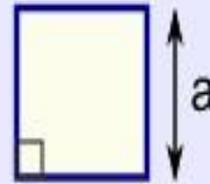
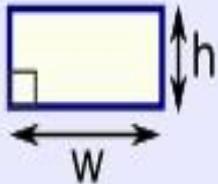


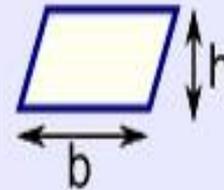
Triangle
Area = $\frac{1}{2} \times b \times h$
b = base
h = vertical height



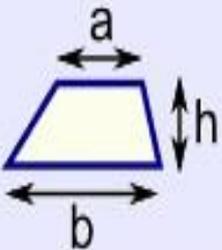
Square
Area = a^2
a = length of side



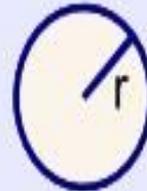
Rectangle
Area = $w \times h$
w = width
h = height



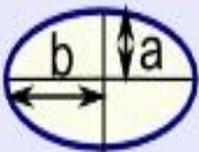
Parallelogram
Area = $b \times h$
b = base
h = vertical height



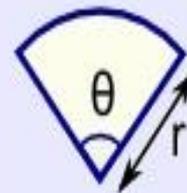
Trapezoid (US)
Trapezium (UK)
Area = $\frac{1}{2}(a+b) \times h$
h = vertical height



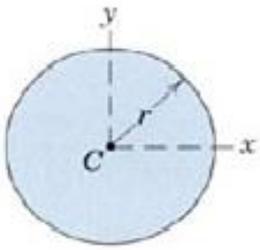
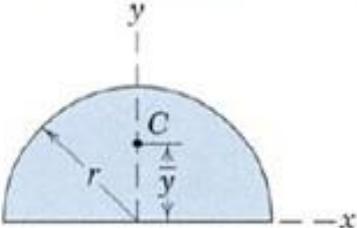
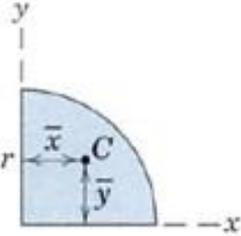
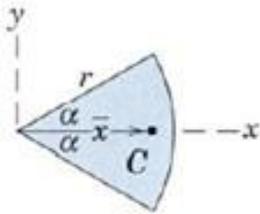
Circle
Area = $\pi \times r^2$
Circumference = $2 \times \pi \times r$
r = radius



Ellipse
Area = πab



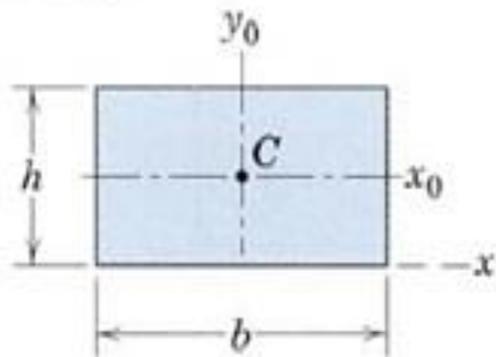
Sector
Area = $\frac{1}{2} \times r^2 \times \theta$
r = radius
 θ = angle in **radians**

FIGURE	CENTROID
<p data-bbox="154 299 347 328">Circular Area</p> 	<p data-bbox="792 278 830 299">—</p>
<p data-bbox="96 599 289 664">Semicircular Area</p> 	$\bar{y} = \frac{4r}{3\pi}$
<p data-bbox="115 928 367 992">Quarter-Circular Area</p> 	$\bar{x} = \bar{y} = \frac{4r}{3\pi}$
<p data-bbox="135 1256 367 1320">Area of Circular Sector</p> 	$\bar{x} = \frac{2}{3} \frac{r \sin \alpha}{\alpha}$

