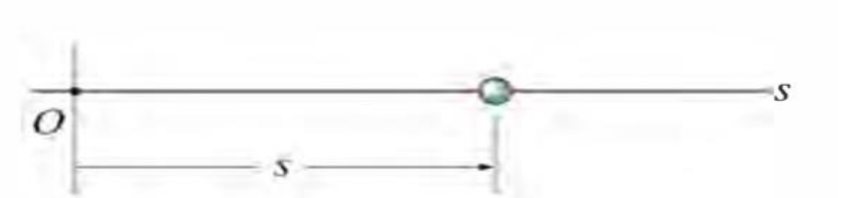
Dynamics

Dynamics

- Kinematics: motion as displacement, velocity and acceleration
- Kinetics: force and motion $F = m \cdot a$

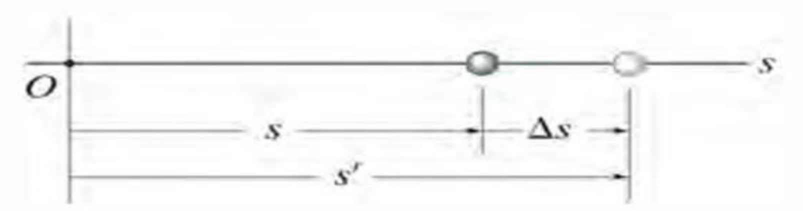
Rectilinear Kinematics

1- Position



Position

2- Displacement [m , ft]



Displacement

$$\Delta s = \acute{s} - s$$

3- Velocity [m/s , ft/s]

The moves of the particles through a displacement Δs during the time interval Δt .

$$v = \frac{ds}{dt}$$

4- Acceleration [m/s^2 , ft/s^2]

The velocity of the particle at two points.

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Ex: The car moves in a straight line such that for a short time its velocity is defined by $v = (3t^2 + 2t)$ ft/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (3t^2 + 2t)$$

$$\int_0^s ds = \int_0^t (3t^2 + 2t)\,dt$$

$$s\Big|_0^s = t^3 + t^2\Big|_0^t$$

$$s = t^3 + t^2$$

When $t = 3$ s,

$$s = (3)^3 + (3)^2 = 36 \text{ ft} \qquad \textit{Ans.}$$

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(3t^2 + 2t)$$

$$= 6t + 2$$

When $t = 3$ s,

$$a = 6(3) + 2 = 20 \text{ ft/s}^2 \rightarrow$$ *Ans.*

Freely Falling Bodies

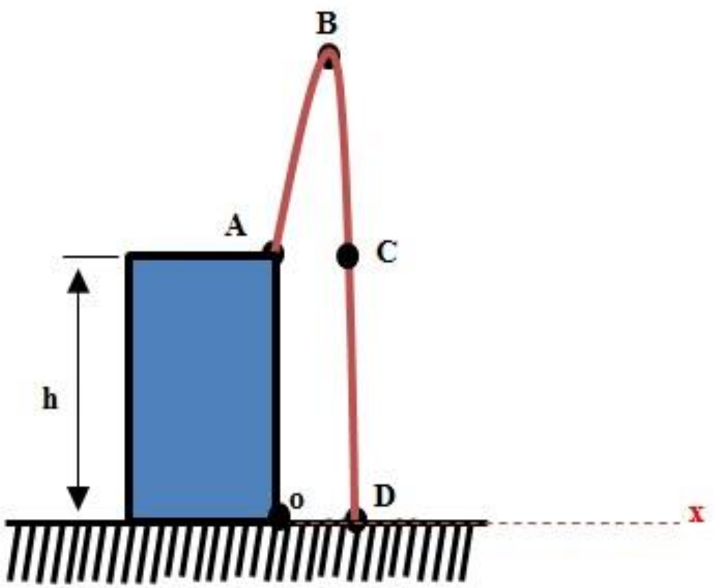
$g = 32.2\ ft/s^2$

$g = 9.81\ m/s^2$

$y_A = h$

$y_D = 0$

$y_c = h$

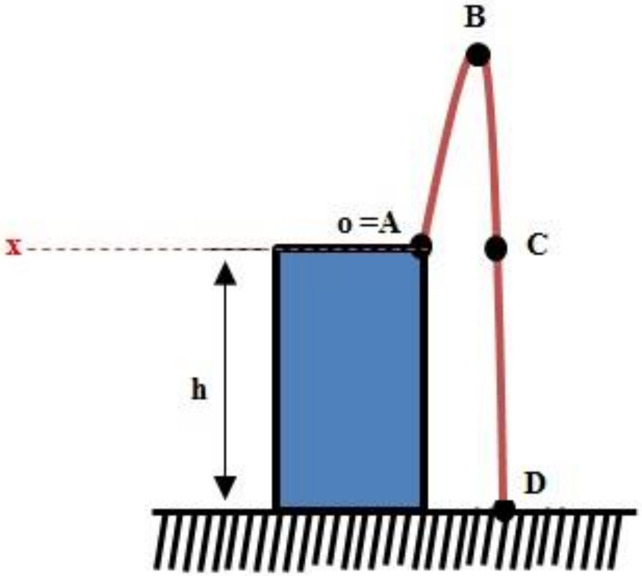


$a = g$ Or $a = -g$

$y_o = y_A = 0$

$y_D = -h$

$y_c = 0$



Ex: a ball is thrown vertically up word from the top of 18 m tower with an initial velocity of 12 m/s . Find?

