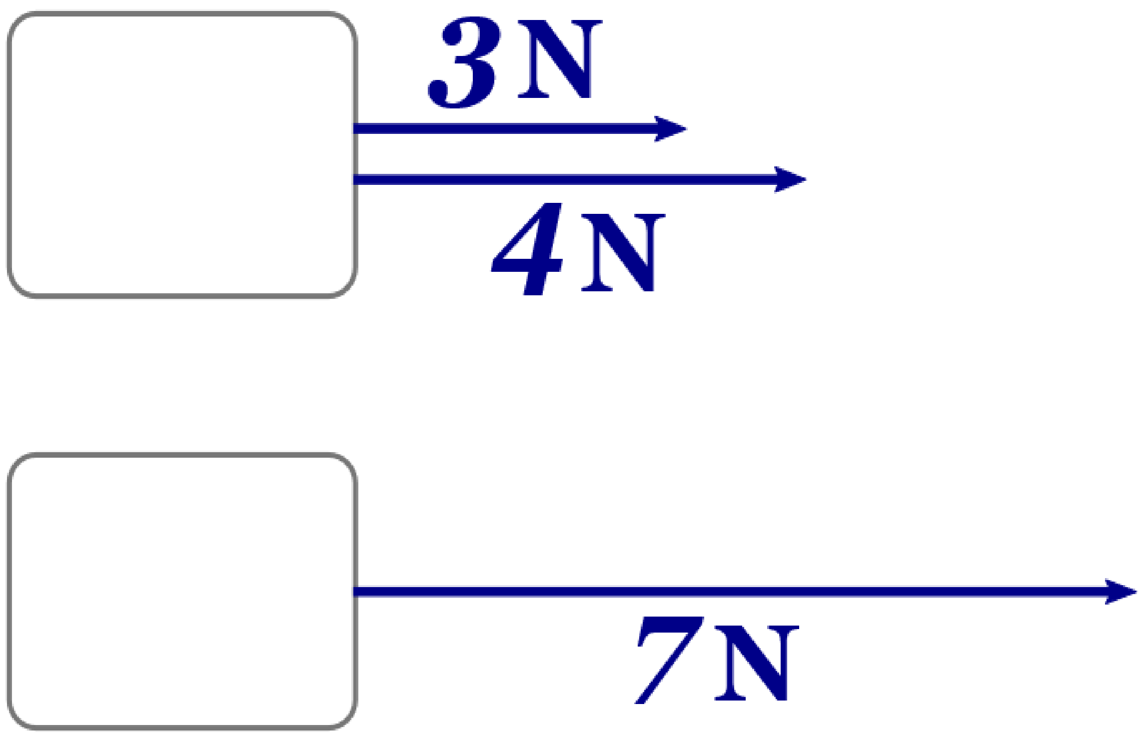
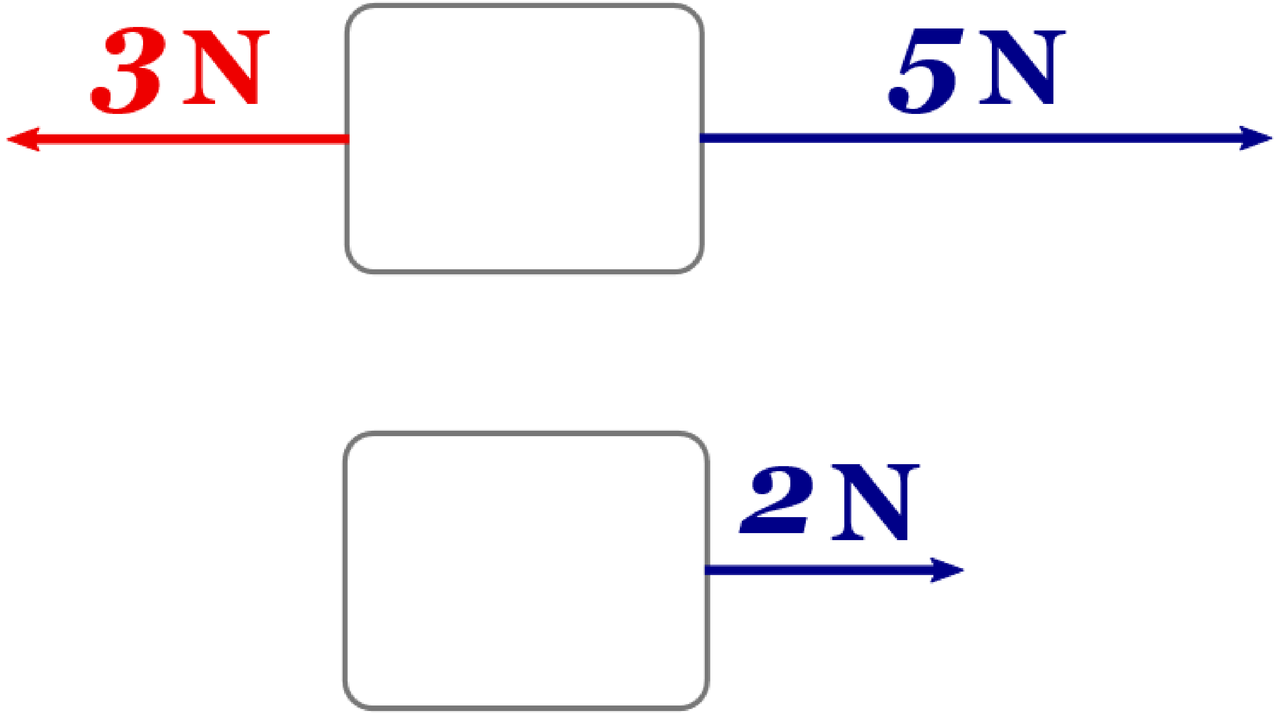
Resultant Force

