Equilibrium of Rigid Bodies

Rigid body will be in equilibrium when:

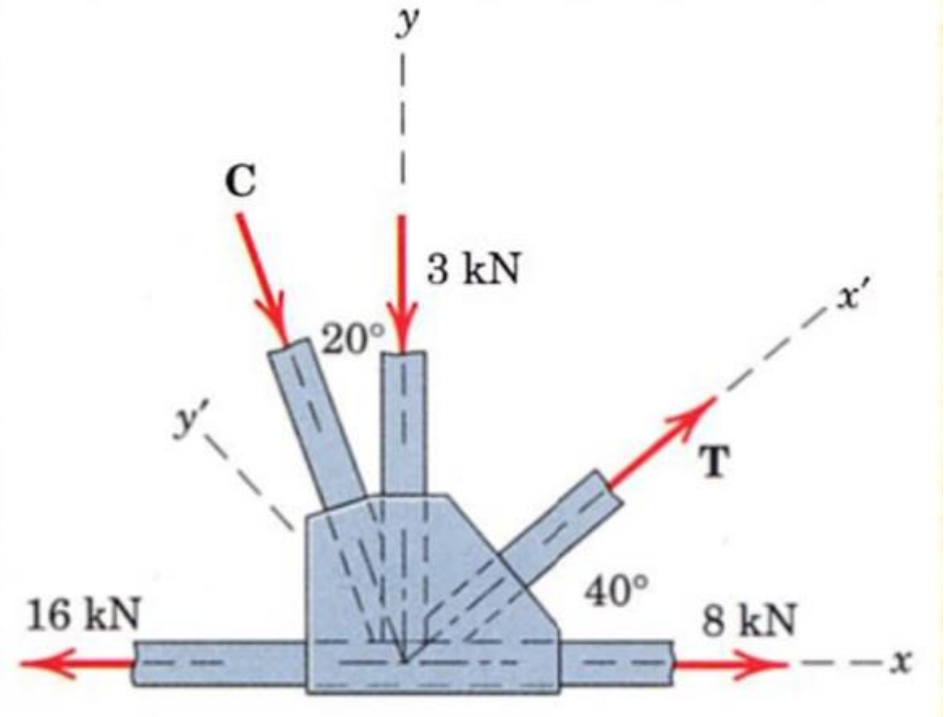
$\mathbf{R} = 0$ and $\mathbf{M} = 0$

$$\left.\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M_? &= 0 \end{aligned}\right\}$$

3 equation's give 3 unknown

? may be any point inside or outside the body

Ex: Determine the magnitudes of the forces C and T, which act on the bridge-truss joint.



Solution

$$\sum F_x = 0$$

8 + T cos 40 + C sin 20 – 16 = 0

0.766 T + 0.342 C = 8 -------- (1)

$$\sum F_y = 0$$

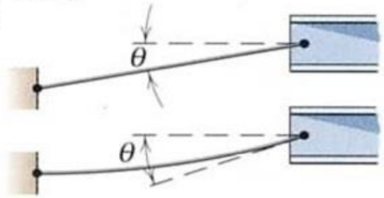
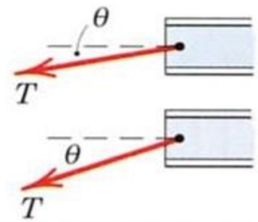
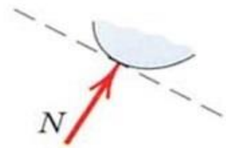
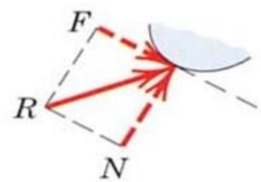
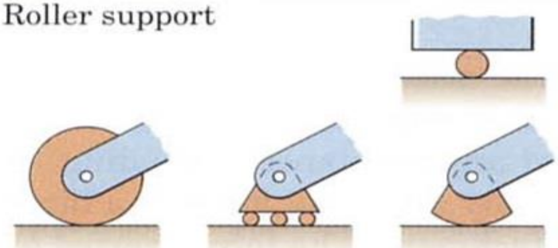
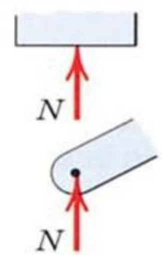
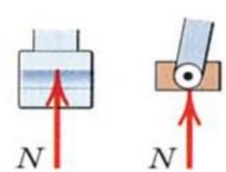
T sin 40 – C cos 20 – 3 = 0

