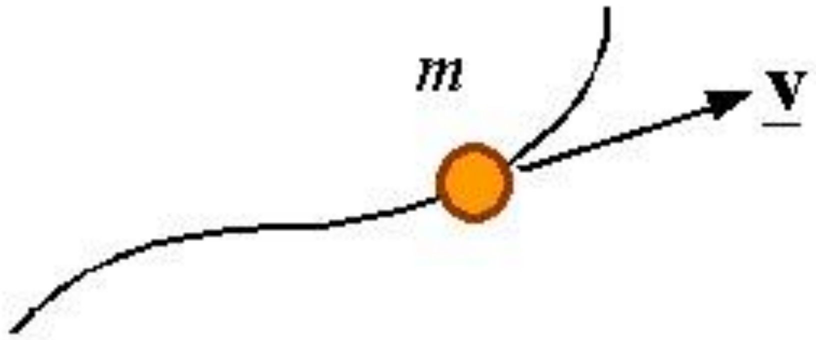
Principle of Linear Impulse and Momentum

Momentum: is a vector quantity that is the product of the mass and the velocity of an object or particle. (Momentum: is mass in motion)
The unit of momentum is (kg.m/s).

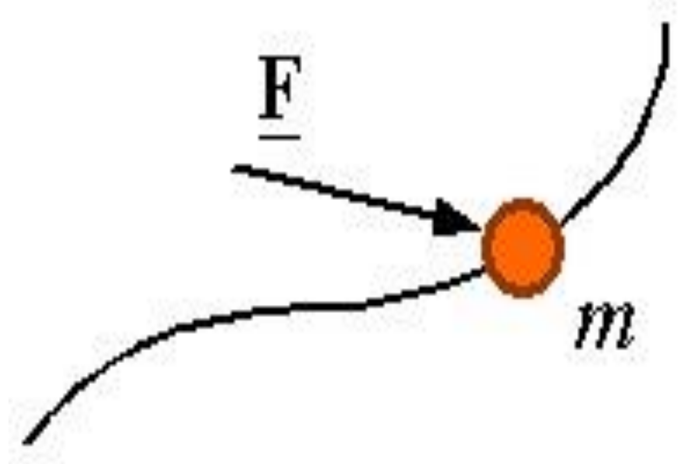
$$p = m * v$$

p = momentum
m = mass
v = velocity



Impulse: is a term that quantifies the overall effect of a force acting over time. It is conventionally given the symbol **I** and expressed in Newton-seconds. For a constant force,

$$I = F.\,\Delta t$$



Impulse of a force from time t1 to t2: The integral of the force over the time interval of concern is its impulse. The impulse of a force is a vector given by the integral

$$\mathbf{I} \equiv \int_{t_1}^{t_2} \mathbf{F}\,dt$$

The Momentum-Impulse Theorem:

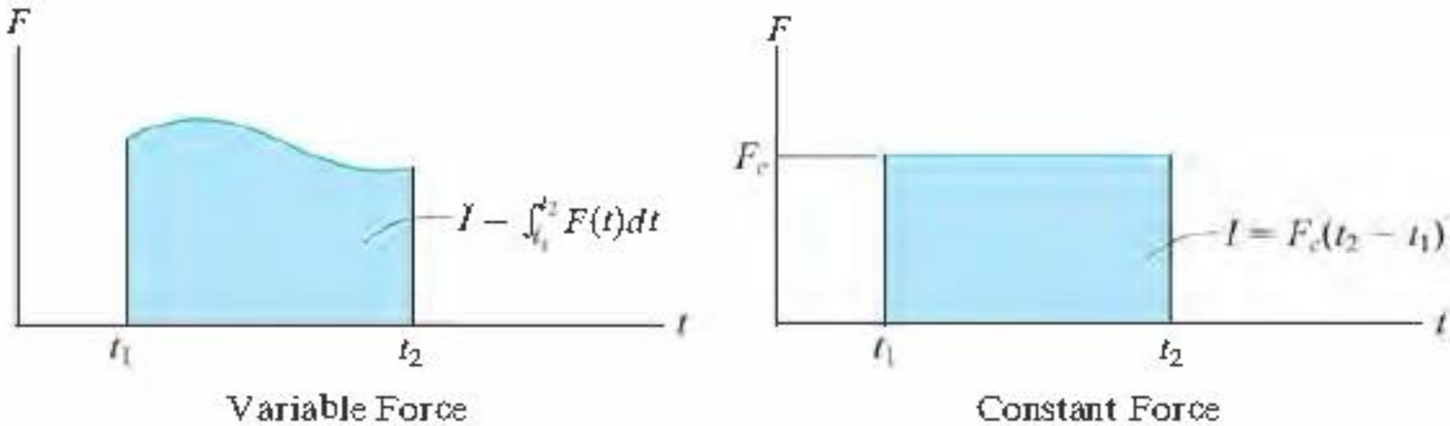
states that in order to change the momentum of an object, one must exert an impulse