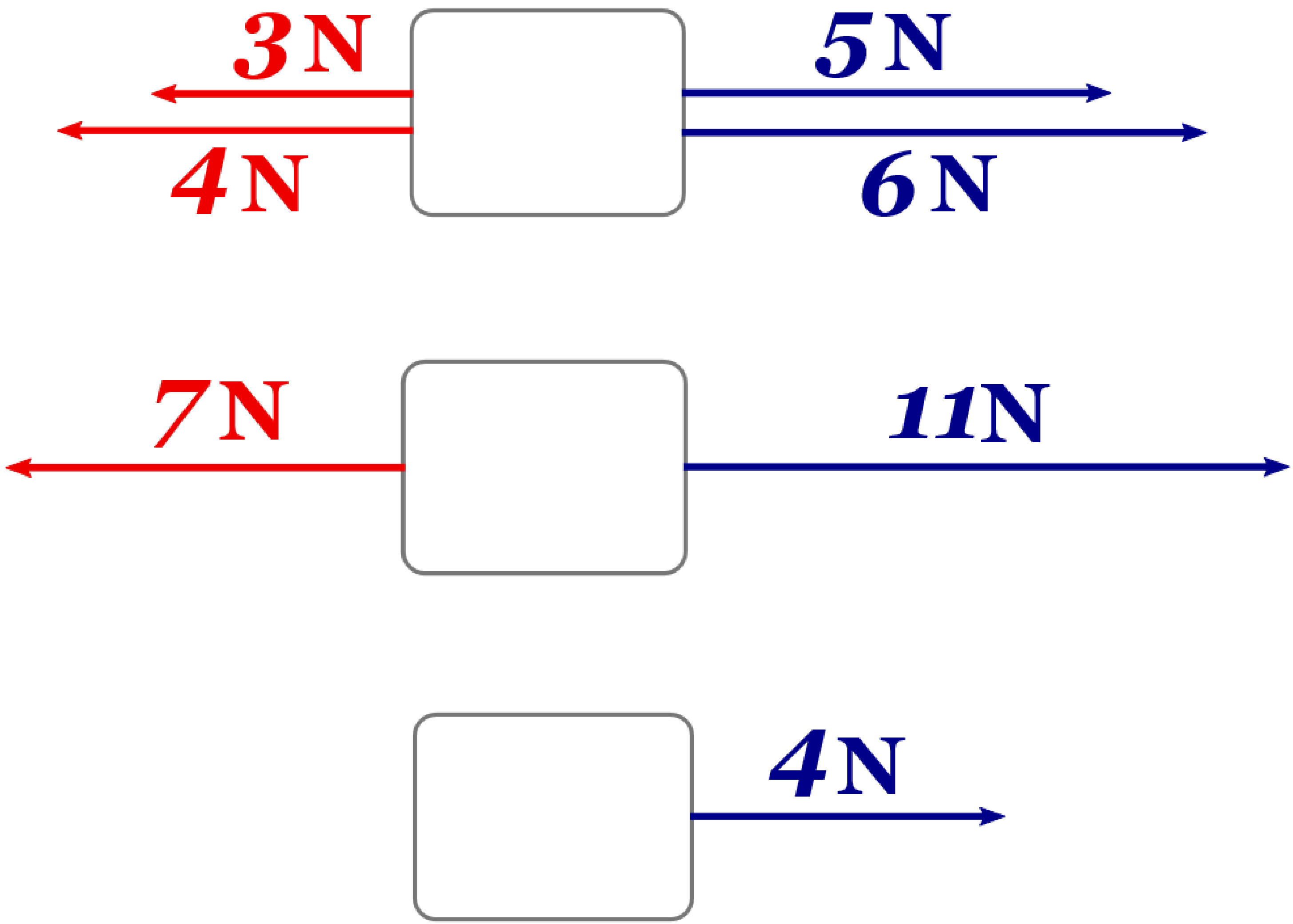
Two forces that act in the same direction