0.643 T – 0.940 C = 3 --------(2)

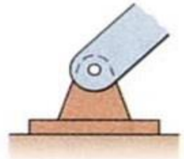
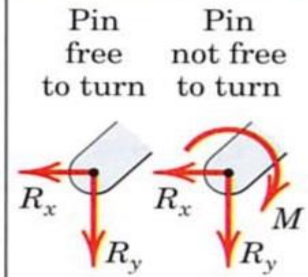
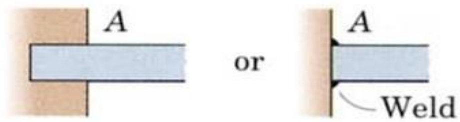
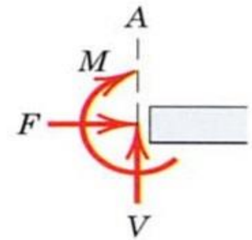
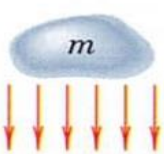
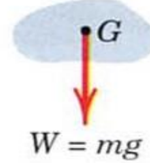
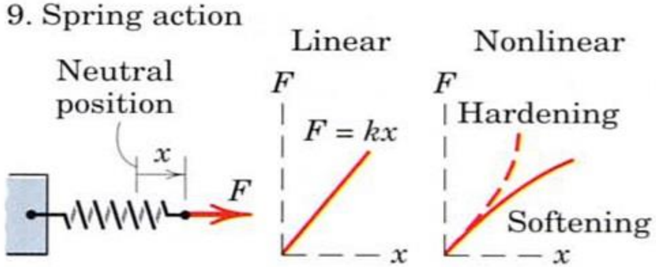
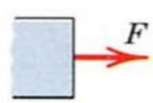
Solve equ. (1) and (2) to find that

