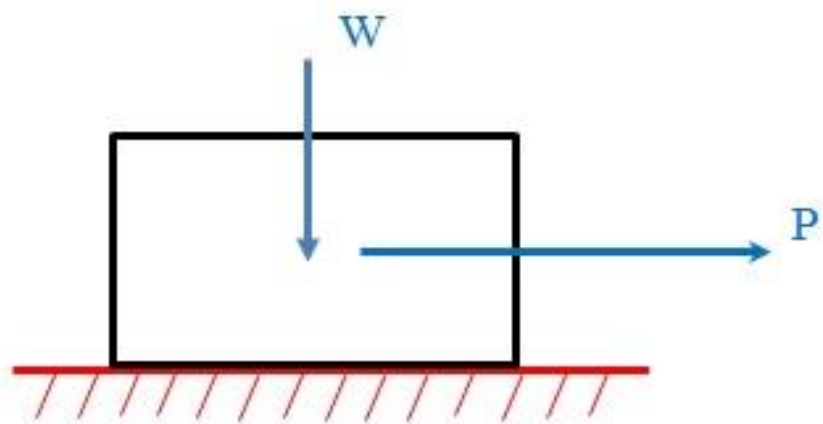


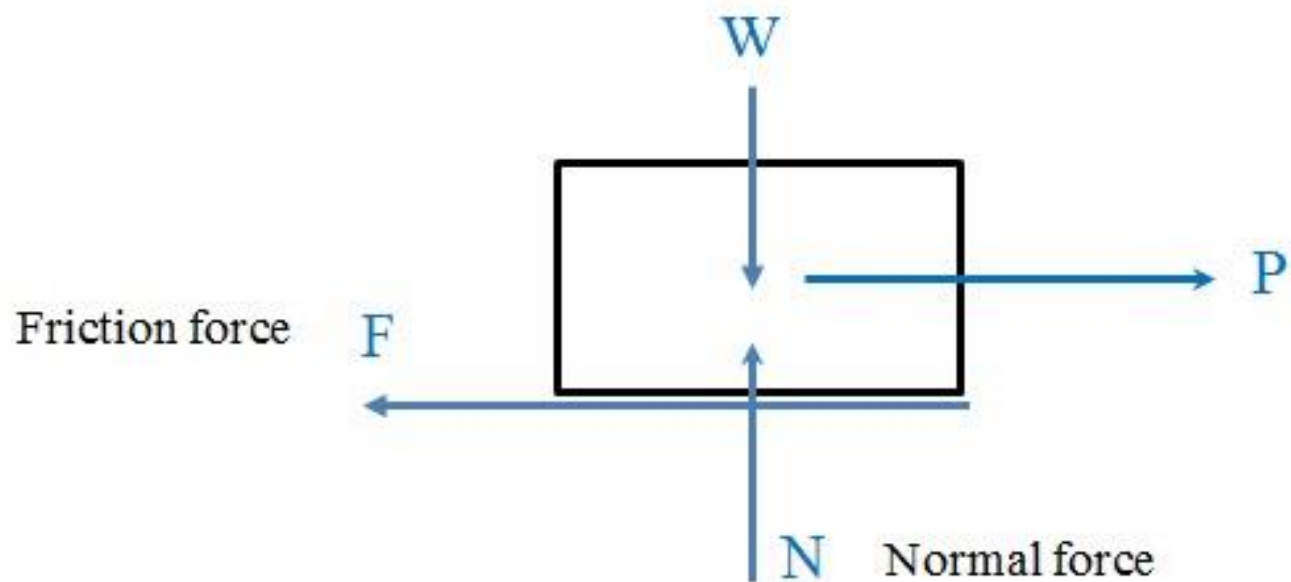
FRICITION

Dry Friction

- 1) The friction is always opposite to the direction of motion.
- 2) $F = M_s N$ only when motion is impending, where: M_s is static coefficient of friction.
- 3) When motion exists $F = M_k N$ = constant independent velocity, where: M_k is kinetic coefficient of friction. $M_k < M_s$ ($M_k \approx \frac{3}{4} M_s$)
- 4) When there are no motion or impending motion $F \neq M_s N \neq M_k N$, F is computed from equilibrium.



