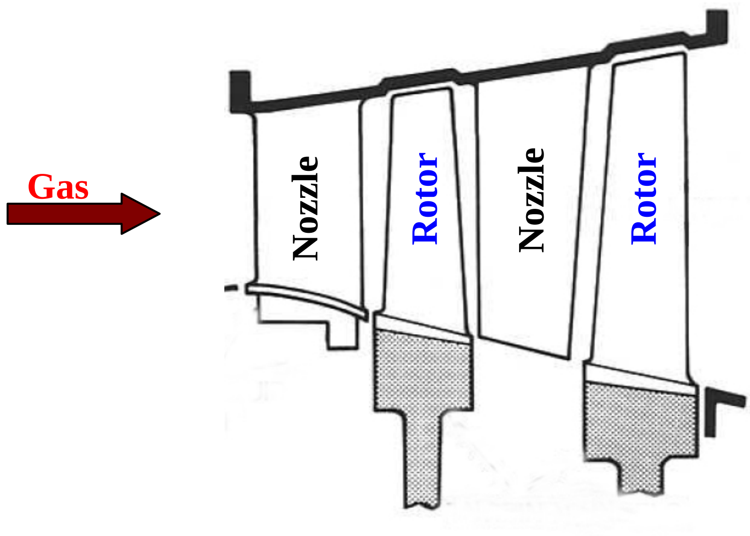
Axial Flow Gas Turbine

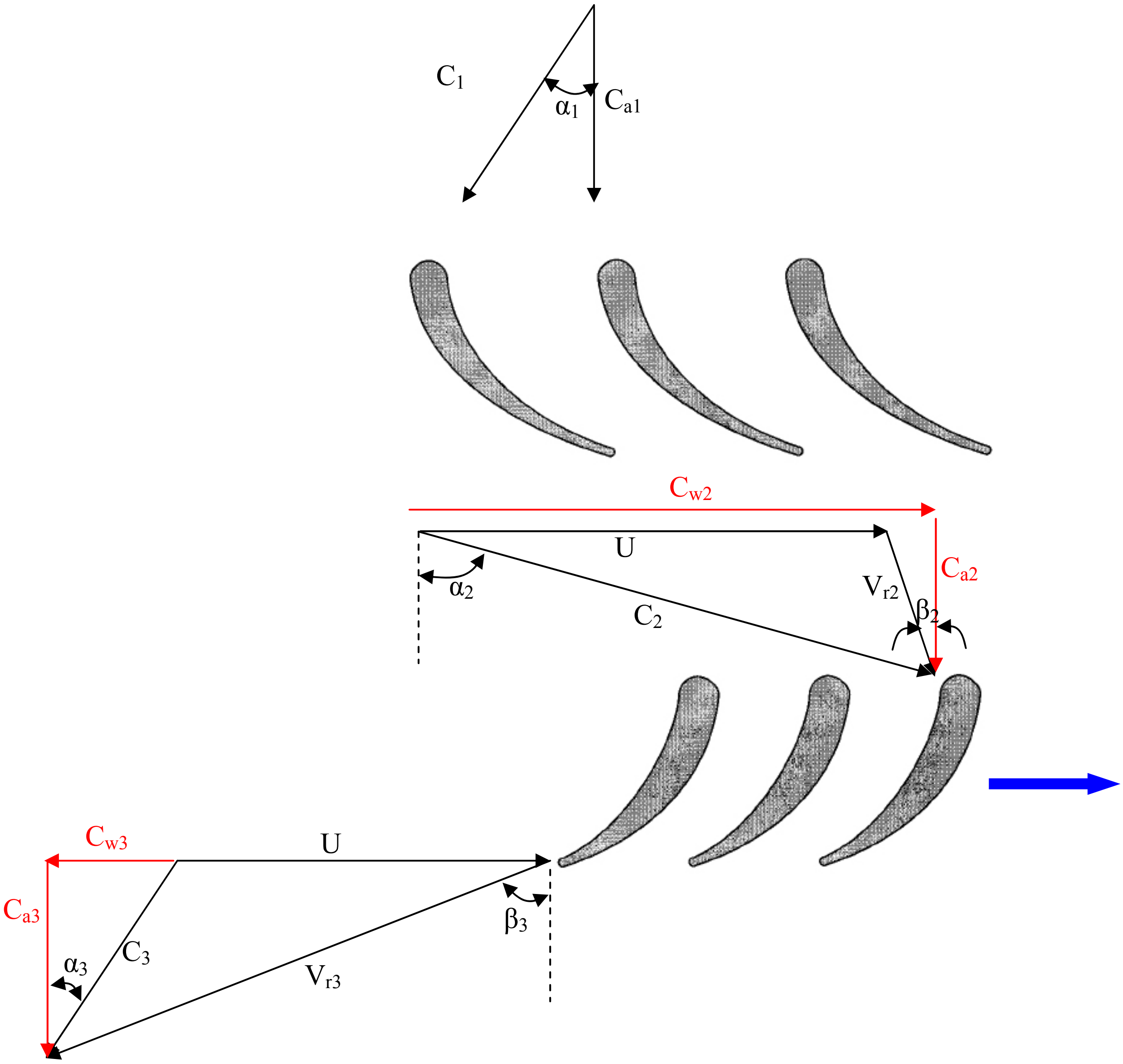


- If the stage is of many in a multi stage turbine, α_1 and C_1 will probably be equal to α_3 and C_3 respectively, so that same blade shapes can used in successive stages: it is then some times called "repeating stage"

- The change in axial components (C_{a2}-C_{a3}) produces an axial thrust on the rotor.