Two forces that act in opposite directions

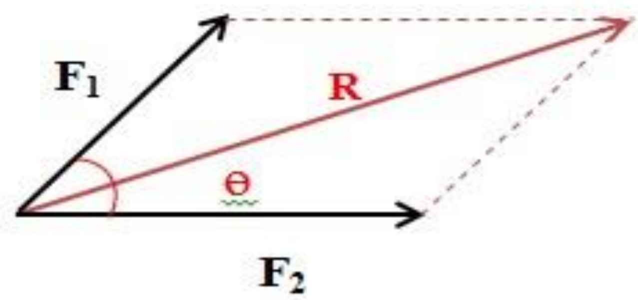


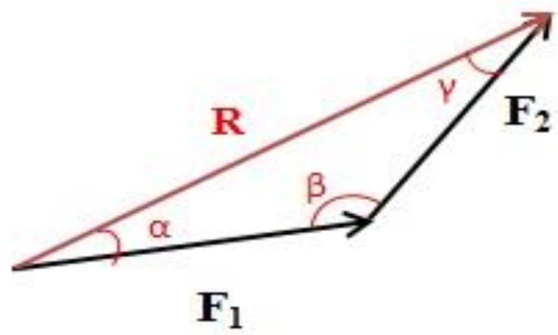
Two forces parallel to one another



Two forces in different directions

Addition of Force





Law of Cosines

$$R^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos\theta$$

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos\beta$$

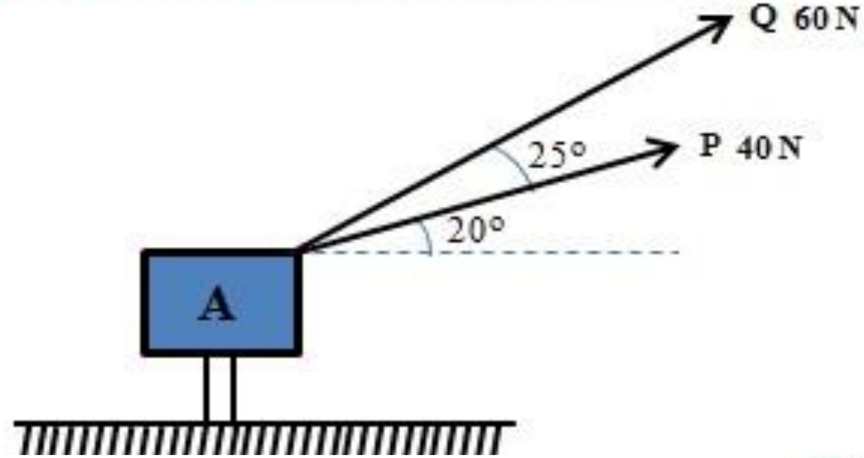
Law of Sines

$$\frac{R}{\sin\beta} = \frac{F_1}{\sin\gamma} = \frac{F_2}{\sin\alpha}$$

Or

$$\frac{\sin\beta}{R} = \frac{\sin\gamma}{F_1} = \frac{\sin\alpha}{F_2}$$

Ex: The two force $\vec{P}$ and $\vec{Q}$ at on bolt **A** as shown in the figure bellow. Determine their resultant?



Analytic Method

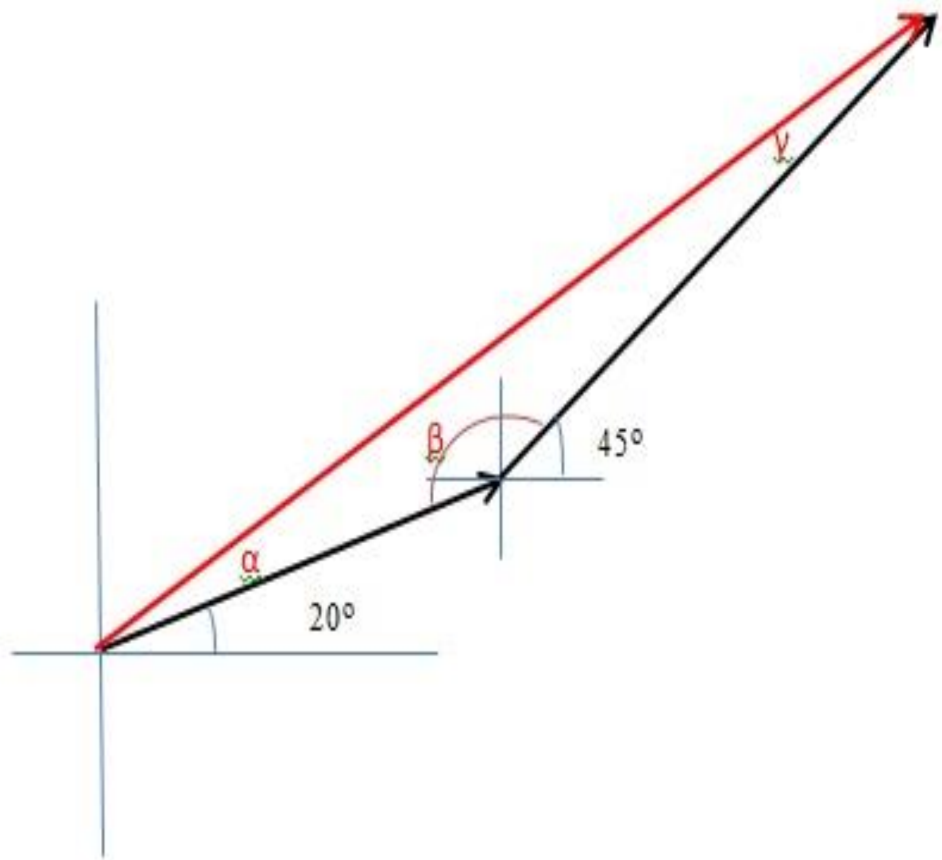
$$R^2 = 40^2 + 60^2 - 2.40.60 \ Cos\,(20 + 135)$$

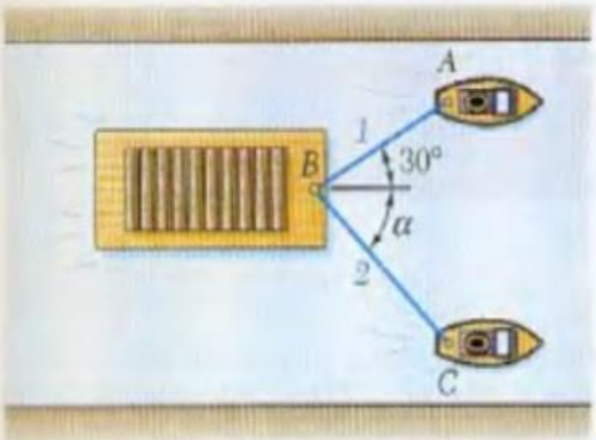
$$= 97.7 \ \ N$$

$$\frac{\text{Sin}\ \alpha}{60} = \frac{\text{Sin}\ 155}{97.7}$$

$\text{Sin}\ \alpha = 0.259$, then $\alpha = 15^{\circ}$

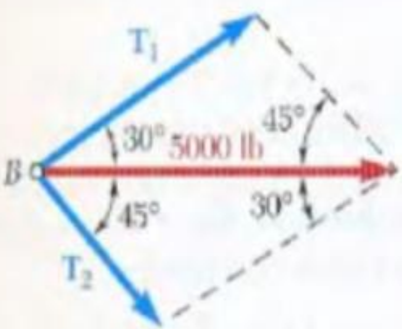
$\vec{R} = 97.7 \ \angle\ 35^{\circ}$ (N)