Rough surface



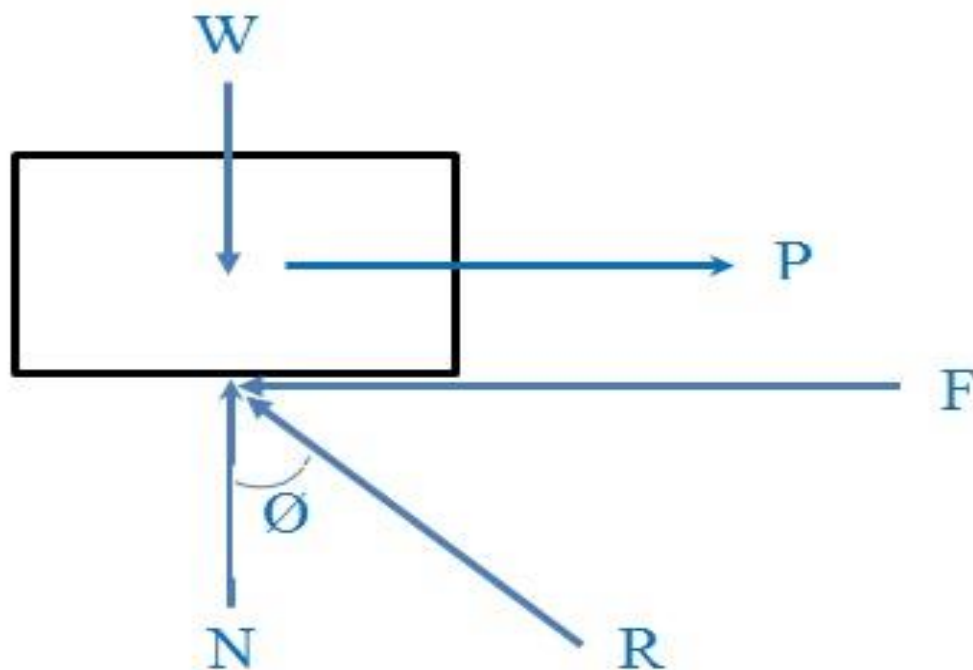
Angle of Friction ϕ

Angle of friction is the angle between N & R , in the case of impending motion

$$\tan \phi_s = \frac{M_s N}{N} = M_s$$

In case of motion

$$\phi_k = \tan^{-1} M_k$$



Types of Problems Involving Friction

1- Case of impending motion

Put $F = M_s N$ and solve for the unknown using equilibrium equations.

2- Case of steady motion

Put $F = M_k N$ and apply equilibrium equations to solve for the unknowns.

3- Unknown case (not clear the body is in equilibrium or in motion)

A. Denote the friction force by F

B. Assume equilibrium, find F

C. Compute $F^* = M_s N$

When $F < F^*$ the body is in equilibrium

But when $F > F^*$ the body is in motion, and the actual friction force is $F = M_k N$

Ex: Determine the maximum angle θ before the block of mass m begins to slip, ($M_s = 0.3$).

Max θ when motion impends

$$F = 0.3 * N$$

$$\sum F_x = 0 ; \quad 0.3 * N - m * g * \sin \theta = 0$$

$$0.3 * N = m * g * \sin \theta \quad \text{----- (1)}$$

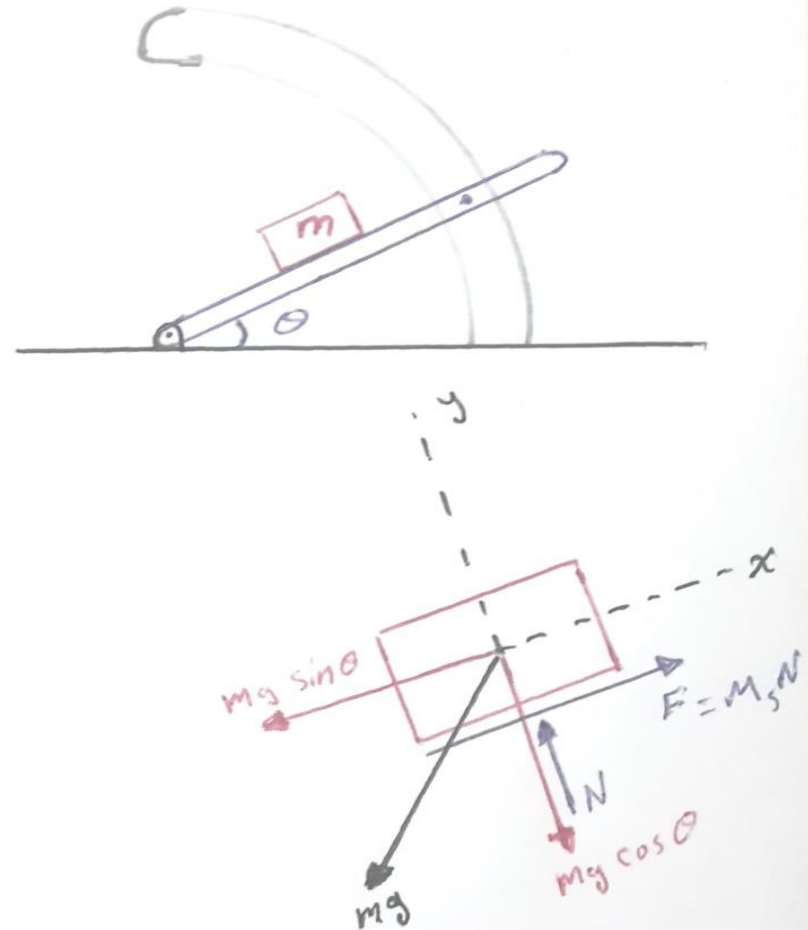
$$\sum F_y = 0 ; \quad N - m * g * \cos \theta = 0$$