T = 9.09 KN

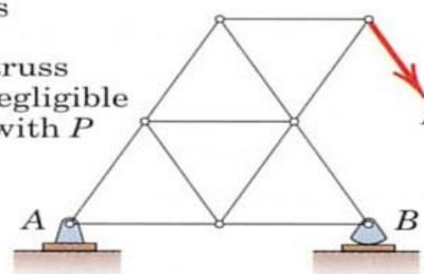
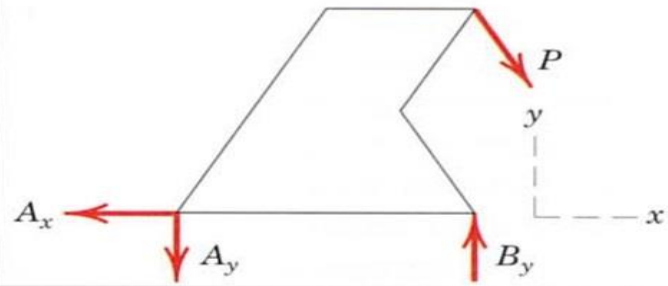
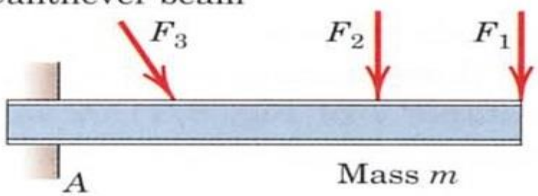
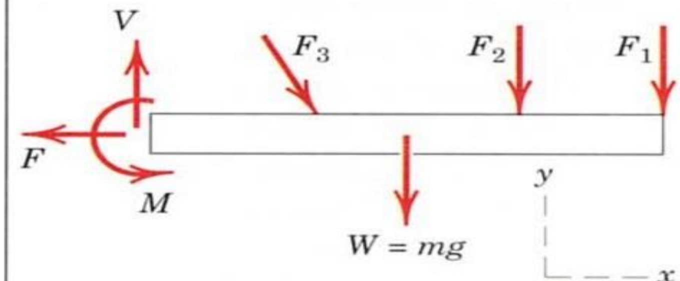
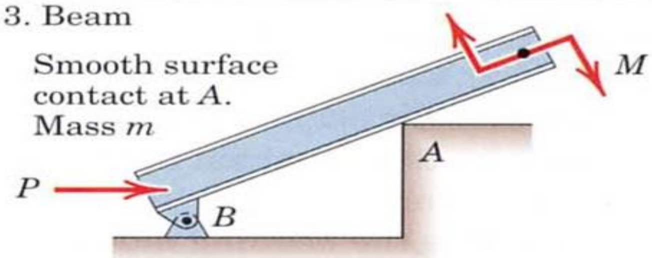
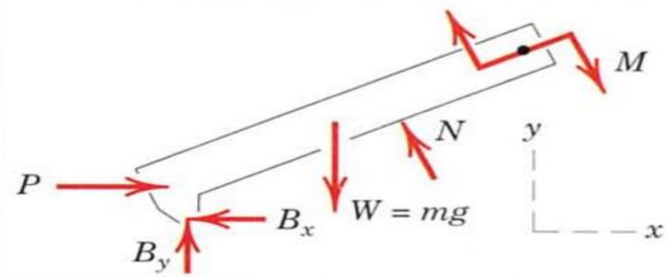
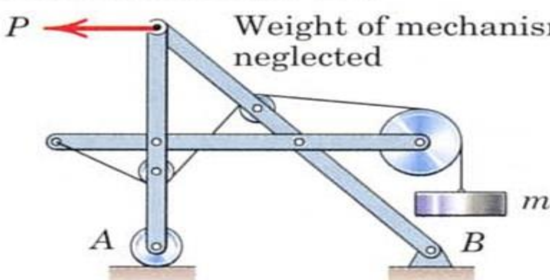
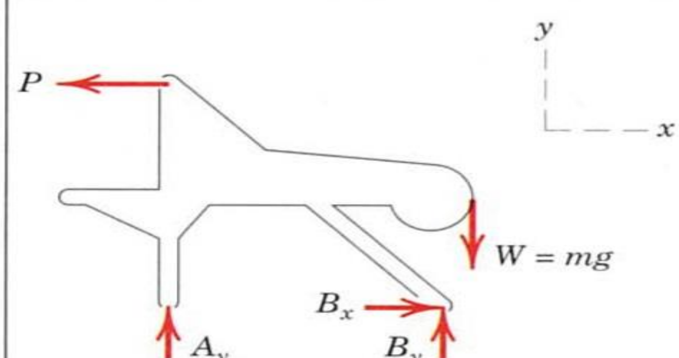
C = 3.03 KN

MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope Weight of cable negligible Weight of cable not negligible 	 Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.