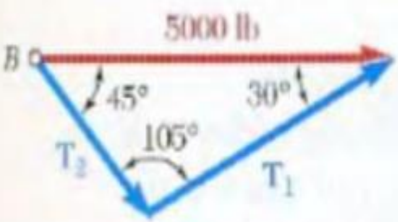


A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$,

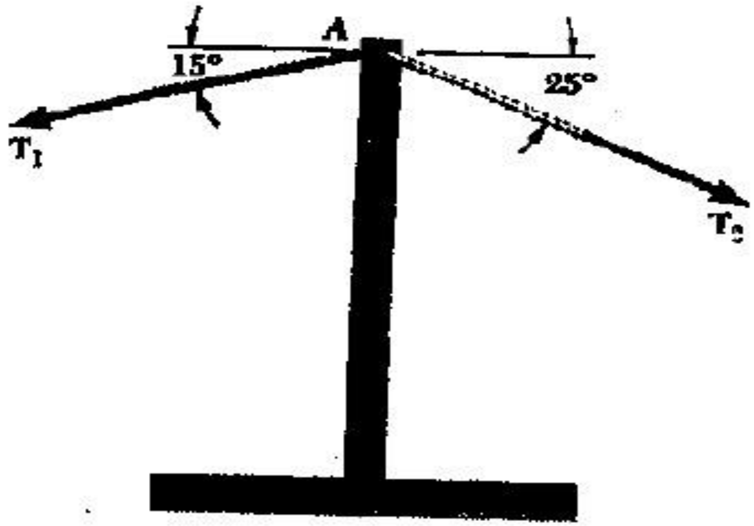
SOLUTION



$$\frac{T_1}{\sin 45°} = \frac{T_2}{\sin 30°} = \frac{5000 \text{ lb}}{\sin 105°}$$



$T_1 = 3660$ lb $\quad T_2 = 2590$ lb



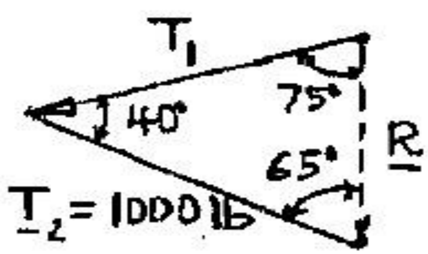
GIVEN:

RESULTANT $\underline{R}$ OF $\underline{T}_1$ AND $\underline{T}_2$ MUST BE VERTICAL AND $T_2 = 1000$ lb

FIND:

(a) T_1

(b) R



TRIANGLE RULE AND LAW OF SINES:

$$\frac{T_1}{\sin 65^\circ} = \frac{1000\text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ}$$

(a) SOLVING FOR T_1:

$$T_1 = (1000\text{ lb})\,\frac{\sin 65^\circ}{\sin 75^\circ} = 938.28\text{ lb}, \quad T_1 = 938\text{ lb}$$

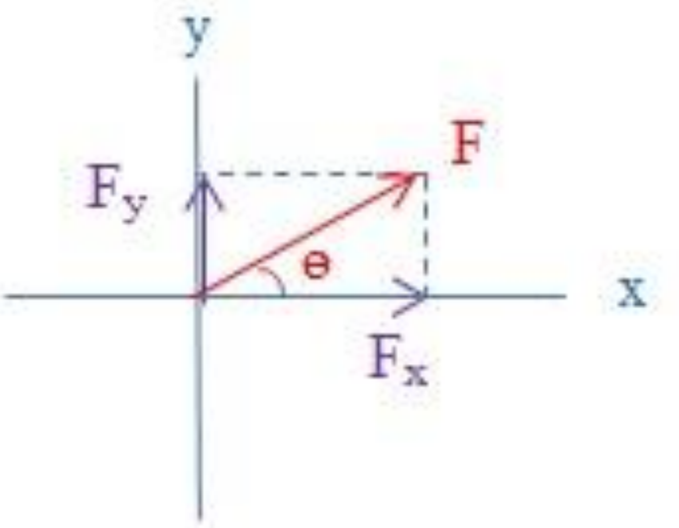
(b) SOLVING FOR R:

$$R = (1000\text{ lb})\,\frac{\sin 40^\circ}{\sin 75^\circ} = 665.46\text{ lb}, \quad R = 665\text{ lb}$$

Rectangular components of a force

$$F = \sqrt{F_x^2 + F_y^2}$$

$$Tan\,\theta = \frac{F_y}{F_x}$$



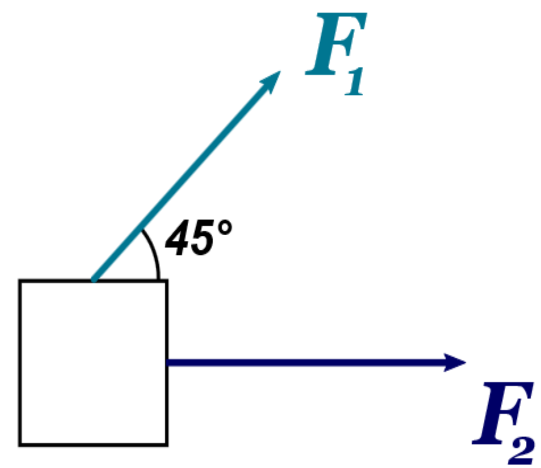
If we have more than one force, then the resultant will be :

$$R_x = \sum F_x \qquad R_y = \sum F_y$$

$$\boldsymbol{R} = \sqrt{\boldsymbol{R_x^2} + \boldsymbol{R_y^2}}$$

$$Tan\,\theta = \frac{R_y}{R_x}$$

Two forces F_1, F_2 of 50N and 60N respectively. Find resultant?



