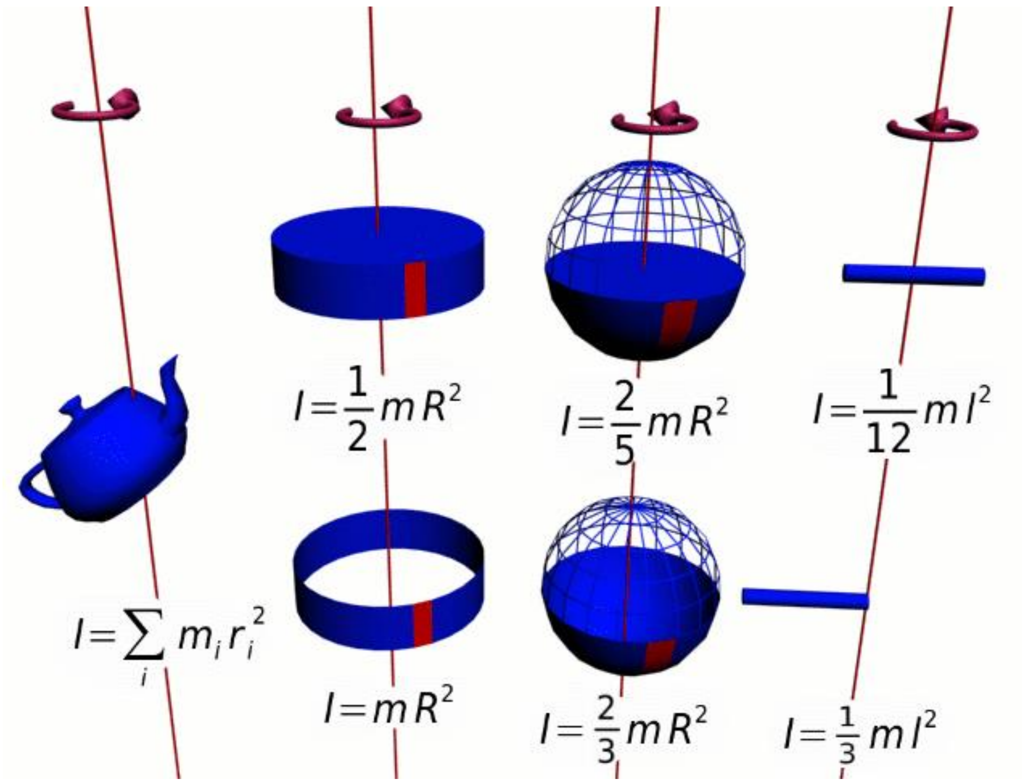
Moment of Inertia

It is a measure of an object’s resistance to changes to its rotation.

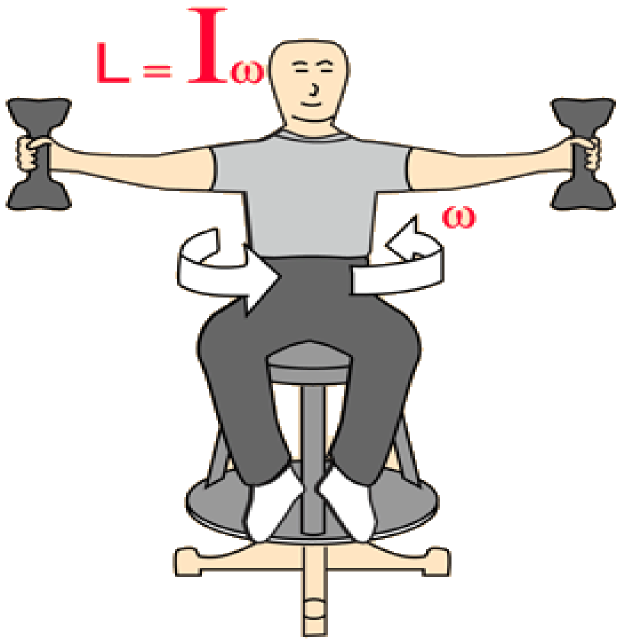
- Also defined as the capacity of a cross-section to resist bending.
- It must be specified with respect to a chosen axis of rotation.
- It is usually quantified in m^4 or $kg.m^2$