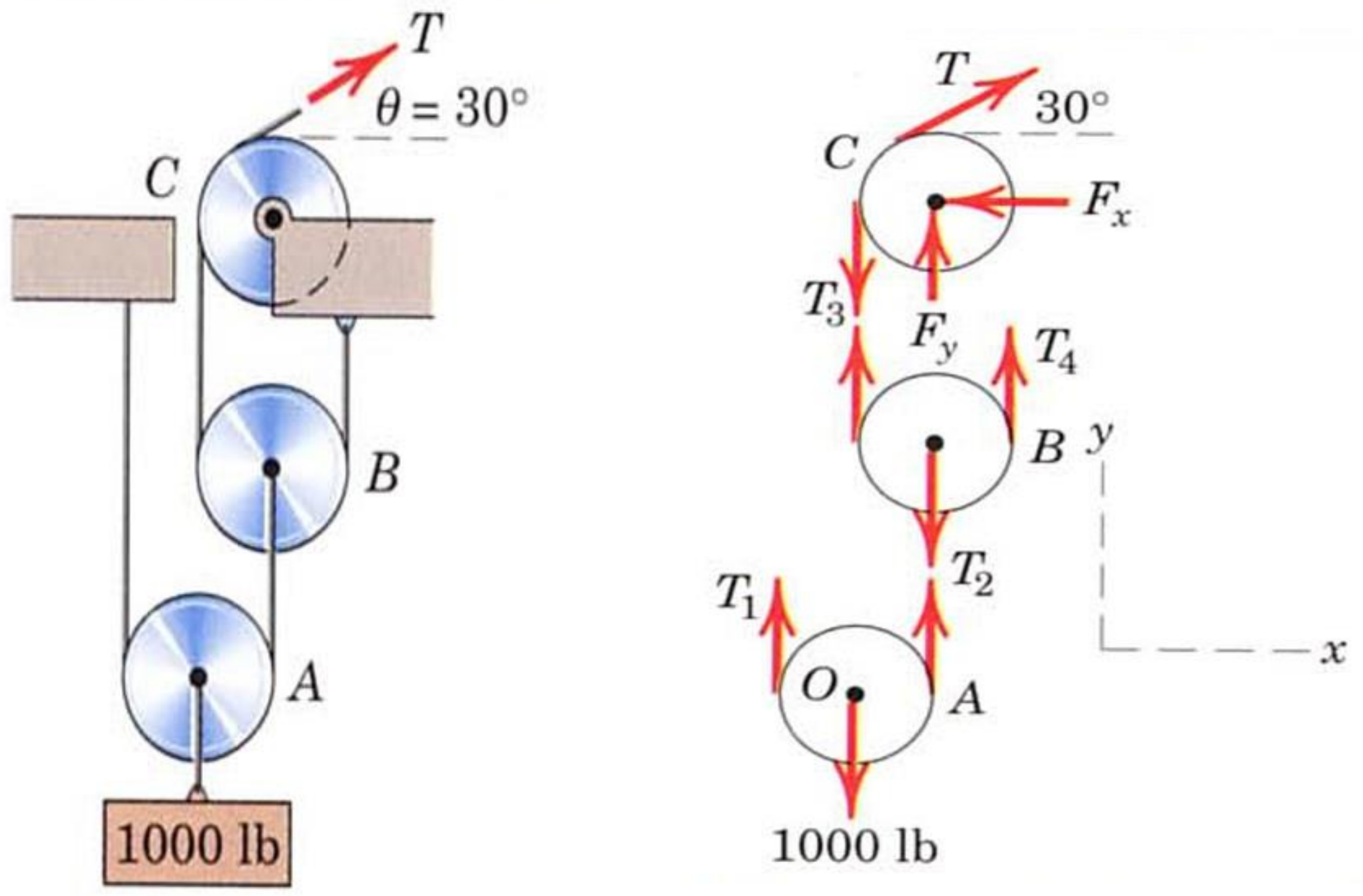
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS (*cont.*)

Type of Contact and Force Origin	Action on Body to Be Isolated	
6. Pin connection 		A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y. A pin not free to turn may also support a couple M.
7. Built-in or fixed support 		A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 		The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 		Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Ex:

SAMPLE FREE-BODY DIAGRAMS	
Mechanical System	Free-Body Diagram of Isolated Body
1. Plane truss Weight of truss assumed negligible compared with P 	
2. Cantilever beam 	
3. Beam Smooth surface contact at A. Mass m 	
4. Rigid system of interconnected bodies analyzed as a single unit Weight of mechanism neglected 	

Ex: Calculate the tension T in the cable which supports the 1000 Ib load with the pulley arrangement shown. Each pulley is free to rotate about the bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.



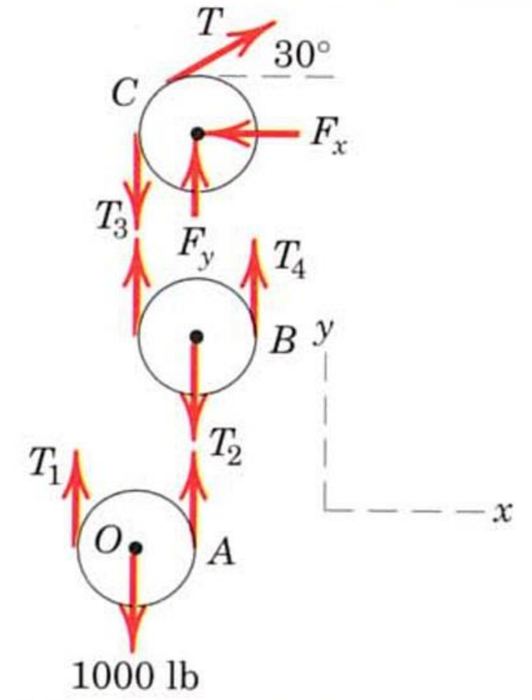
Solution:

At pulley A

$$\sum M_o = 0$$

$T_1\, r - T_2\, r = 0$ then $T_1 = T_2$

$$\sum F_y = 0$$



$T_1 + T_2 - 1000 = 0$

$2\,T_1 = 1000$ then $T_1 = T_2 = 500$ lb

At pulley B

$\sum M = 0$ then $T_3\, r - T_4\, r = 0$ then $T_3 = T_4$

$\sum F_y = 0$ then $T_3 + T_4 - 500 = 0$

$2\,T_3 = 500$ then $T_3 = T_4 = 250$ lb

At pulley C

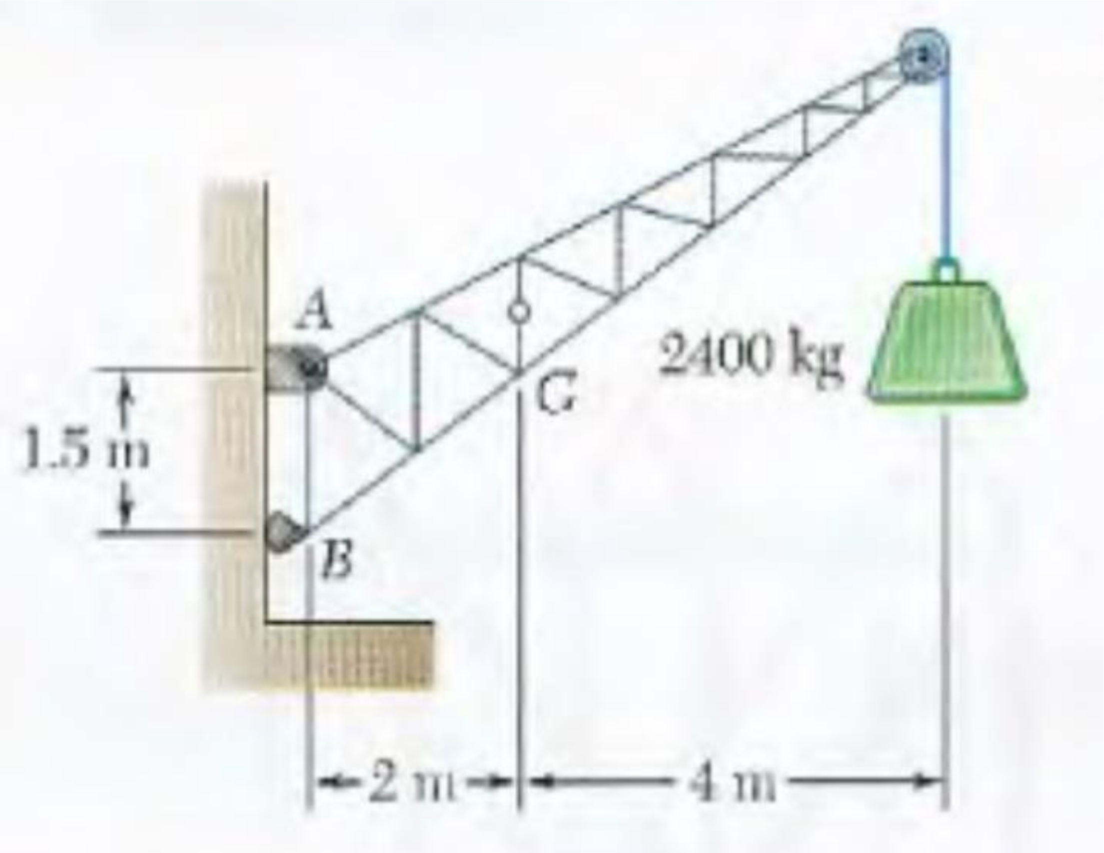
In equilibrium $T = T_3$ then $T = 250$ lb