$F_{1x} = F_1 \cos 45°$

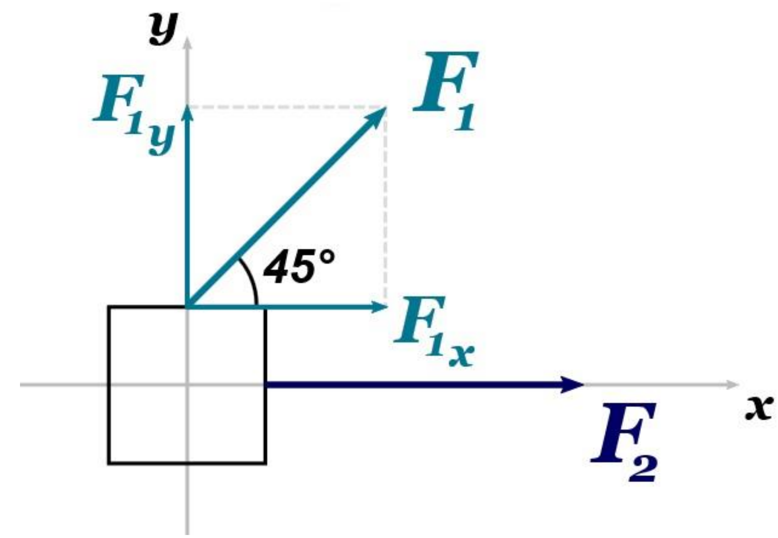
$F_{1y} = F_1 \sin 45°$

$F_{2x} = F_2$

$F_{2y} = 0$

$R_x = F_1 \cos 45° + F_2$

$\mathbf{R_x = 95N}$



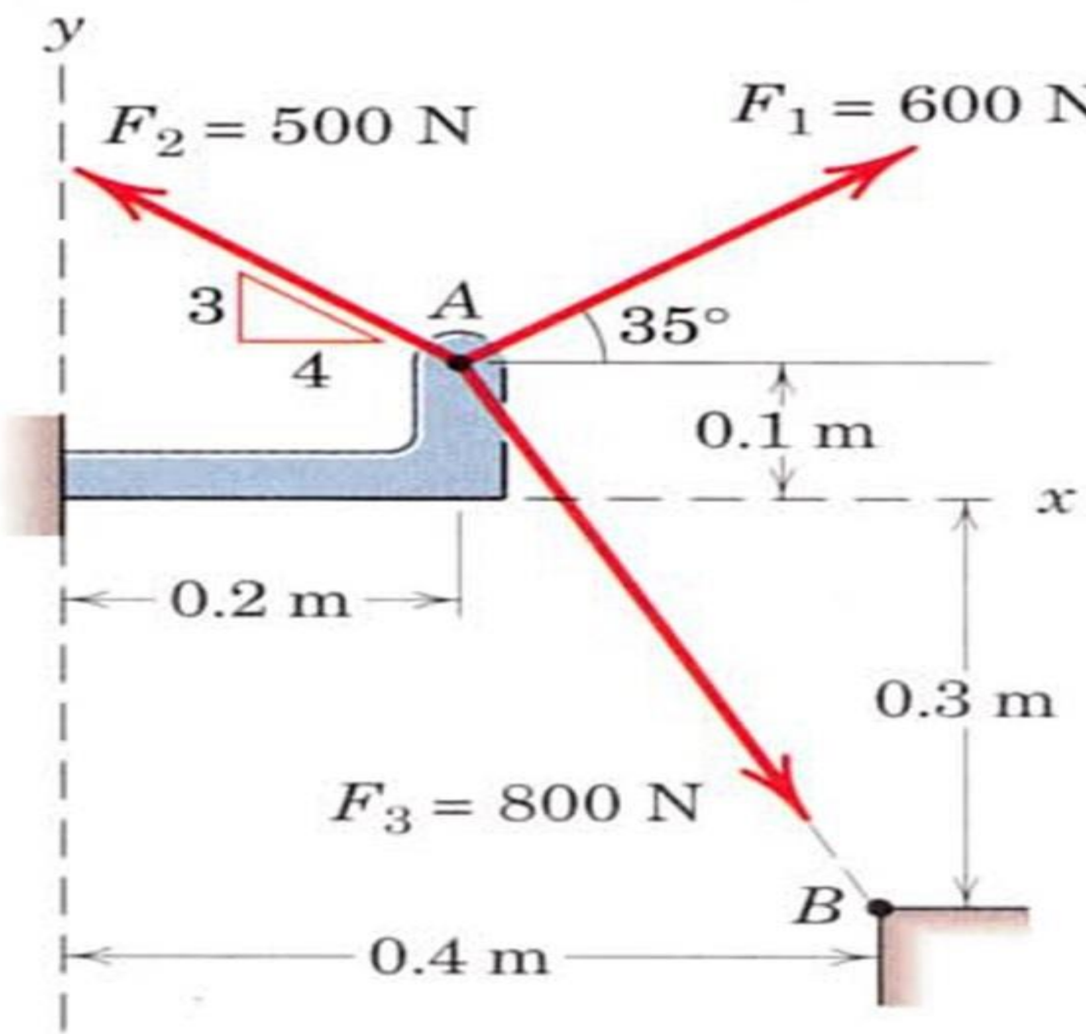
$R_y = F_1 \sin 45°$

$\mathbf{R_y = 35N}$

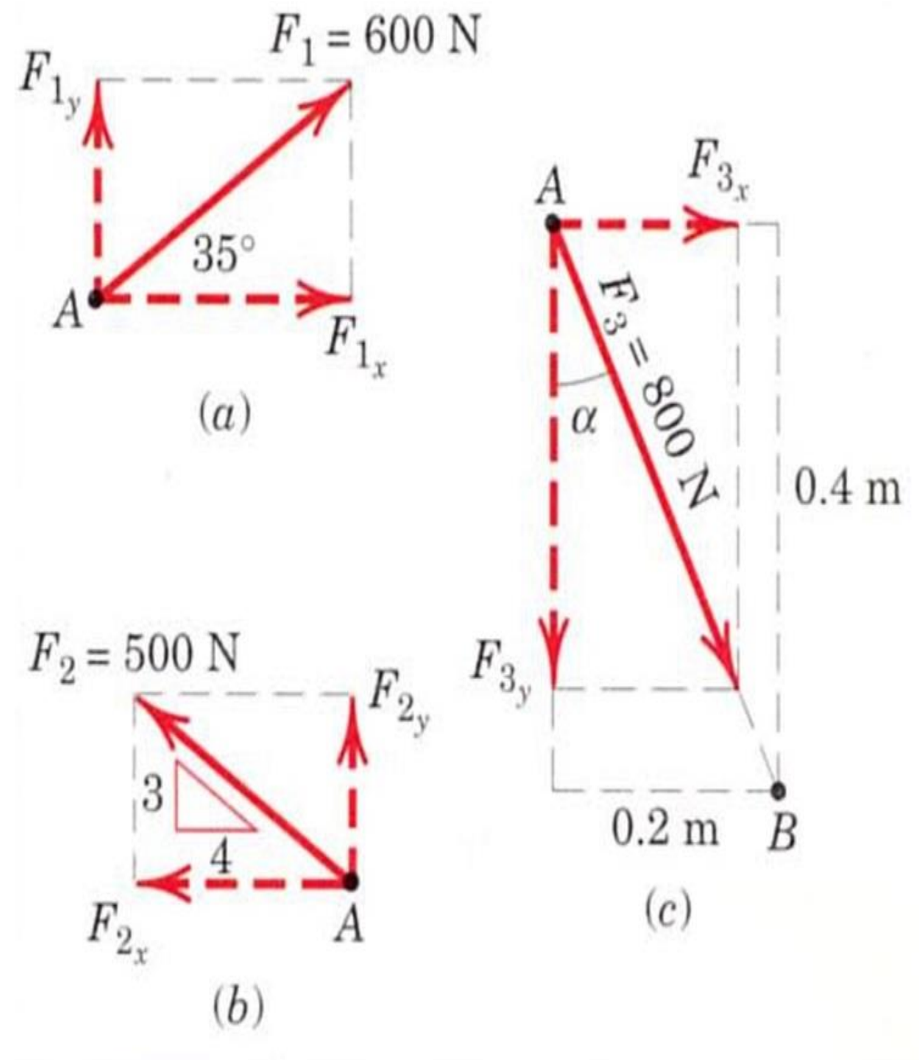
$R = \sqrt{95^2 + 35^2} = 100 \text{ N}$

$\theta = 20°$

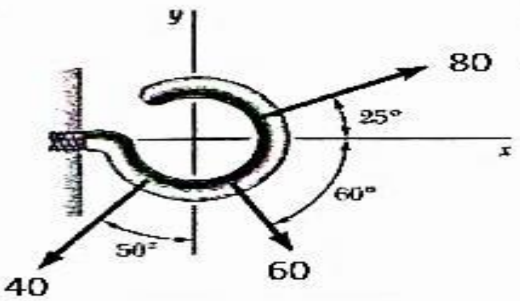
Ex: the forces F_1, F_2 and F_3 all which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components and the resultant R.



$$F_{1x} = 600\cos 35^o = 491\ N$$