$$\text{(change in momentum)} = \text{(impulse)}$$

$$p_{\text{final}} - p_{\text{initial}} = \text{(force)} * \text{(time)}$$

$$m*v_{\text{final}} - m*v_{\text{initial}} = \text{(force)} * \text{(time)}$$

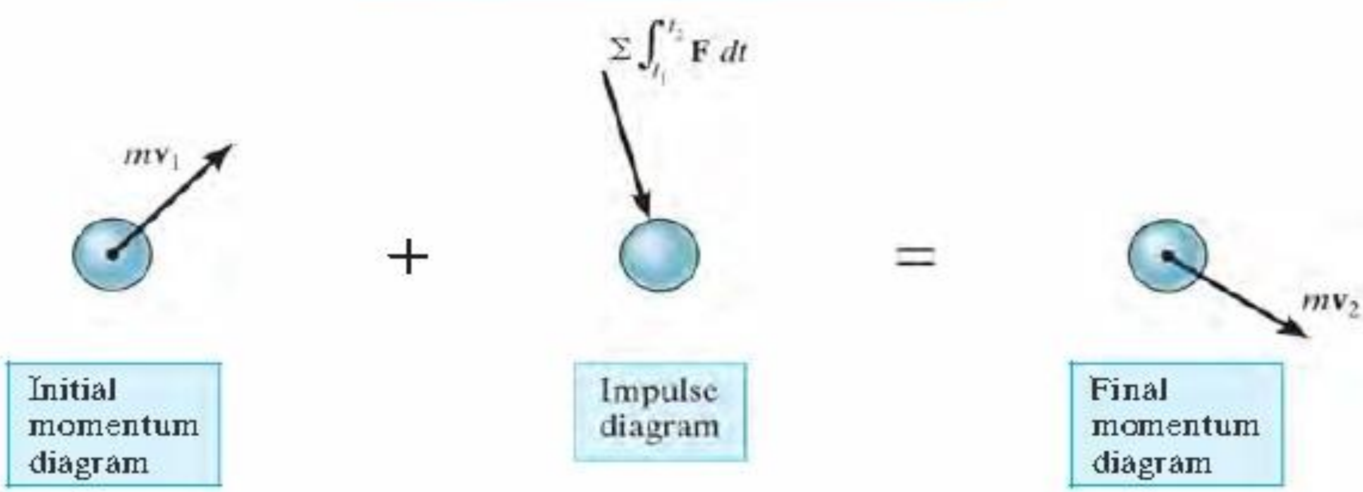
$$\mathbf{m}\,\mathbf{v}_1 + \Sigma \int_{t_1}^{t_2} \mathbf{F}\,dt = \mathbf{m}\,\mathbf{v}_2$$



If each of the vectors is resolved into its x, y components, we can write the following two scalar equations of linear impulse and momentum.

$$m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \Sigma \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2$$



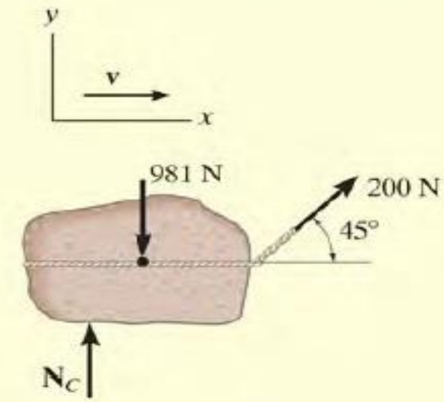
EXAMPLE 15.1

The 100-kg stone shown in Fig. 15–4*a* is originally at rest on the smooth horizontal surface. If a towing force of 200 N, acting at an angle of 45°, is applied to the stone for 10 s, determine the final velocity and the normal force which the surface exerts on the stone during this time interval.

