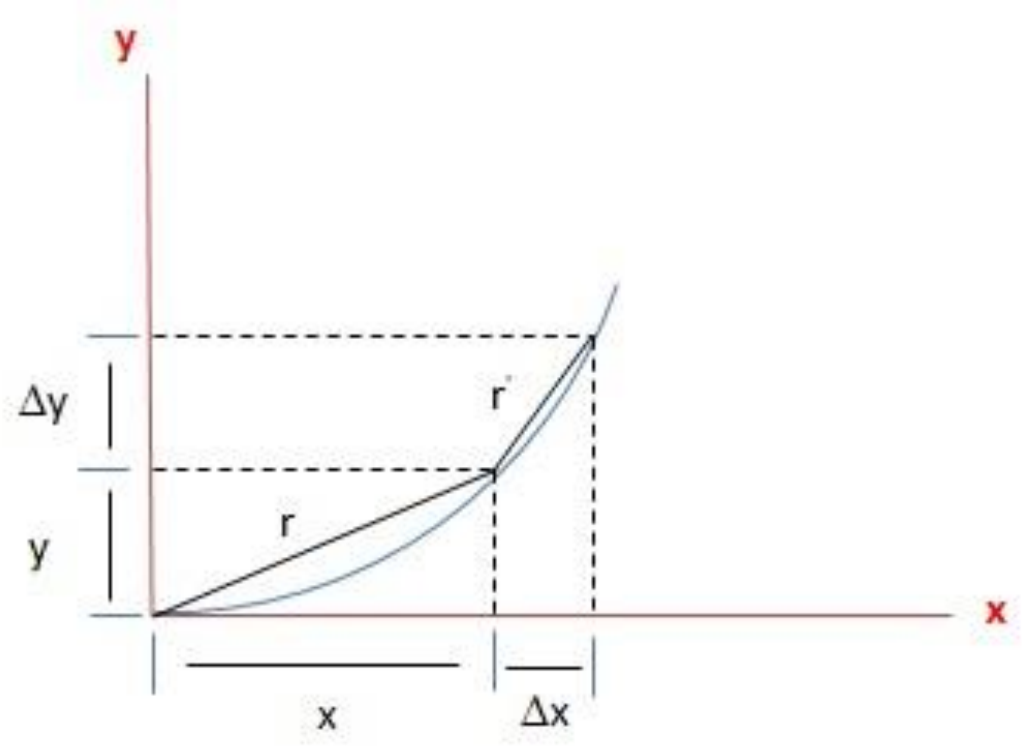
Curvilinear Motion

The motion a long a path other than a straight line is called a curvilinear motion



Displacement :

In x-direction : x = f(t)

In y-direction : y = g(t)

$r^2 = x^2 + y^2$

$\theta = \tan^{-1} y/x$

$r^>$ displacement from o is called the position vector.

Velocity :

In x-direction

$$v_x = \frac{dx}{dt} = x'$$

In y-direction

$$v_y = \frac{dy}{dt} = y'$$

$$v^{>} = v_x^{>} + v_y^{>} \quad ; \qquad v = \sqrt{v_x^2 + v_y^2}$$

$$\theta = tan^{-1}\frac{v_y}{v_x} = tan^{-1}\frac{dy}{dx}$$

Acceleration :

$$a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = x''$$

$$a_y = \frac{dv_y}{dt} = \frac{d^2y}{dt^2} = y''$$

$$a^{>} = a_x^{>} + a_y^{>} \quad ; \qquad a = \sqrt{a_x^2 + a_y^2}$$

$$\theta = tan^{-1}\frac{d^2y}{d^2x}$$

Ex: A particle is moving along a curve such that $x=(t+1)^2$ and $y=(t+1)^{-2}$ (x and y in m, t in sec). Find v and a at t = 1 sec.

$V_x = dx/dt = x' = 2(t+1)$

Then $a_x = 2$

$V_y = dy/dt = y' = -2\ (t+1)^{-3}$

Then $a_y = 6\ (t+1)^{-4}$

at t = 1 sec

$v_x = 2(1+1) = 4$ m/s

$v_y = -2\ (1+1)^{-3} = -1/4$ m/s

$v = \sqrt{4^2 + (\frac{-1}{4})^2}$, v = 4 m/s

$\Theta = \tan^{-1} dy/dx = \tan^{-1} 1/16 = 3.75^o$

$a_x = 2$ m/s^2

$a_y = 6\ (1+1)^{-4} = 3/8$ m/s^2

$a = \sqrt{2^2 + (\frac{3}{8})^2}$, v = 2 m/ s^2

$\Theta = \tan^{-1} d^2 y/d^2 x = \tan^{-1} 3/16 = 10.62^o$

Ex: If $x = 5t^3$ and $y = 4t^2$ at time t, find the magnitude and direction of the velocity when $t = 10$.