$$F_{1y} = 600\sin 35^o = 344\ N$$
$$F_{2x} = -500\left(\frac{4}{5}\right) = -400\ N$$
$$F_{2y} = 500\left(\frac{3}{5}\right) = 300\ N$$
$$Tan\ \alpha = \left(\frac{0.2}{0.4}\right) \rightarrow \quad \alpha = 26.6^o$$
$$F_{3x} = 800\sin 26.6^o = 358\ N$$
$$F_{3y} = -800\cos 26.6^o = -716\ N$$
$$R_x = 491 - 400 + 358 = 449\ N$$
$$R_y = 344 + 300 - 716 = -72\ N$$
$$R = \sqrt{449^2 + (-72)^2} = 454.74\ N$$



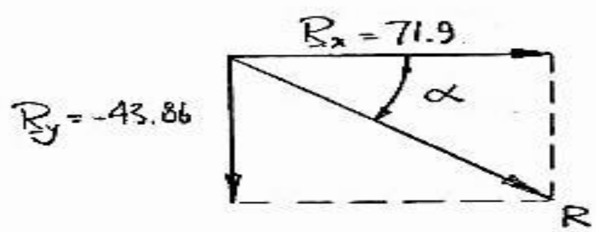
Determine the x and y components of each of the forces. Determine the resultant of the three forces.