FIGURE

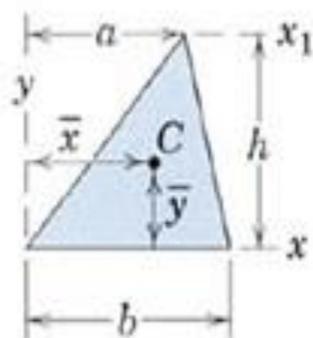
CENTROID

Rectangular Area



—

Triangular Area



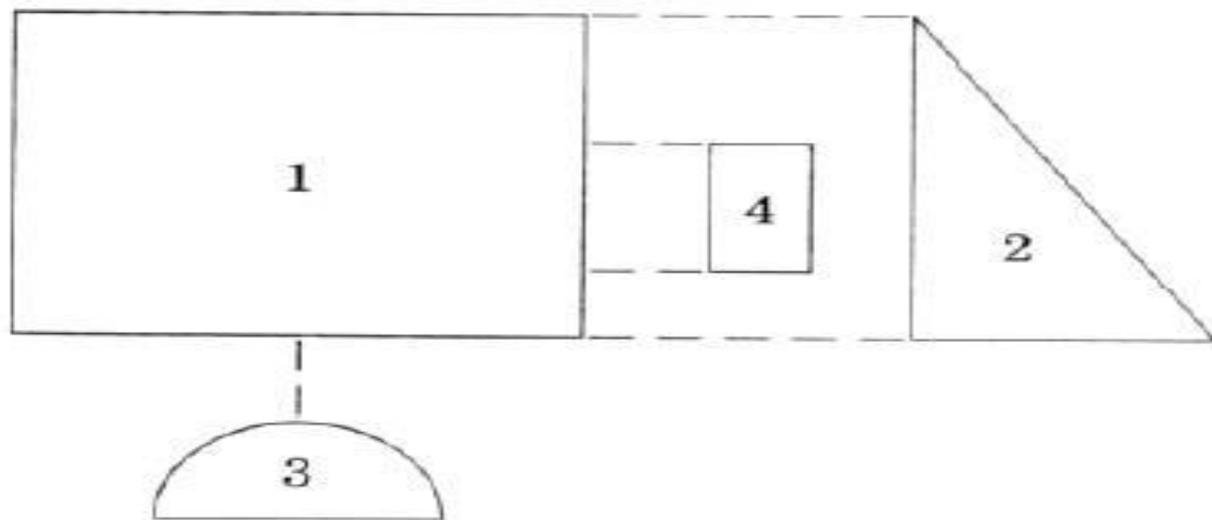
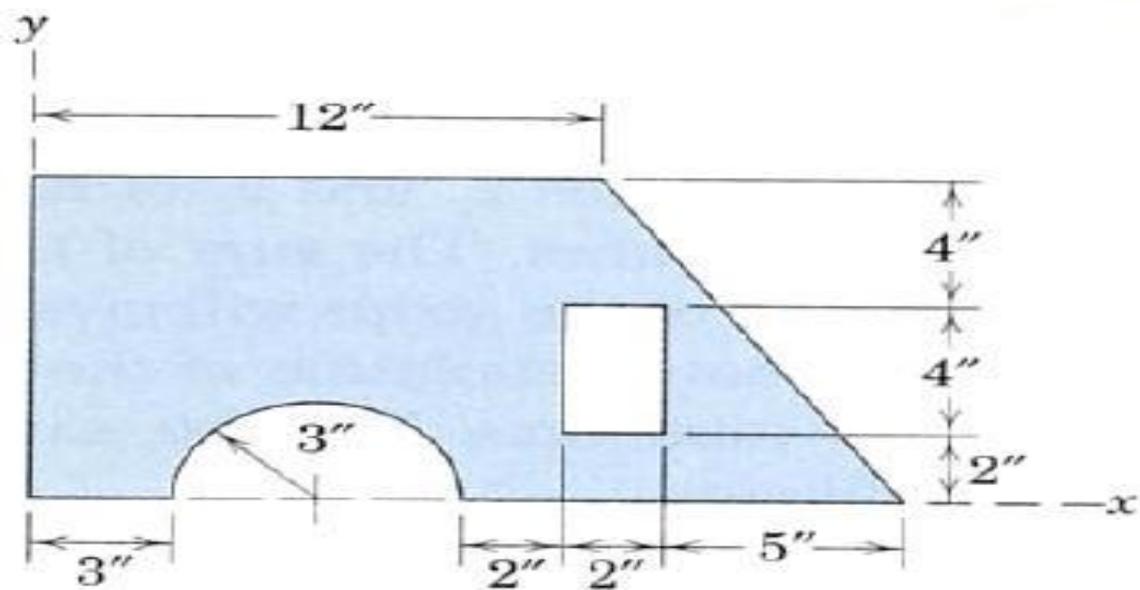
$$\bar{x} = \frac{a+b}{3}$$

$$\bar{y} = \frac{h}{3}$$

Composite Figures

$$\bar{X} = \frac{\sum A \bar{x}}{\sum A} \quad \bar{Y} = \frac{\sum A \bar{y}}{\sum A}$$

ملاحظة : الاجزاء المقطوعة سواء كانت مساحة او طول تعطى اشارة سالبة



Problem

Locate the centroid of the shaded area.