$x = 5t^3$ So $dx/dt = 15t^2$

At $t = 10$, the velocity in the x-direction is given by:

$dx/dt = v_x = 15(10)^2 = 1500$ m/s

Also, $y = 4t^2$ so the velocity in the y-direction is:

$dy/dt = 8t$

When $t = 10$, the velocity in the y-direction is:

$dy/dt = v_y = 8(10) = 80$ m/s

So the magnitude of the velocity will be:

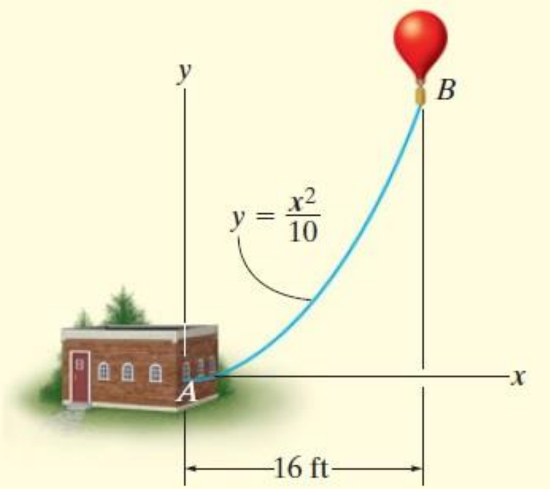
$v = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{1500^2 + 80^2} = 1502.1$ m/s

Now for the direction of the velocity (it is an angle, relative to the positive x-axis):

$\tan \theta_v = v_y / v_x$ So $\theta_v = 3.05^\circ$.

EXAMPLE

At any instant the horizontal position of the weather balloon is defined by $x = (8t)$ ft, where t is in seconds. If the equation of the path is $y = x^2/10$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.

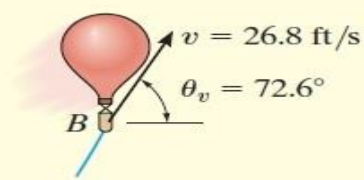


SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(8t) = 8 \text{ ft/s} \rightarrow$$

$$v_y = \dot{y} = \frac{d}{dt}(x^2/10) = 2x\dot{x}/10 = 2(16)(8)/10 = 25.6 \text{ ft/s} \uparrow$$



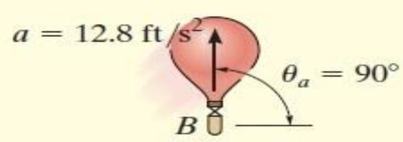
(b)

When $t = 2$ s

$$v = \sqrt{(8 \text{ ft/s})^2 + (25.6 \text{ ft/s})^2} = 26.8 \text{ ft/s} \qquad \textit{Ans.}$$

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{25.6}{8} = 72.6° \qquad \textit{Ans.}$$

Acceleration.



(c)

$$a_x = \dot{v}_x = \frac{d}{dt}(8) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/10) = 2(\dot{x})\dot{x}/10 + 2x(\ddot{x})/10$$

$$= 2(8)^2/10 + 2(16)(0)/10 = 12.8 \text{ ft/s}^2 \uparrow$$

$$a = \sqrt{(0)^2 + (12.8)^2} = 12.8 \text{ ft/s}^2 \qquad \textit{Ans.}$$

$$\theta_a = \tan^{-1}\frac{12.8}{0} = 90° \qquad \textit{Ans.}$$

EXAMPLE

For a short time, the path of the plane is described $y = (0.001x^2)$ m. If the plane is rising with a constant velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it is at $y = 100$ m.

SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$

$$x = 316.2 \text{ m.}$$

$$v_y = 10 \text{ m/s,}$$

$$y = v_y t; \quad \text{then} \quad 100 \text{ m} = (10 \text{ m/s})\, t \quad \text{we have} \quad t = 10 \text{ s}$$

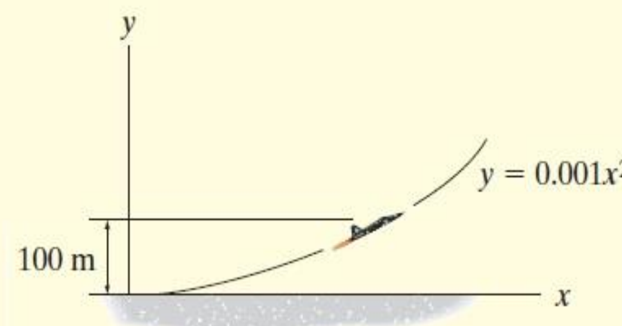
Velocity.

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x$$

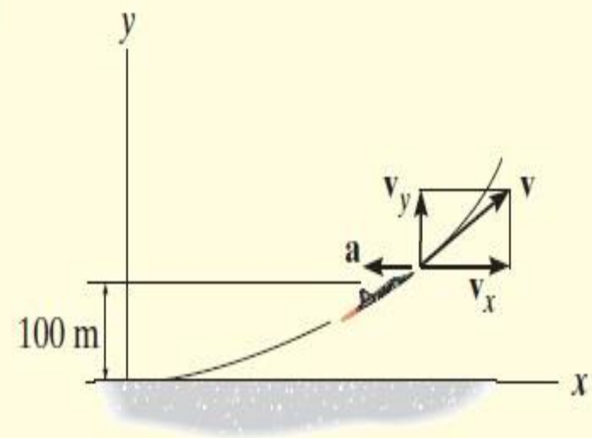
Thus

$$10 \text{ m/s} = 0.002(316.2 \text{ m})(v_x)$$

$$v_x = 15.81 \text{ m/s}$$



$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81\text{ m/s})^2 + (10\text{ m/s})^2} = 18.7\text{ m/s} \quad \textit{An}$$



(b)

Acceleration.

$$a_y = \dot{v}_y = 0.002\dot{x}v_x + 0.002x\dot{v}_x = 0.002(v_x^2 + xa_x)$$

When $x = 316.2\text{ m}$, $v_x = 15.81\text{ m/s}$, $\dot{v}_y = a_y = 0$,

$$0 = 0.002((15.81\text{ m/s})^2 + 316.2\text{ m}(a_x))$$
$$a_x = -0.791\text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791\text{ m/s}^2)^2 + (0\text{ m/s}^2)^2}$$
$$= 0.791\text{ m/s}^2 \quad \textit{Ans.}$$