SOLUTION

The components of the forces

Force	x comp.	y comp.
40	−30.6	−25.7
60	30	−51.96
80	72.5	33.8
	$R_x = 71.9$	$R_y = -43.86$



$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j}$$

$$= (71.9)\mathbf{i} - (43.86)\mathbf{j}$$

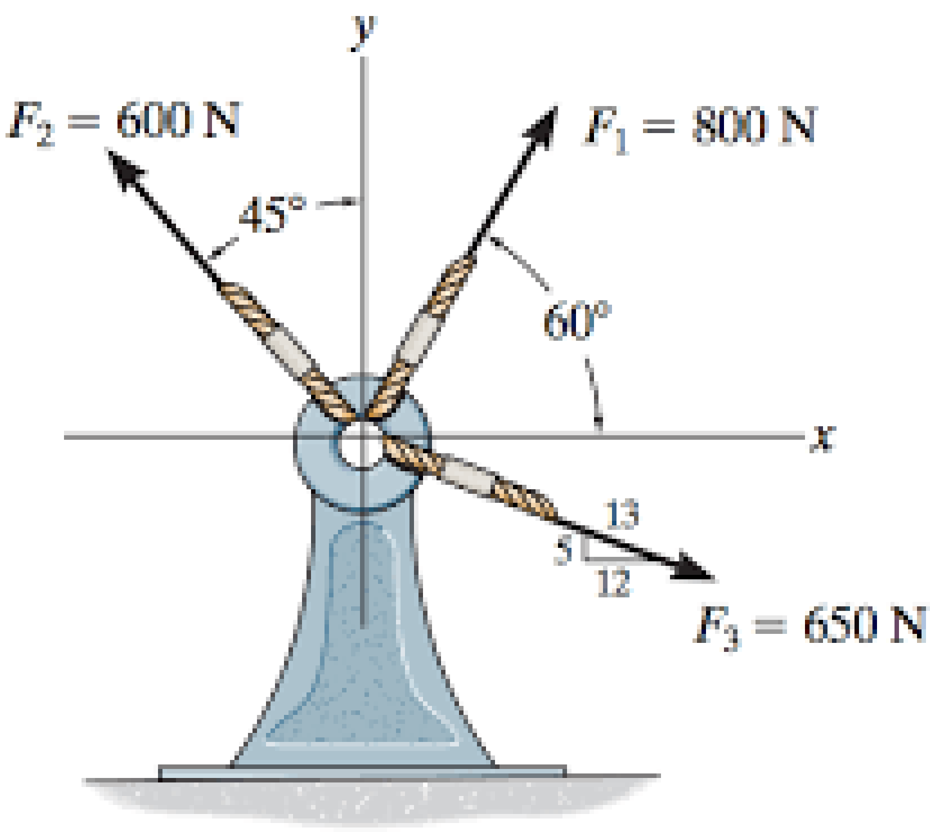
$$\tan\alpha = \frac{43.86}{71.9}$$

$$\alpha = 31.38°$$

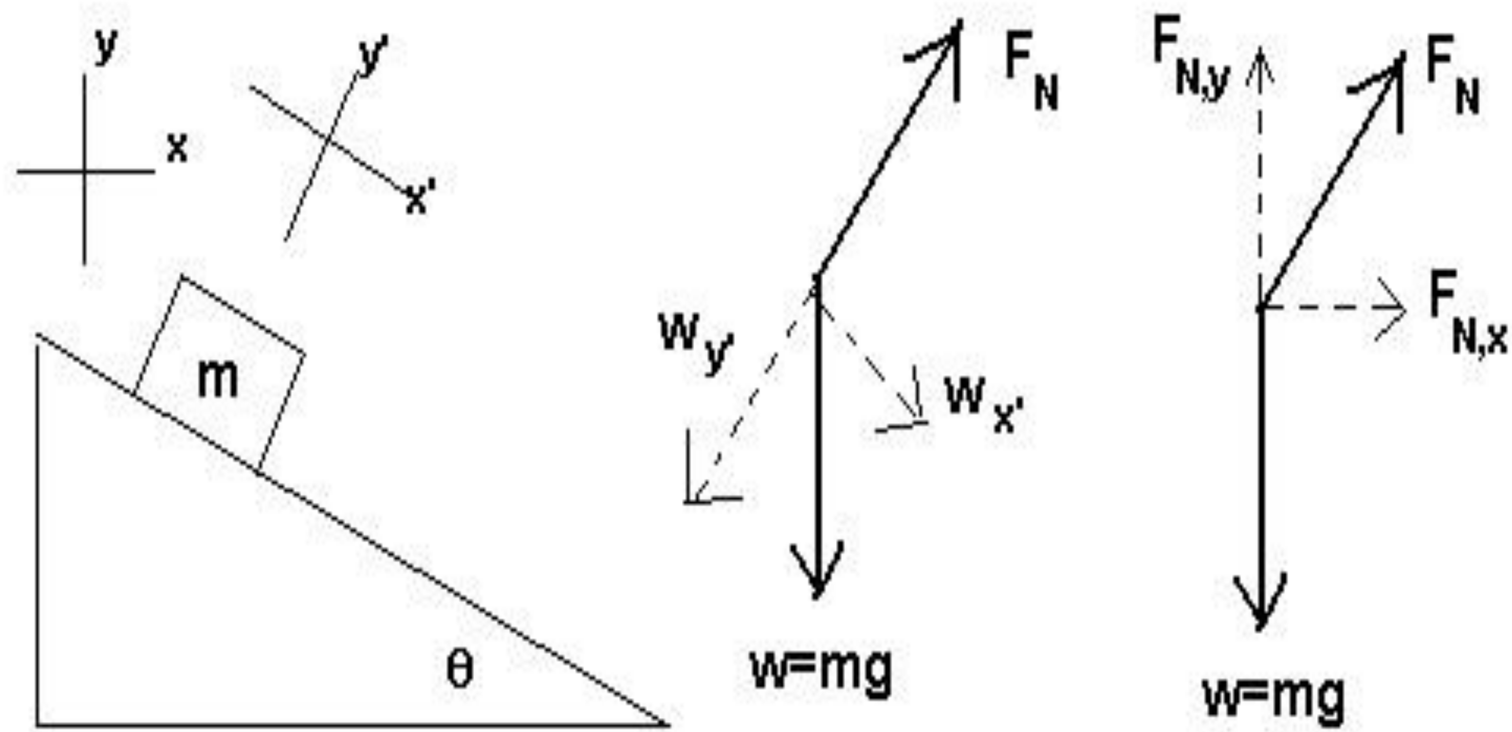
$$R = \sqrt{(71.9)^2 + (-43.86)^2}$$