Solution.

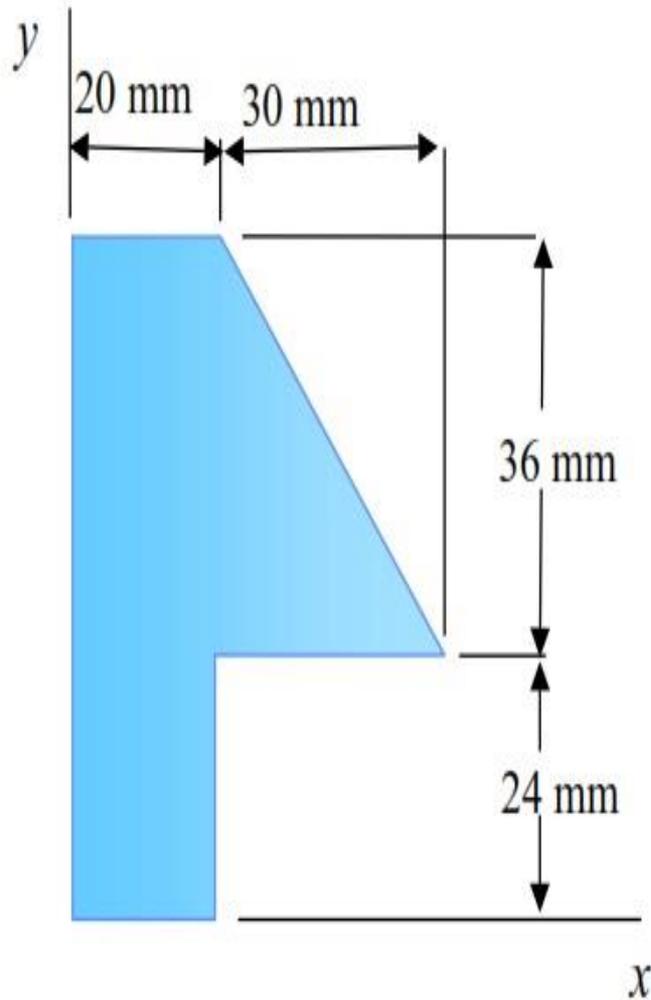
PART	A in. ²	\bar{x} in.	\bar{y} in.	$\bar{x}A$ in. ³	$\bar{y}A$ in. ³
1	120	6	5	720	600
2	30	14	10/3	420	100
3	-14.14	6	1.273	-84.8	-18
4	-8	12	4	-96	-32
TOTALS	127.9			959	650

$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A} \right]$$

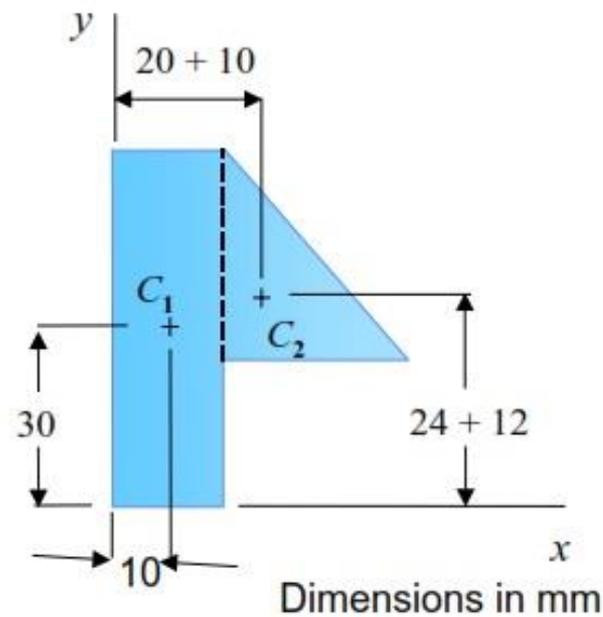
$$\bar{X} = \frac{959}{127.9} = 7.50 \text{ in.}$$

