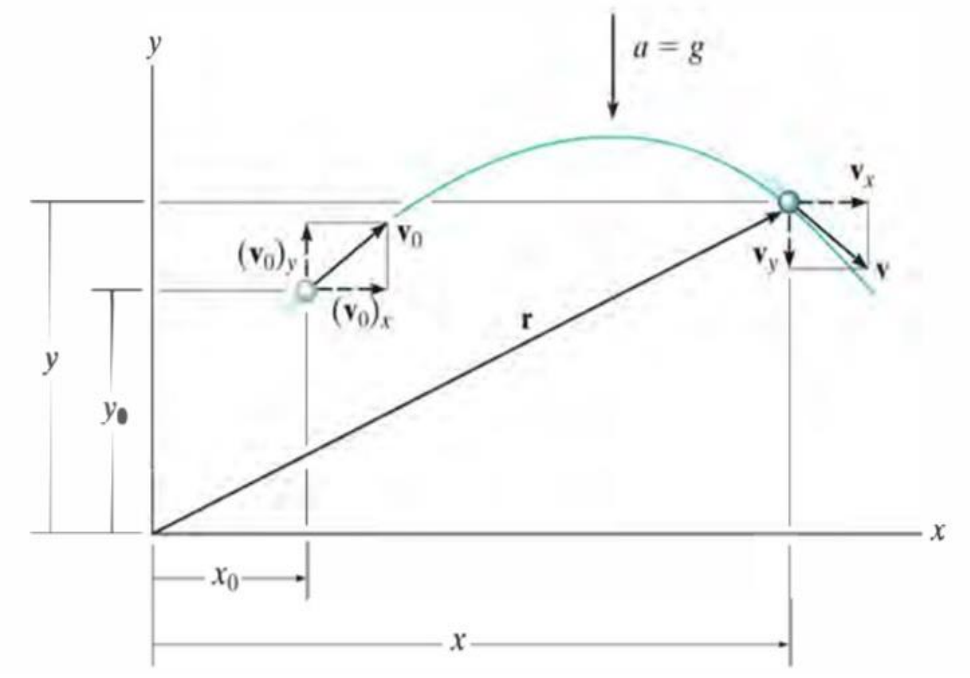
Motion of a Projectile

When air resistance is neglected, the only force acting on the projectile is its weight, which causes the projectile to have a constant downward acceleration of:

$$a_c = g = 9.81\ m/s^2 \quad \text{or} \quad g = 32.2\ ft/s^2$$



In x-direction :

Neglecting air resistance $a_x = 0$

$V_x = \text{constant} = V_{ox} = V_o \cos\alpha$

$X = X_o + V\,t$

$X = V_o \cos\alpha\, t \qquad (\text{ when } x_o = 0)$

In y-direction :

$a_y = -g$

$V = V_o + a\,t$

$y = y_0 + V_o\, t + ½\, a\, t^2$

$V^2 = V_o^2 + 2\,a\,(y - y_o)$

When the $y_o = 0$ then the equation will be:

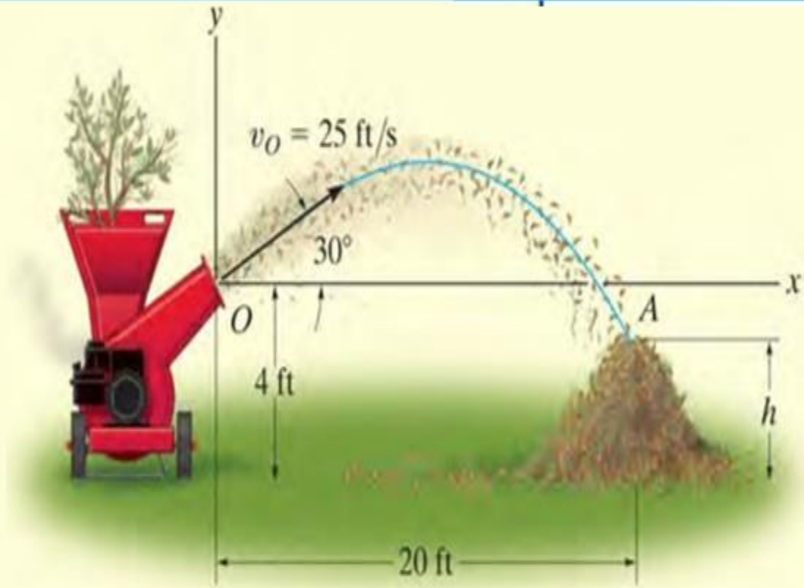
$V_y = V_o \sin\alpha - g\,t$

$y = V_o \sin\alpha\, t - ½\, g\, t^2$

$V_y^2 = (V_o \sin\alpha)^2 - 2\,g\,y$

EXAMPLE

The chipping machine is designed to eject wood chips at $v_O = 25$ ft/s as shown If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 20 ft from the tube.