$$= 84.23$$

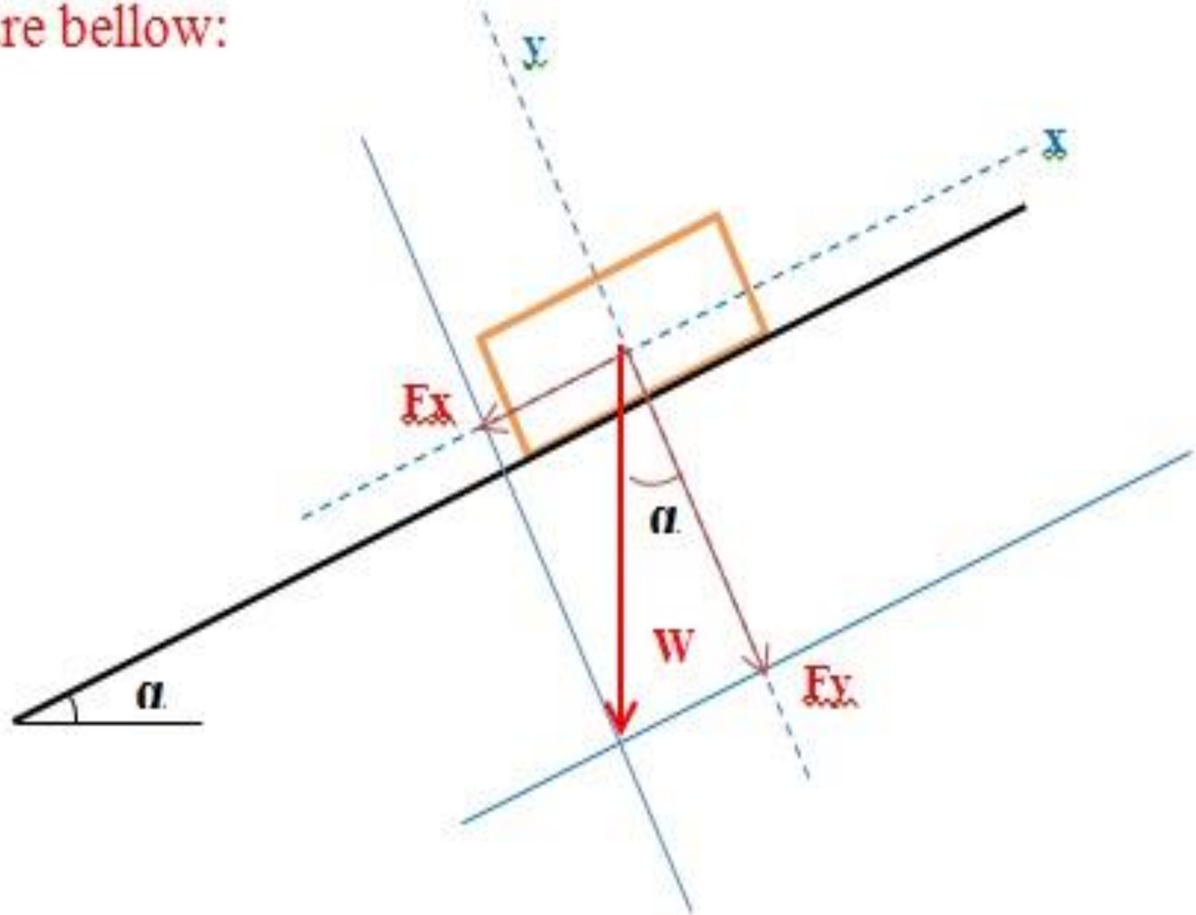
$\mathbf{R} = 84.2$ ⦪ $31.4°$


y
$F_2 = 600$ N
$F_1 = 800$ N
45°
60°
x
13
5
12
$F_3 = 650$ N

Inclined Coordinates



Ex: Find the components of the weight W along and perpendicular the incline that shown in figure bellow:



$$F_x = W\ Sin\ \alpha$$

$$F_y = W\ Cos\ \alpha$$

اذا كان هناك مستقيمان بينهما زاوية فالعمودان عليهما بينهما نفس الزاوية