$$N = m * g * \cos \theta \quad \text{----- (2)}$$

$$\frac{0.3 N}{N} = \frac{m g \sin \theta}{m g \cos \theta}$$

$$\tan \theta = 0.3$$

$$\text{Then } \theta = 16.7$$



Ex: Given $M_s = 0.25$ and $M_k = 0.2$, determine whether the block is in equilibrium or not, also find the value of the friction force.

- 1) The friction force F
- 2) Assume equilibrium

$$\sum F_x = 0$$

$$-F + 100 - 180 = 0$$

$$\text{Then } F = -80 \text{ (N)}$$

Tendency of motion is down

$$F^* = M_s N$$

$$\sum F_y = 0$$

$$N = 300 \cos 36.87$$

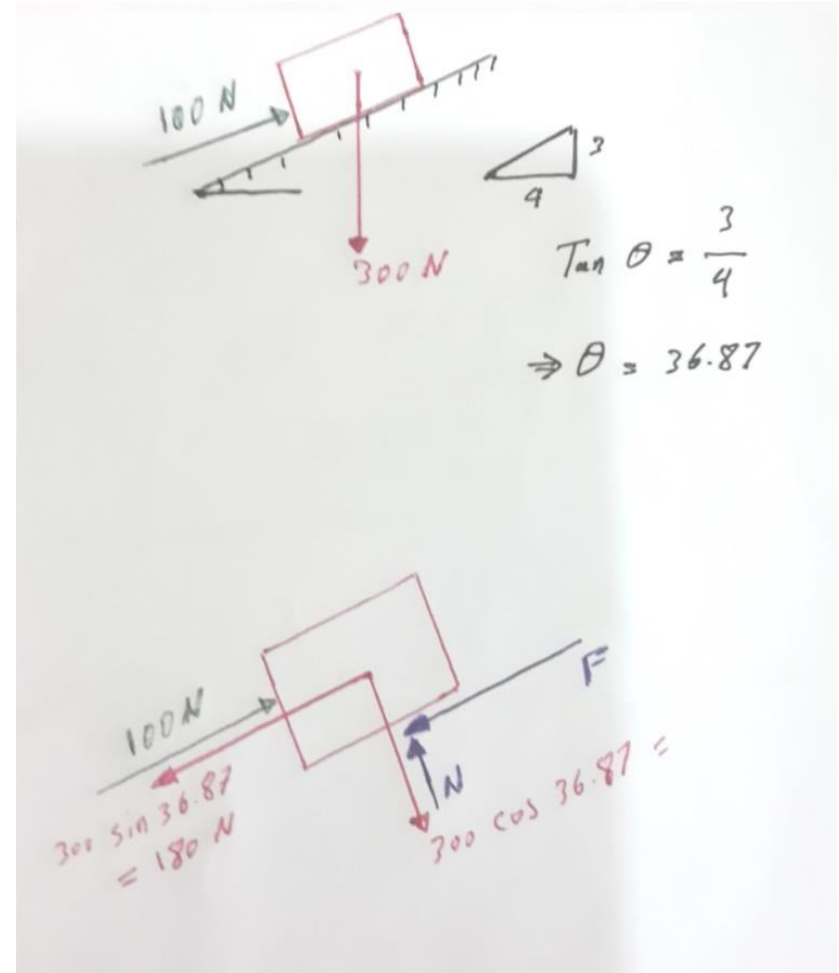
$$\text{Then } N = 240 \text{ (N)}$$

$$F^* = M_s N = 0.25 * 240 = 60 \text{ (N)}$$

Since $F > F^*$ the body is moving downwards,

then the friction force is $F = M_k N$

$$F = 0.2 * 240 = 48 \text{ (N)}$$



EX: Determine the max value of P before any slipping teak place.