$X = 20$ ft , $\alpha = 30^o$, $V_o = 25$ ft/s , $a = -32.2$ ft/s^2

$X = V_o \cos\alpha \, t$

$20 = 25 \cos 30 \; t$ then $t = 0.9238$ sec (زمن السقوط)

$y = V_o \sin\alpha \, t - ½ \, g \, t^2$

$= 25 \sin 30 * 0.9238 - ½ * 32.2 * 0.9238^2$

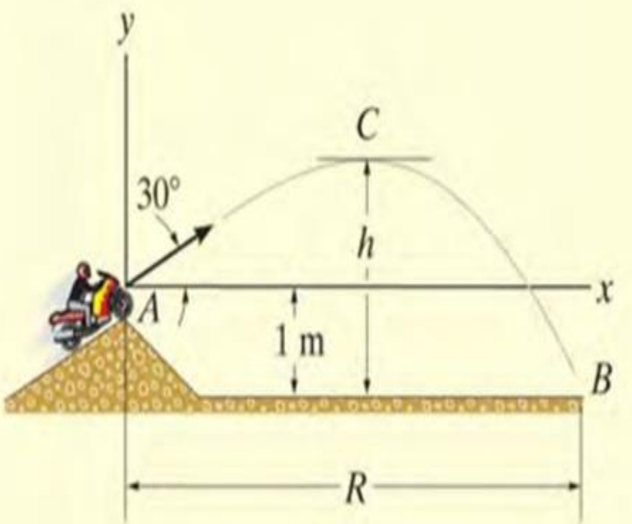
Then $y = -2.19$ ft

$h = 4 - 2.19$

$= 1.81$ ft

EXAMPLE

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider remained in mid air for 1.5 s. Determine
the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.



$\alpha = 30^o$, $y = -1$ m , $t = 1.5$ sec

$x = R = ?$, $h_{max} = ?$, $V_y = ?$

$y = V_o \sin \alpha \, t - ½ \, g \, t^2$

$-1 = V_o \sin 30 * 1.5 - ½ * 9.81 * 1.5^2$

Then $V_o = 13.382$ m/s

$X = V_o \sin \alpha \, t$

$R = 13.382 \cos 30 * 1.5$

Then $R = 17.384$ m

y_{max} at $V_y = 0$

$V_y^2 = (V_o \sin \alpha)^2 - 2 \, g \, y_{max}$

$0 = (13.382 \sin 30)^2 - 2 * 9.81 * y_{max}$

Then $y_{max} = 2.282$ m

$h_{max} = 2.282 + 1$

$= 3.282$ m

Ex: a projectile is shot with an initial velocity of 800 ft/s at a target B located 2000 ft above the gun A and at a horizontal distance 12000 ft . Neglect air resistance, determine firing angle α :

$X = 800 \cos\alpha\, t$

$12000 = 800 \cos\alpha\, t$

$t = 15/\cos\alpha$ ------(1)

$y = 800 \sin\alpha\, t - ½ * 32.2\, t^2$

at point B $2000 = 800 \sin\alpha\, t - 16.1\, t^2$ -----(2)

equ. (1) in (2)

$2000 = 800 \sin\alpha\, (15/\cos\alpha) - 16.1\, (15/\cos\alpha)^2$

$2000 = 800 * 15 \tan\alpha - 16.1 * 15^2 . \sec^2\alpha$

$2000 = 800 * 15 \tan\alpha - 16.1 * 15^2 . (1 + \tan^2\alpha)$

$3622 * \tan^2\alpha - 12000 \tan\alpha + 5622 = 0$

$$\tan\propto = \frac{12000 \mp \sqrt{12000^2 - 4 * 5622 * 3622}}{2 * 3622}$$

$\tan\alpha = 1.565$ then $\alpha = 29.5^o$

Or

$\tan\alpha = 2.75$ then $\alpha = 70^o$