$\sum F_x = 0$ then $250 \cos 30 - F_x = 0$ then $F_x = 217$ lb

$\sum F_y = 0$ then $F_y + 250 \sin 30 - 250 = 0$ then $F_y = 125$ lb

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{217^2 + 125^2} \quad then \quad F = 250\, lb$$

Ex: A fixed crane has a mass of 1000 Kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and rocker at B. the center of gravity of the crane is located at C. Determine the components of the reactions at A and B.



Solution

$g = 9.81\ m/s^2$

m*g

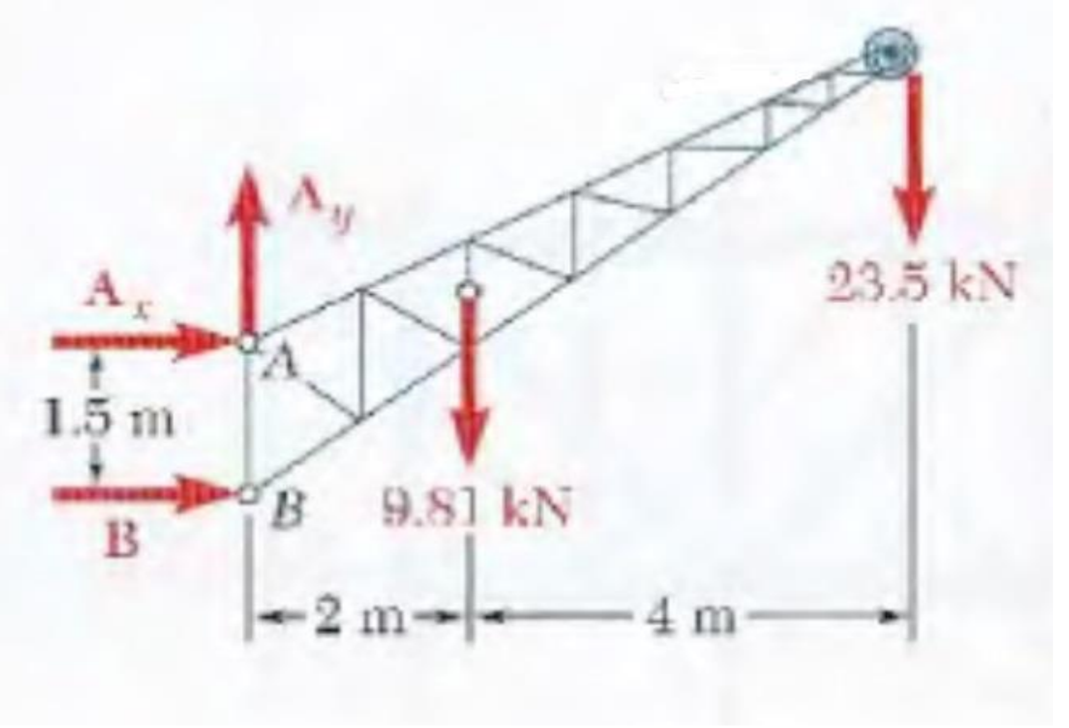
The equivalent of the mass is 9.81 KN

The equivalent of the weight is 23.5 KN

$$\sum M_A = 0$$

B (1.5) – 9.81 (2) – 23.5 (6) = 0

Then B = 107.1 KN



$$\sum F_x = 0$$

$A_x + B = 0$

$A_x + 107.1 = 0$

then $A_x = - 107.1$ KN

$$\sum F_y = 0$$

$A_y - 9.81 – 23.5 = 0$

then $A_y = 33.3$ KN

قاعدة : فكر في اختيار عزم او مركز عزوم يمثل نقطة التقاء قوتين مجهولة

H. W. :

CHAPTER THREE

2 , 4 , 10 , 17 , 20 , 26 , 37 , 54