$m = 0.3$ for A, $m = 0.4$ for B , $m = 0.45$ for C

Solution

Impending motion $f = m_s N$

Case (1)

Block C fixed, block B impends motion

For block A

$$\Sigma F_y = 0 ; N_1 = 30 g \cos 30 \text{ then } N_1 = 255 \text{ (N)}$$

For block B

$$\Sigma F_y = 0 ; N_2 = N_1 + 50 g \cos 30$$

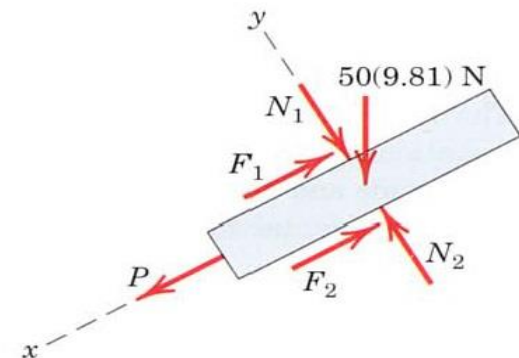
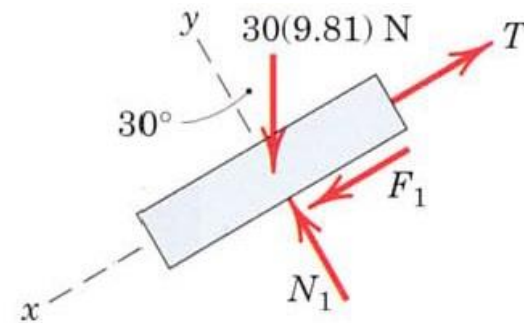
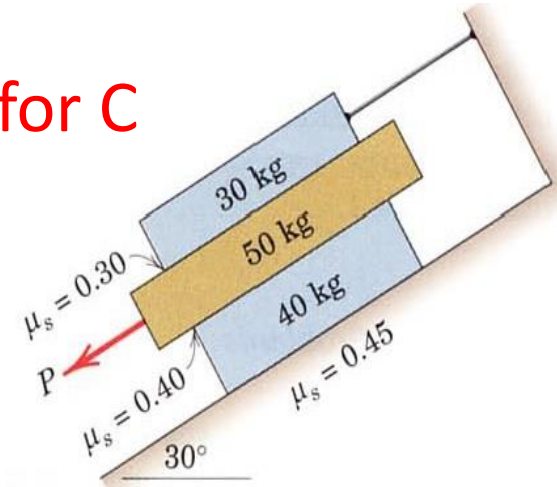
$$= 255 + 50 * 9.81 * 0.866$$

$$N_2 = 680 \text{ (N)}$$

$$\Sigma F_x = 0 ; P = 0.3 N_1 + 0.4 N_2 - 50 g \sin 30$$

$$= 0.3 * 255 + 0.4 * 680 - 50 * 9.81 * 0.5$$

$$P = 103.1 \text{ (N)}$$



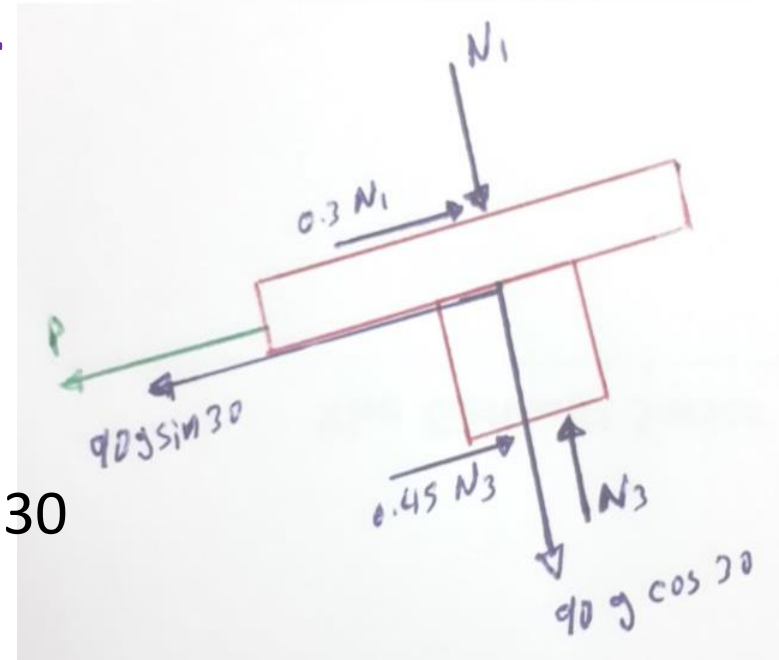
Case (2)