(a)

SOLUTION

Principle of Impulse and Momentum.



(b)

$$(\xrightarrow{+}) \qquad m(v_x)_1 + \Sigma\int_{t_1}^{t_2} F_x\,dt = m(v_x)_2$$

$$0 + 200\text{ N}\cos 45°(10\text{ s}) = (100\text{ kg})v_2$$

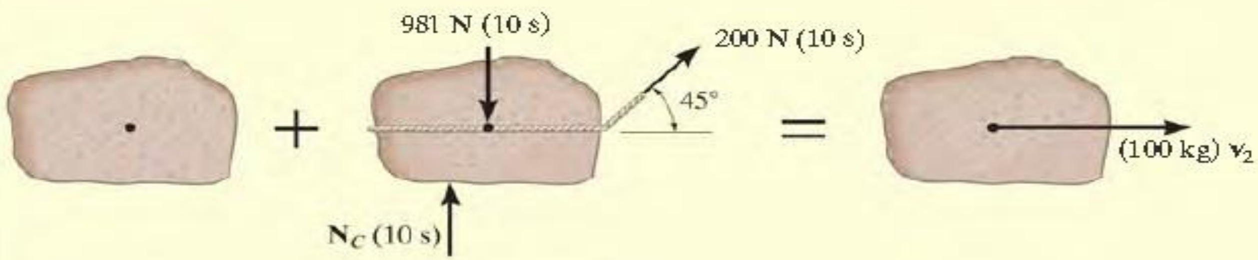
$$v_2 = 14.1\text{ m/s} \qquad \textit{Ans.}$$

$$(+\uparrow) \qquad m(v_y)_1 + \Sigma\int_{t_1}^{t_2} F_y\,dt = m(v_y)_2$$

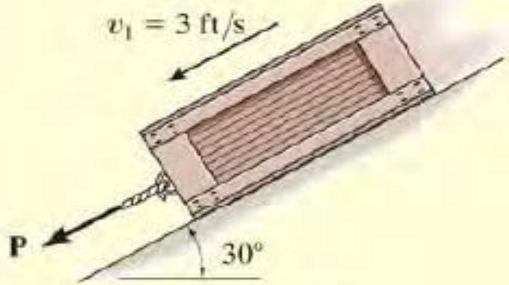
$$0 + N_C(10\text{ s}) - 981\text{ N}(10\text{ s}) + 200\text{ N}\sin 45°(10\text{ s}) = 0$$

$$N_C = 840\text{ N} \qquad \textit{Ans.}$$

NOTE: Since no motion occurs in the *y* direction, direct application of the equilibrium equation $\Sigma F_y = 0$ gives the same result for N_C.



EXAMPLE 15.2



(a)

The 50-lb crate shown in Fig. 15–5*a* is acted upon by a force having a variable magnitude $P = (20t)$ lb, where t is in seconds. Determine the crate's velocity 2 s after **P** has been applied. The initial velocity is $v_1 = 3$ ft/s down the plane, and the coefficient of kinetic friction between the crate and the plane is $\mu_k = 0.3$.

SOLUTION

Principle of Impulse and Momentum. Applying Eqs. 15–4 in the x direction, we have

$$(+\swarrow) \qquad m(v_x)_1 + \Sigma \int_{t_1}^{t_2} F_x\, dt = m(v_x)_2$$

$$\frac{50 \text{ lb}}{32.2 \text{ ft/s}^2}(3 \text{ ft/s}) + \int_0^{2\text{ s}} 20t\, dt - 0.3N_C(2 \text{ s}) + (50 \text{ lb}) \sin 30°(2 \text{ s}) = \frac{50 \text{ lb}}{32.2 \text{ ft/s}^2} v_2$$

$$4.658 + 40 - 0.6N_C + 50 = 1.553v_2$$

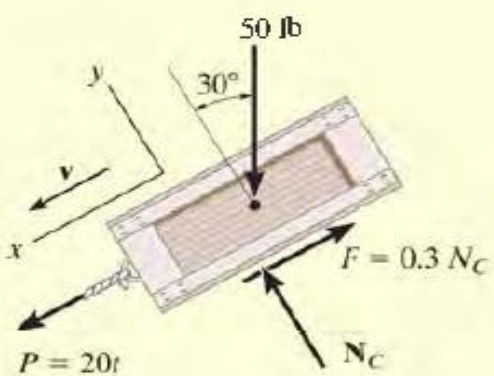
The equation of equilibrium can be applied in the y direction.