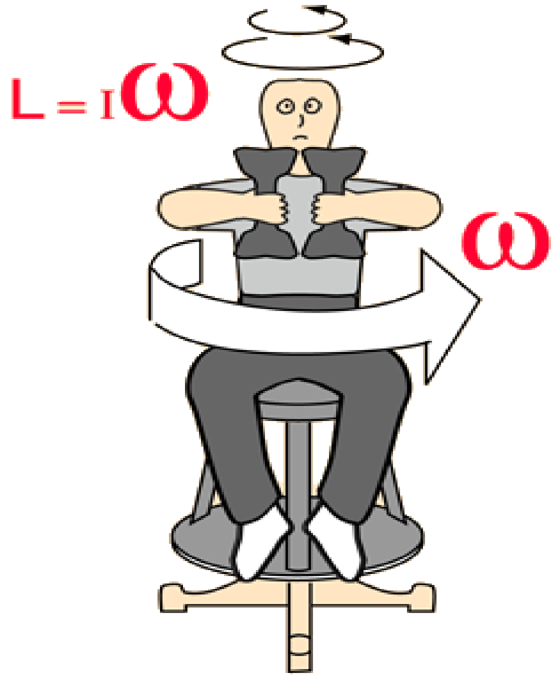


عزم القصور الذاتي :

هو مقياس مقاومة الجسم للتغيرات في معدل دورانه

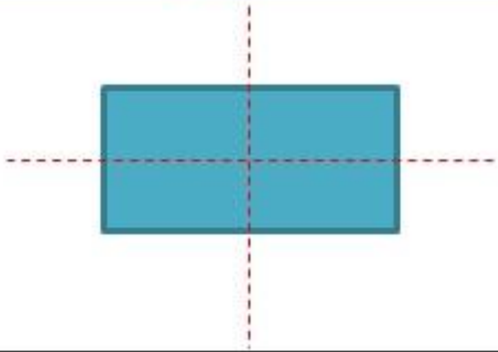
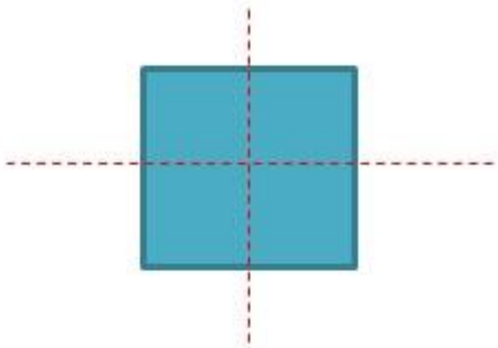
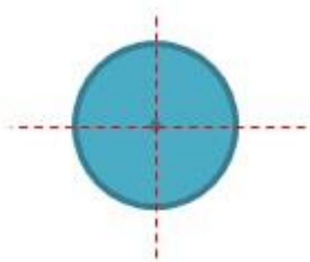
Rotating Stool