1) The velocity and displacement at any time.
2) The highest elevation
3) The time when the ball reached the ground and the velocity at that time.

1)

dv/dt = - 9.81

$$\int_{12}^{v} dv = \int_{0}^{t} - 9.81\, dt$$

v = - 9.81

v – 12 = - 9.81 t

v = 12 – 9.81 t (Instantaneous velocity)

dy/ dt = 12 – 9.81 t

$$\int_{y_o=0}^{y} dy = \int_{0}^{t} (12 - 9.81t)dt$$

$y = 12\, t - \frac{9.81}{2}\, t^2$ (Instantaneous displacement)

2) At highest elevation

v = 0

so 0 = 12-9.81 t then t = 1.22 sec

$y_{at\ 1.22} = 12 * 1.22 - (9.81/2) * (1.22)^2$

$= 7.3$ m

y = 7.3 m from the top

y = 7.3 + 18 = 25.3 m from the ground

3) when the ball hits the ground y = - 18

$-18 = 12\,t - 4.905\,t^2$

$4.905\,t^2 - 12\,t - 18 = 0$

$$t = \frac{12 \mp \sqrt{12^2 + 4 * 18 * 4.905}}{2 * 4.905}$$

Then t = - 10.5 sec neglected

or t = 3.5 sec then $v_{at\ 3.5} = 12 - 9.81 * 3.5 = -22.3$ m/s

Types of problems involving motion

1. x= f(t)

2. a= f(t)

$$\frac{dv}{dt} = f(t)$$

$$\int dv = \int f(t)\ dt$$

v = g(t) + c

c is found from the initial condition at t = 0 and v = v_o

$\frac{dx}{dt}$ = g (t) + c

$$\int dx = \int (g(t) + c)dt$$

Or $\int_0^v dv = \int_0^t f(t)dt$

Or $\int_{v_o}^v dv = \int_{t_o}^t f(t)dt$

3. a = f(v)

$$\frac{dv}{dt} = f(v)$$

$\int_{v_o}^{v} \frac{dv}{f(v)} = \int_{t_o}^{t} dt$ or $a = v\,\frac{dv}{dx}$

$$v\,\frac{dv}{dx} = f(v)$$

$$\int_{v_o}^{v} v\,\frac{dv}{f(v)} = \int_{x_o}^{x} dx$$

4. a= f(x)

$$v\,\frac{dv}{dx} = f(x)$$

$$\int_{v_o}^{v} v\,dv = \int_{x_o}^{x} f(x)dx$$

v = g(x)

EXAMPLE

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)\ \text{m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

Velocity.

$a = f(v)$ with $v_0 = 60\ \text{m/s}$ when $t = 0$

$(+\downarrow)$
$$a = \frac{dv}{dt} = -0.4v^3$$
$$\int_{60\ \text{m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$
$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\Bigg|_{60}^{v} = t - 0$$
$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$
$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}\ \text{m/s}$$

When $t = 4$ s, $v = 0.559\ \text{m/s}\downarrow$ *Ans.*

Position.

from $v = ds/dt$ condition $s = 0$, when $t = 0$,

$(+\downarrow)$

$$v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\Bigg|_0^t$$

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\} \text{ m}$$

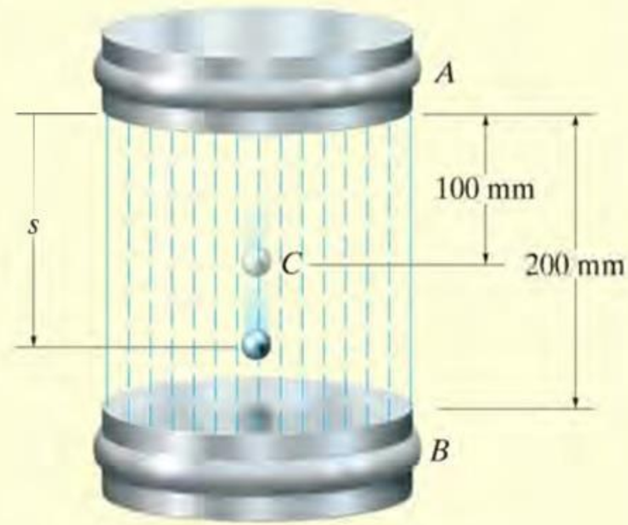
When $t = 4$ s,

$$s = 4.43 \text{ m}$$

Ans.