$$+\nwarrow \Sigma F_y = 0; \qquad N_C - 50 \cos 30° \text{ lb} = 0$$

Solving,

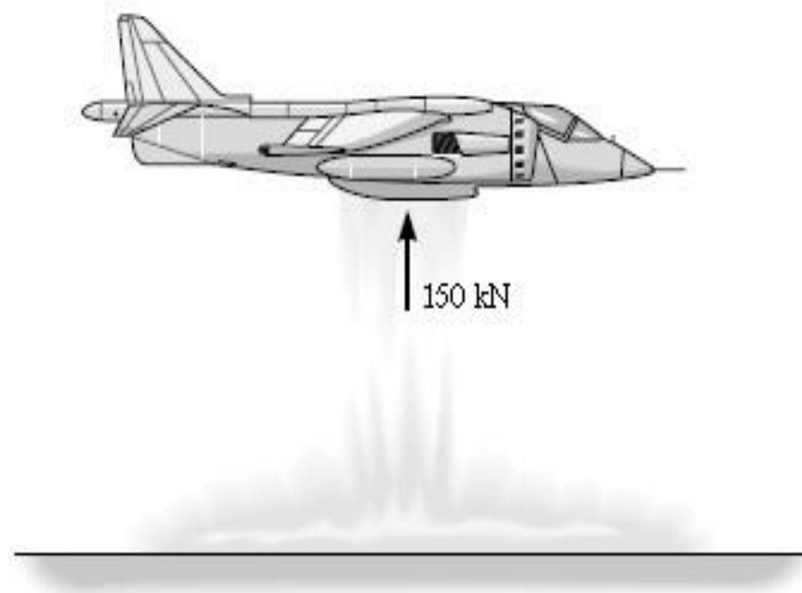
$$N_C = 43.30 \text{ lb}$$

$$v_2 = 44.2 \text{ ft/s} \swarrow \qquad \textit{Ans.}$$



(b)

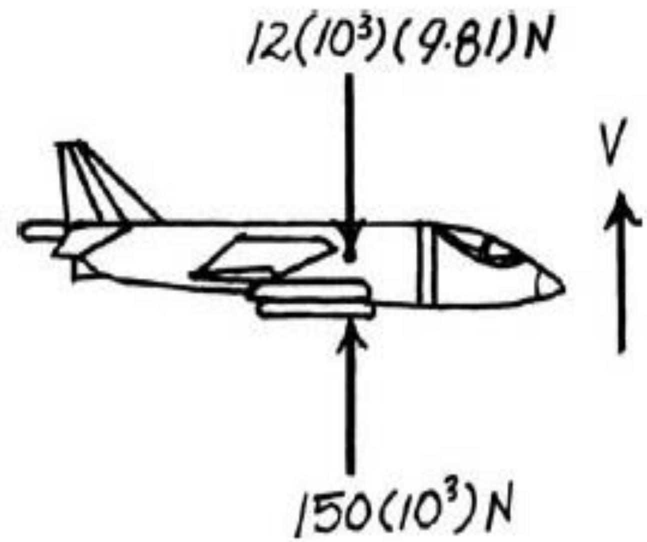
15–2. The 12-Mg "jump jet" is capable of taking off vertically from the deck of a ship. If its jets exert a constant vertical force of 150 kN on the plane, determine its velocity and how high it goes in $t = 6$ s, starting from rest. Neglect the loss of fuel during the lift.



$(+\uparrow) \qquad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$

$$0 + 150(10^3)(6) - 12(10^3)(9.81)(6) = 12(10^3)v$$

$$v = 16.14 \text{ m/s} = 16.1 \text{ m/s}$$ **Ans.**



$(+\uparrow) \qquad v = v_0 + a_c t$

$$16.14 = 0 + a(6)$$

$$a = 2.690 \text{ m/s}^2$$

$(+\uparrow) \qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$s = 0 + 0 + \frac{1}{2}(2.690)(6)^2$$

$$s = 48.4 \text{ m}$$ **Ans.**