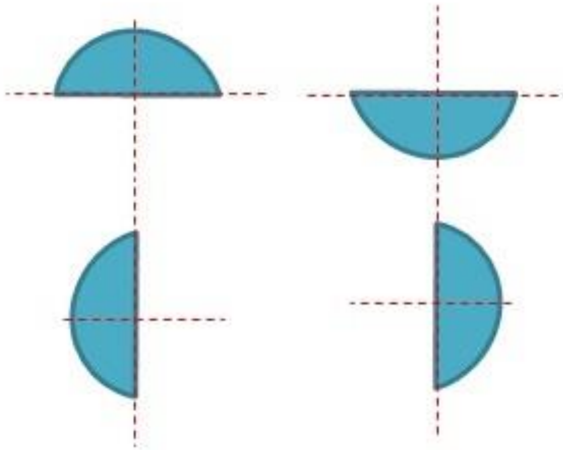
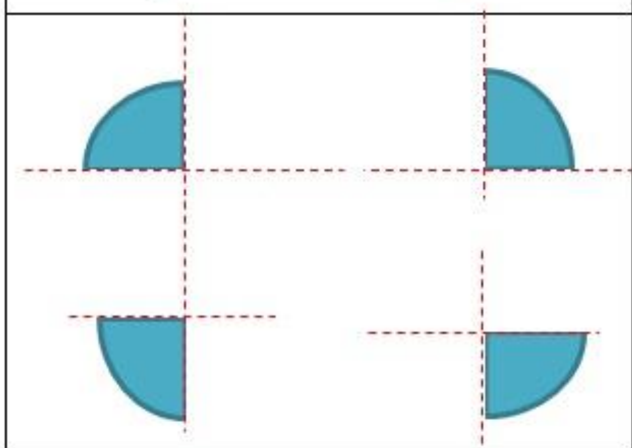
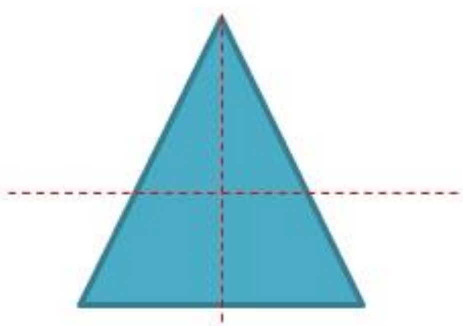




The moment of inertia is large with the masses held out. For a given angular momentum, the angular velocity is relatively low.	If the masses are pulled in, the moment of inertia is considerably decreased. Conservation of angular momentum dictates that the angular velocity must increase.

* Moment of Inertia always positive

Figure	Moment of Inertia
	$\bar{I}_x = \dfrac{b\,h^3}{12}$ $\bar{I}_y = \dfrac{h\,b^3}{12}$
	$\bar{I}_x = \bar{I}_y = \dfrac{b\,h^3}{12}$
	$\bar{I}_x = \bar{I}_y = \dfrac{\pi\,r^4}{4}$ $\bar{J}_o = \dfrac{\pi\,r^4}{2}$

	$\bar{I}_x = \bar{I}_y = \dfrac{\pi r^4}{8}$
	$\bar{I}_x = \bar{I}_y = \dfrac{\pi r^4}{16}$
	$\bar{I}_x = \dfrac{b h^3}{12}$ $\bar{I}_y = \dfrac{h b^3}{12}$

Parallel Axis Theorem

The moment of area of an object about any axis parallel to the centroidal axis is the sum of MI about its centroidal axis and the product of area with the square of distance of from the reference axis.

$$I = \overline{I} + A\,d^2$$

d: is the perpendicular distance between the centroidal axis and the parallel axis.