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} \right]$$

$$\bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.}$$



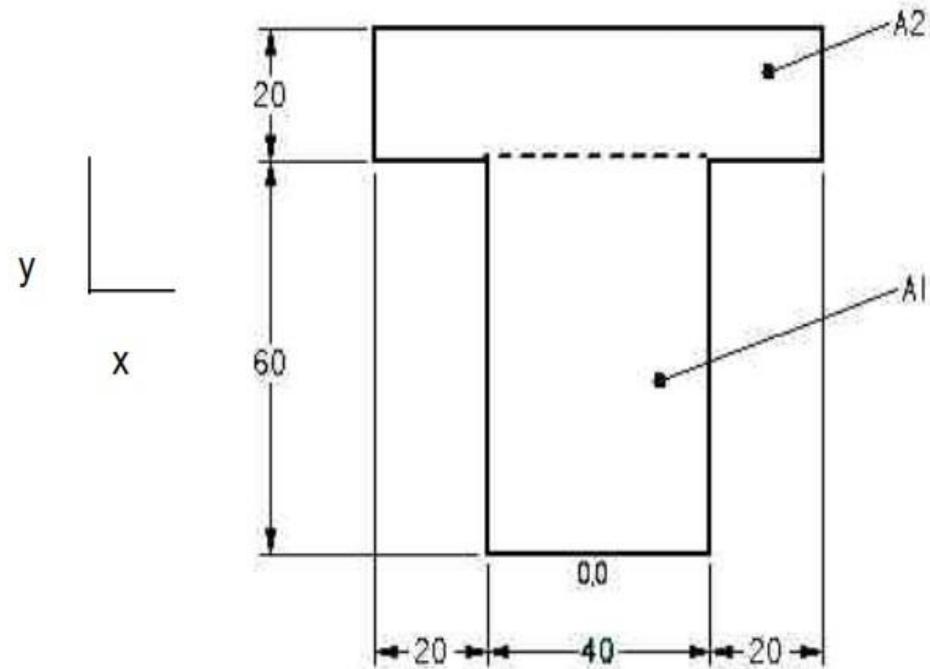
Locate the centroid of the plane area shown.



	$A, \text{ mm}^2$	$\bar{x}, \text{ mm}$	$\bar{y}, \text{ mm}$	$\bar{x}A, \text{ mm}^3$	$\bar{y}A, \text{ mm}^3$
1	$20 \times 60 = 1200$	10	30	12 000	36 000
2	$(1/2) \times 30 \times 36 = 540$	30	36	16 200	19 440
Σ	1740			28 200	55 440

Then $\bar{X} \Sigma A = \Sigma \bar{x}A$
 $\bar{X} (1740) = 28,200$
 or $\bar{X} = 16.21 \text{ mm}$

and $\bar{Y} \Sigma A = \Sigma \bar{y}A$
 $\bar{Y} (1740) = 55,440$
 or $\bar{Y} = 31.9 \text{ mm}$



the areas of the sub-regions

$$A_1 = 40\text{mm} \cdot 60\text{mm} = 2400\text{mm}^2$$

$$A_2 = 20\text{mm} \cdot 40\text{mm} = 800\text{mm}^2$$

$$\Sigma A_T = A_1 + A_2 = 3200\text{mm}^2$$

Find $\Sigma \bar{y}A$

$$\bar{y}_1 \cdot A_1 = 30\text{mm} \cdot 2400\text{mm}^2$$

$$= 7.20 \cdot 10^4 \text{mm}^3$$

$$\bar{y}_2 \cdot A_2 = 70\text{mm} \cdot 800\text{mm}^2$$

$$= 5.60 \cdot 10^4 \text{mm}^3$$

$$\Sigma \bar{y}A = \bar{y}_1 \cdot A_1 + \bar{y}_2 \cdot A_2$$

$$= 1.28 \cdot 10^5 \text{mm}^3$$

H W: Chapter 5

45 , 48 , 47 , 53 , 58