Block B and C impend motion together

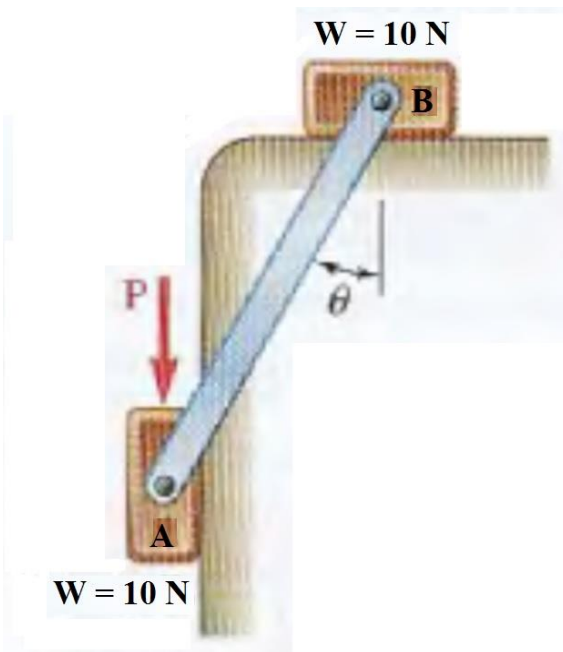
$$\begin{aligned}\sum F_y = 0; \quad N_3 &= N_1 + 90 \text{ g} \cos 30 \\ &= 1019.6 \text{ (N)}\end{aligned}$$

$$\begin{aligned}\sum F_x = 0; \quad P &= 0.3 N_1 + 0.45 N_3 - 90 \text{ g} \sin 30 \\ P &= 93.8 \text{ (N)}\end{aligned}$$

$$P_{\max} = 93.8 \text{ (N)}$$

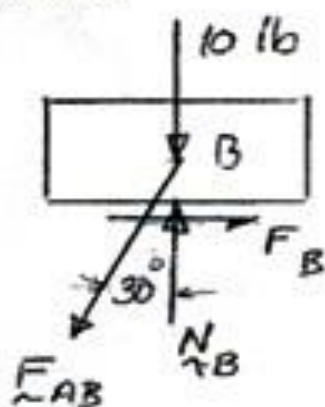


Determine the smallest force **P** that made block **A** impending motion. The two blocks have a weight of $W = 10 \text{ N}$, if the coefficients of static friction for all surfaces are $\mu_s = 0.3$. Neglecting the rod weight, the rod angle is $\theta = 30^\circ$.

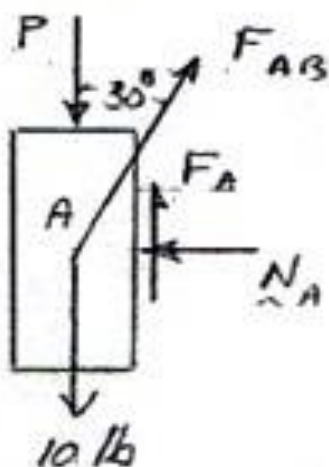


SOLUTION

FBD block B:



FBD block A:



(b) For P_{max} , motion impends at both surfaces

$$B: \quad \uparrow \Sigma F_y = 0: N_B - 10 \text{ lb} - F_{AB} \cos 30^\circ = 0$$

$$N_B = 10 \text{ lb} + \frac{\sqrt{3}}{2} F_{AB} \quad (1)$$

$$\text{Impending motion:} \quad F_B = \mu_s N_B = 0.3 N_B$$

$$\rightarrow \Sigma F_x = 0: F_B - F_{AB} \sin 30^\circ = 0$$

$$F_{AB} = 2F_B = 0.6 N_B \quad (2)$$

$$\begin{aligned} \text{Solving (1) and (2)} \quad N_B &= 10 \text{ lb} + \frac{\sqrt{3}}{2} (0.6 N_B) \\ &= 20.8166 \text{ lb} \end{aligned}$$

$$\text{Then} \quad F_{AB} = 0.6 N_B = 12.4900 \text{ lb}$$

$$A: \quad \rightarrow \Sigma F_x = 0: F_{AB} \sin 30^\circ - N_A = 0$$

$$N_A = \frac{1}{2} F_{AB} = \frac{1}{2} (12.4900 \text{ lb}) = 6.2450 \text{ lb}$$

$$\text{Impending motion:} \quad F_A = \mu_s N_A = 0.3 (6.2450 \text{ lb}) = 1.8735 \text{ lb}$$

$$\uparrow \Sigma F_y = 0: F_A + F_{AB} \cos 30^\circ - P - 10 \text{ lb} = 0$$

$$P = F_A + \frac{\sqrt{3}}{2} F_{AB} - 10 \text{ lb}$$

$$= 1.8735 \text{ lb} + \frac{\sqrt{3}}{2} (12.4900 \text{ lb}) - 10 \text{ lb} = 2.69 \text{ lb}$$

$$P = 2.69 \text{ lb} \blacktriangleleft$$

Since $P = 2.69 \text{ lb}$ to initiate motion,

For the figure below, determine the minimum value of P that makes block C sliding down. The coefficient of friction between all surfaces is 0.4 .

Case (1) A&B moves up and C move down

$$N_1 = 29.74 \text{ N}$$

$$F_1 = 11.9 \text{ N}$$

$$T = 29.1 \text{ N}$$

$$N_2 = 63.72 \text{ N}$$

$$P = 45.88 \text{ N}$$

Case (2) A moves up and C&B move down

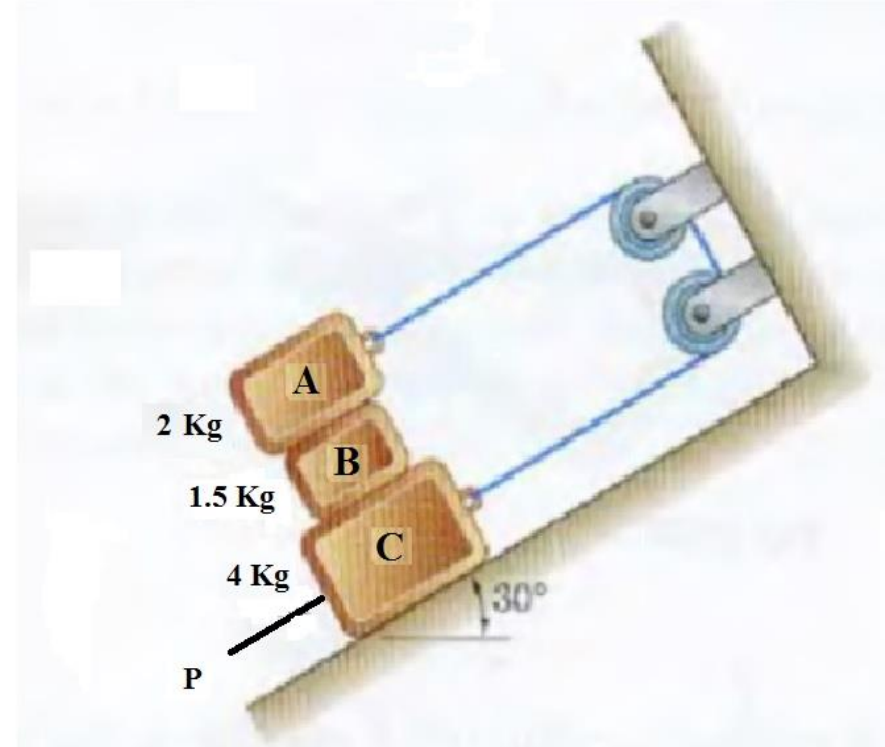
$$N_1 = 17 \text{ N}$$

$$F_1 = 6.8 \text{ N}$$

$$T = 16.6 \text{ N}$$

$$N_2 = 63.73 \text{ N}$$

$$P = 21.9 \text{ N}$$



H.W :

Chapter 6:

3, 5, 7, 8,22, 24