Moment of Inertia for Composite Figures

$$I_x = I_{x1} + I_{x2} + I_{x3} + -----$$

$$I_y = I_{y1} + I_{y2} + I_{y3} + -----$$

Ex: Find the moment of inertia for this figure

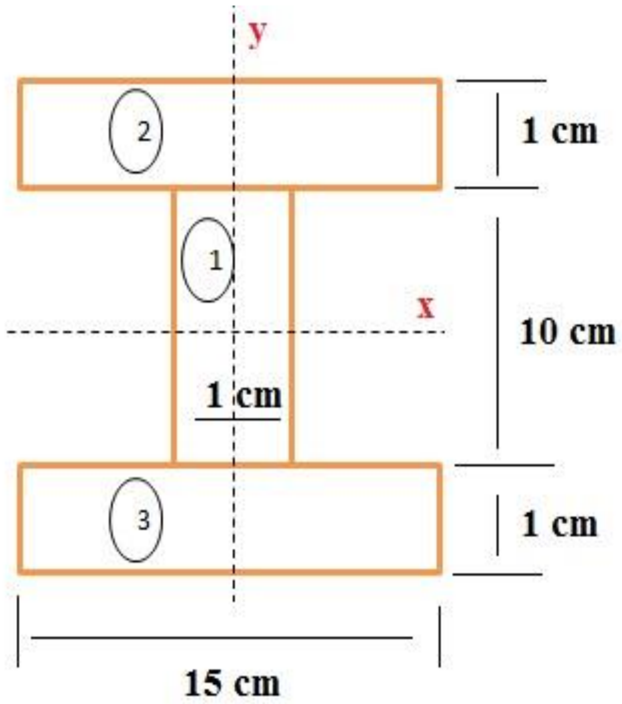


Figure 1

$$I_x = \overline{I}_x + A\,d^2 = \frac{b\,h^3}{12} + (bh) * (0)^2 = \frac{1 * 10^3}{12}$$

$$= 83.334\ \text{cm}^4$$

$$I_y = \overline{I}_y + A\,d^2 = \frac{h\,b^3}{12} + (bh) * (0)^2 = \frac{10 * 1^3}{12}$$

$$= 0.8334\ \text{cm}^4$$

Figure 2

$$I_x = \bar{I}_x + A\,d^2 = \frac{b\,h^3}{12} + (bh) * (5.5)^2 = \frac{15 * 1^3}{12} + (15 * 1) * (5.5)^2$$

$= 455\ \text{cm}^4$

$$I_y = \bar{I}_y + A\,d^2 = \frac{h\,b^3}{12} + (bh) * (0)^2 = \frac{1 * 15^3}{12}$$

$= 281.25$

Figure 3

Is similar to figure 2

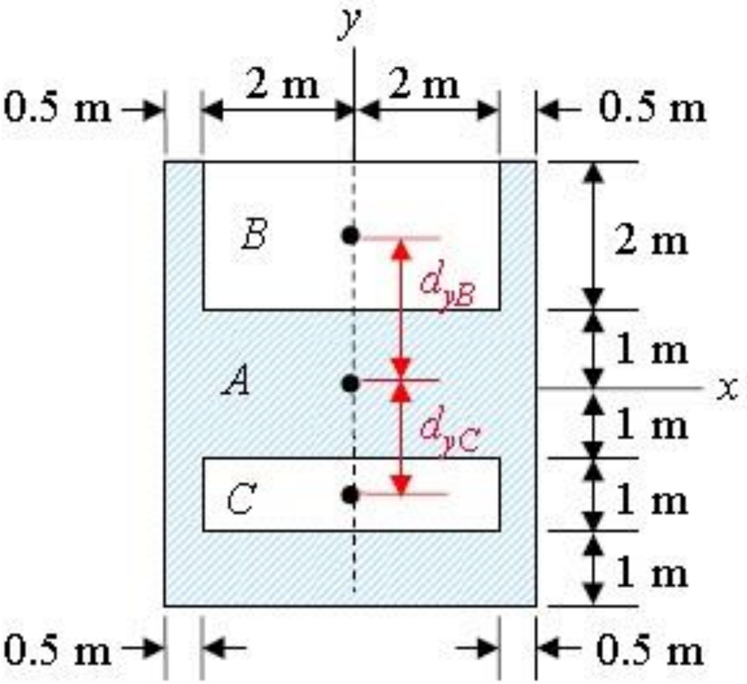
$I_x = 455$

$I_y = 281.25$

For the composite figure

$$I_x = I_{x1} + I_{x2} + I_{x3} = 83.33 + 2 * 455 = 993.3\ cm^4$$

$$I_y = I_{y1} + I_{y2} + I_{y3} = 0.833 + 2 * 281.25 = 563.3\ cm^4$$



$$I_x = (\bar{I}_x + A\cancelto{0}{d_y}^2)_{A5\times6} - (\bar{I}_x + Ad_y^{\,2})_{B4\times2} - (\bar{I}_x + Ad_y^{\,2})_{C4\times1}$$

