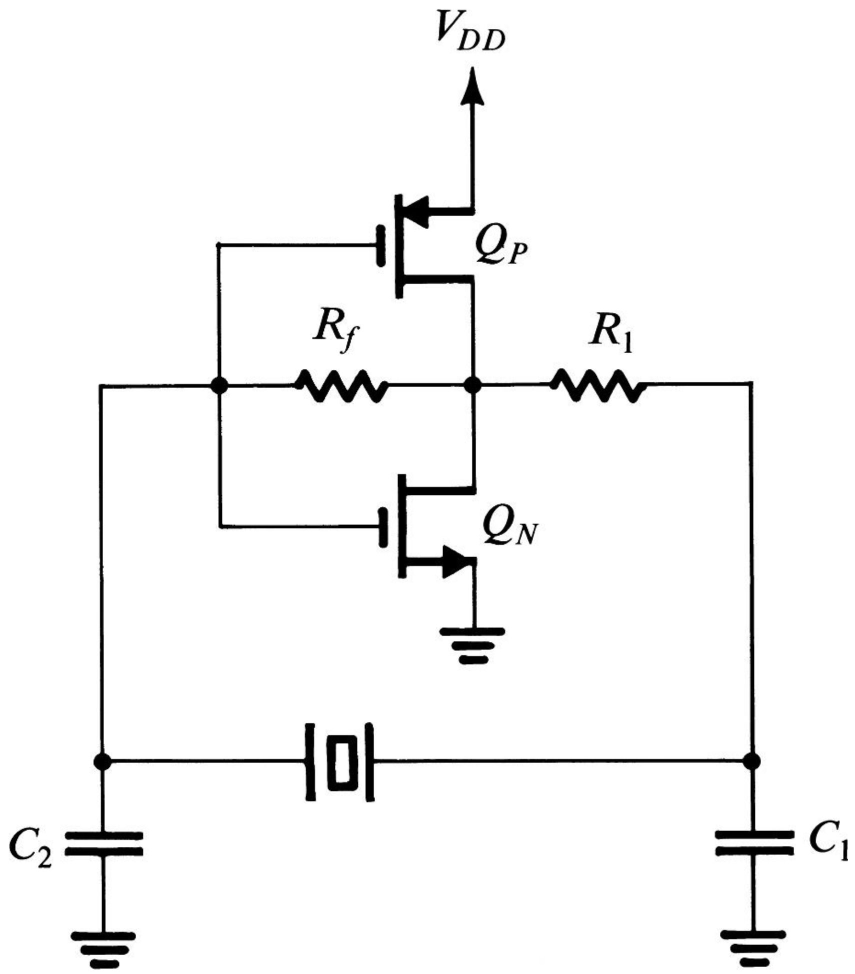


**Figure 3.9** A Pierce crystal oscillator utilizing a CMOS inverter as an amplifier.