EXAMPLE

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B, Fig. 12–5. If the particle is released from rest at the midpoint C, $s = 100$ mm, and the acceleration is $a = (4s)$ m/s^2, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200$ mm, and the time it takes to travel from C to B.



SOLUTION

Velocity.

Since $a = f(s)$,

$v\,dv = a\,ds$. using $v = 0$ at $s = 0.1$ m

$$(+\downarrow) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_{0.1\text{ m}}^{s} 4s\,ds$$

$$\frac{1}{2}v^2\Big|_0^v = \frac{4}{2}s^2\Big|_{0.1\text{ m}}^{s}$$

$$v = 2(s^2 - 0.01)^{1/2}\text{ m/s}$$

At $s = 200$ mm $= 0.2$ m,

$$v_B = 0.346\text{ m/s} = 346\text{ mm/s}\downarrow \qquad \textit{Ans.}$$

Time. The time for the particle to travel from C to B can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1$ m when $t = 0$. From Appendix A,

$$(+\downarrow) \qquad ds = v\,dt$$

$$= 2(s^2 - 0.01)^{1/2}dt$$

$$\int_{0.1}^{s} \frac{ds}{(s^2 - 0.01)^{1/2}} = \int_0^t 2\,dt$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right)\Big|_{0.1}^{s} = 2t\Big|_0^t$$

$$\ln\left(\sqrt{s^2 - 0.01} + s\right) + 2.303 = 2t$$

At $s = 0.2$ m,

$$t = \frac{\ln\left(\sqrt{(0.2)^2 - 0.01} + 0.2\right) + 2.303}{2} = 0.658 \text{ s} \qquad \textit{Ans.}$$

Special Cases of Motion

1) Uniform motion

v = constant , a = 0

$$\frac{dx}{dv} = v = b$$

$$\int_{x_o}^{x} dx = \int_{0}^{t} b\ dt$$

$x - x_o = b\ t$

$\mathbf{x = x_o + bt}$

2) Uniformly accelerated motion

a = constant

$$\frac{dv}{dt} = a$$

$$\int_{v_o}^{v} dv = \int a\,dt = a \int_0^t dt$$

$v - v_o = a\,t$

$v = v_o + a\,t$

$$\frac{dx}{dt} = v_o + a\,t$$

$$\int_{x_o}^{x} dx = \int_0^t (v_o + a\,t)dt$$

$$x - x_o = v_o\,t + a\,\frac{t^2}{2}$$

$$x = x_o + v_o\,t + \frac{1}{2}\,a\,t^2$$

a = constant

$$v\,\frac{dv}{dx} = a$$

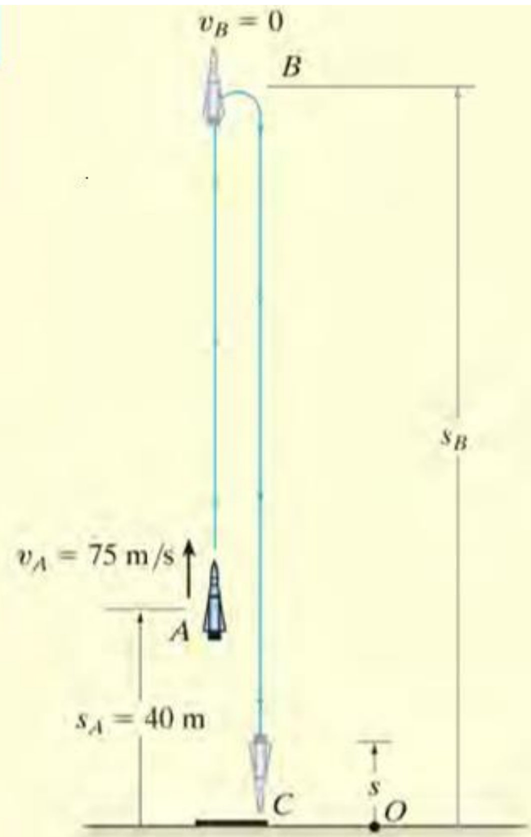
$$\int_{v_o}^{v} v\,dv = \int_{x_o}^{x} a\,dx$$

$$\frac{v^2}{2} = a\,(x - x_o)$$

$$v^2 = v_o^2 + 2\,a\,(x - x_o)$$

EXAMPLE

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.



SOLUTION

Maximum Height.

$v_B = 0.$ the maximum height $s = s_B$

$v_A = +75\text{m/s}$ when $t = 0$. At

Since a_c is *constant* $a_c = -9.81 \text{ m/s}^2$

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$

$$s_B = 327 \text{ m}$$ *Ans.*

Velocity. To obtain the velocity of the rocket just before it hits the ground,

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$

$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow$$ *Ans.*

The negative root was chosen since the rocket is moving downward. Similarly, Eq. 12–6 may also be applied between points A and C,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$

$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$

$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow$$ *Ans.*

H. W. :

Chapter 12: 1, 4, 5, 7, 9, 11, 28, 29, 40