$$= [\frac{1}{12}(5)(6)^3 + 0]_A - [\frac{1}{12}(4)(2)^3 + (2\times4)(2)^2]_B$$

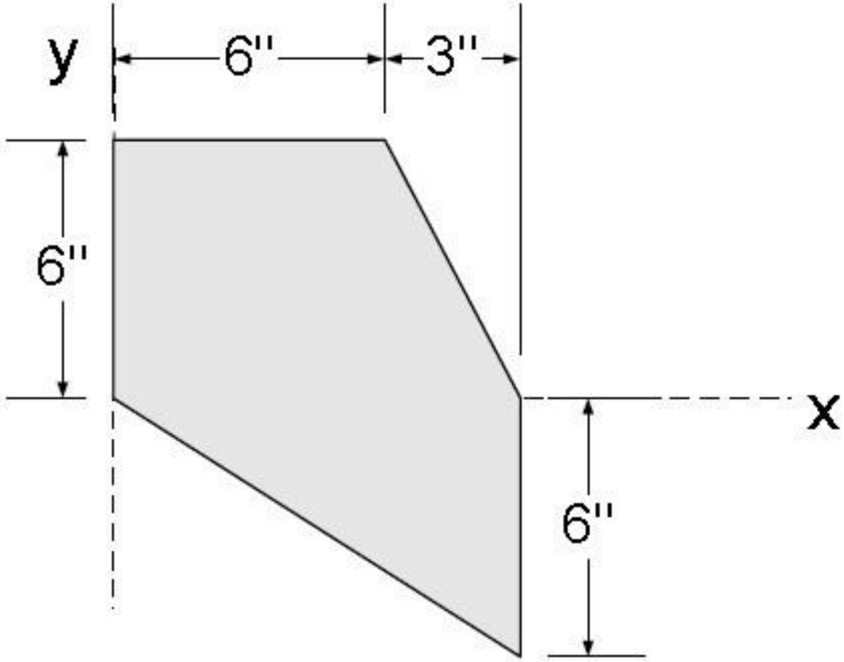
$$-[\frac{1}{12}(4)(1)^3 + (4\times1)(1.5)^2]_C$$

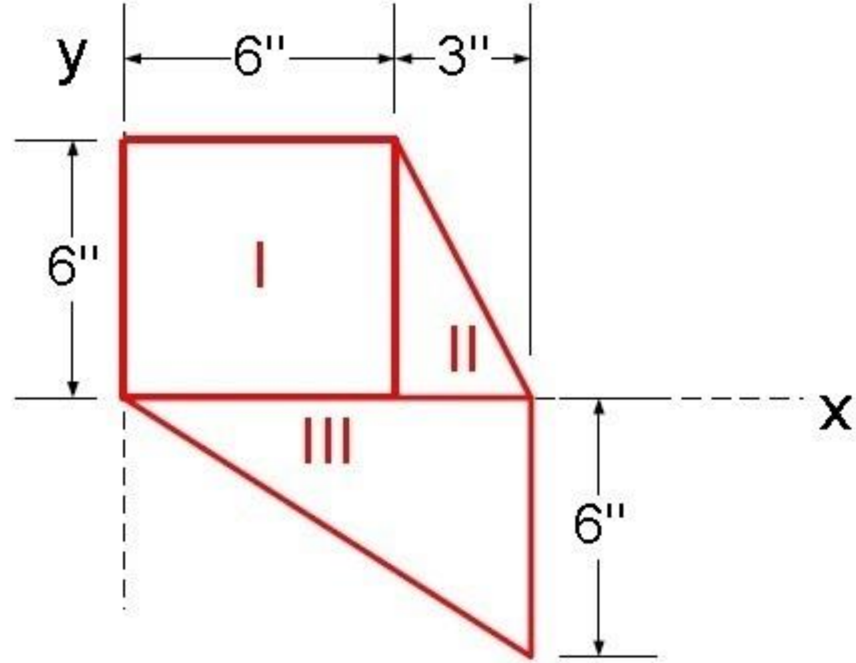
$$I_x = 46\ \text{m}^4$$

$$I_y = (\bar{I}_y + A\cancelto{0}{d_x}^2)_A - (\bar{I}_y + A\cancelto{0}{d_x}^2)_B - (\bar{I}_y + A\cancelto{0}{d_x}^2)_C$$

$$= [\frac{1}{12}(6)(5)^3]_A - [\frac{1}{12}(2)(4)^3]_B - [\frac{1}{12}(1)(4)^3]_C$$

$$I_y = 46.5\ \text{m}^4$$





ID	Area (in^2)	d_x (in)	d_y (in)
I	36	3	3
II	9	7	2
III	27	6	2

$$\bar{I}_x = \frac{b h^3}{12}$$ rectangle

$$\bar{I}_x = \frac{b h^3}{36}$$ triangle

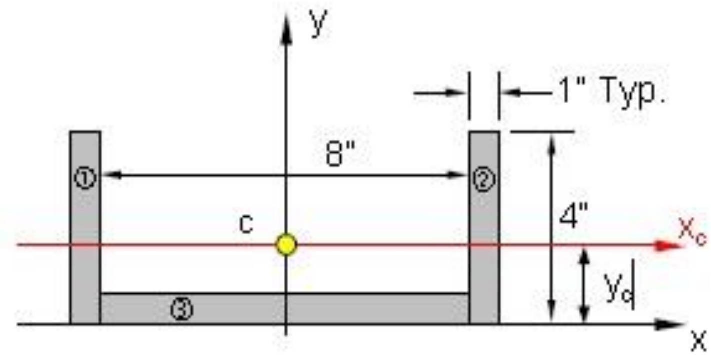
Sub-Area	Area (in^2)	d_x (in)	I_{ybar} (in^4)	$A(d_x)^2$ (in^4)	$I_{ybar} + A(d_x)^2$ (in^4)
I	36	3	108	324	432
II	9	7	4.5	441	445.5
III	27	6	121.5	972	1093.5
					1971

Sub-Area	Area (in^2)	d_y (in)	I_{xbar} (in^4)	$A(d_y)^2$ (in^4)	$I_{xbar} + A(d_y)^2$ (in^4)
I	36	3	108	324	432
II	9	2	18	36	54
III	27	-2	54	108	162
					648

Example :

Determine the location of the centroid ('c') of the beam's cross section and the moment of inertia about the centroidal x-axis.

Part	Dimensions	Area	y	yA
1	4 x 1	4	2	8
2	4 x 1	4	2	8
3	1 x 8	8	0.5	4
Total	---------	**16 in^2**	----	**20 in^3**



$$\bar{y}_c = \sum \frac{\bar{y} \cdot A}{A} = \frac{20}{16} = 1.25 \text{ in}$$

Part	Area	I_{ci}	dy	$d^2y(A)$	$I_{xci} = I_{ci} + d^2y(A)$
1	4	$1 \cdot (4)^3/12$	2 - 1.25	2.25	5.33 + 2.25 = 7.58
2	4	$1 \cdot (4)^3/12$	2 - 1.25	2.25	5.33 + 2.25 = 7.58
3	8	$8 \cdot (1)^3/12$	1.25 - 0.5	4.50	0.666 + 4.50 = 5.16
Total	**16 in^2**	----	----	----	**20.32 in^4**

- H. W.
- Appendix A page (462)

- 35 , 44 , 51 , 54