- In gas turbine the net thrust on the turbine rotor will be partially balanced by the thrust on the compressor rotor and no large thrust bearing are required.

- The axial component Ca is always designed to be constant through the rotor. Hence, the gas flow passage must take an annulus flared to compensate the decrease in density as gas expands through stage.



Annulus area	**Nozzle Throat Area**
$A_{Annulus} = \dfrac{\dot{m}}{\rho C_{a2}}$	$A_{Nozzle} = \dfrac{\dot{m}}{\rho C_2}$

From velocity diagram :

$$\tan \alpha_2 = \frac{C_{w2}}{C_a} , \quad \tan \beta_2 = \frac{C_{w2} - U}{C_a}$$

$$\tan \alpha_2 - \tan \beta_2 = \frac{U}{C_a}$$

Also $\tan \alpha_3 = \dfrac{C_{w3}}{C_a}$, $\tan \beta_3 = \dfrac{U + C_{w3}}{C_a}$

$$\tan \beta_3 - \tan \alpha_3 = \frac{U}{C_a}$$

Hence : $\tan \alpha_2 - \tan \beta_2 = \tan \beta_3 - \tan \alpha_3 = \dfrac{U}{C_a}$

Momentum Balance

On gas: $T = \dot{m} \left(-C_{w3} r - C_{w2} r \right)$

On shaft: $T = \dot{m} r \left(C_{w3} + C_{w2} \right)$

$$\text{power} = \omega T = \frac{U}{\cancel{r}} \dot{m} \cancel{r} \left(C_{w3} + C_{w2} \right)$$

$$= \dot{m} U C_a \left(\tan \alpha_3 + \tan \alpha_2 \right)$$

$$\text{Work} = \frac{\text{Power}}{\dot{m}} = W$$

$\therefore$ $\text{Work} = U C_a \left(\tan \alpha_3 + \tan \alpha_2 \right)$

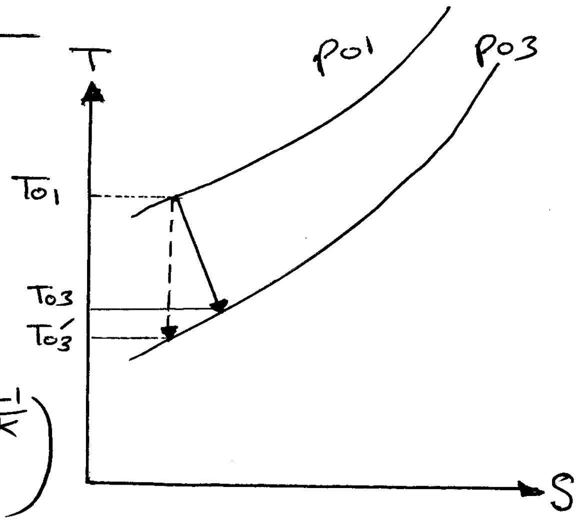
$W = C_p \, \Delta T_{os}$

$$\therefore \Delta T_{os} = \frac{U C_a (\tan\alpha_3 + \tan\alpha_2)}{C_p}$$

ΔT_{os} : stagnation temp. drop

$$\left.\begin{aligned} \Delta T_{os} &= \Delta T_s \\ T_{o1} - T_{o3} &= T_1 - T_3 \end{aligned}\right\} \text{Assuming } C_1 = C_3$$

Stage pressure Ratio



$$\frac{P_{o3}}{P_{o1}} = \left(\frac{T_{o3}'}{T_{o1}}\right)^{\frac{K}{K-1}}$$

$$\frac{T_{o1} - T_{o3}}{T_{o1} - T_{o3}'} = \eta_s$$

$$\therefore T_{o1} - T_{o3} = \eta_s \left(T_{o1} - T_{o1}\left(\frac{P_{o3}}{P_{o1}}\right)^{\frac{K-1}{K}}\right)$$

$$\therefore \Delta T_{os} = \eta_s T_{o1} \left[1 - \left(\frac{P_{o3}}{P_{o1}}\right)^{\frac{K-1}{K}}\right]$$

Non-Dimensional Parameters

① Nozzle loss coefficient :

$$\lambda_N = \frac{T_2 - T_2'}{(C_2^2 / 2C_p)}$$

② Blade loading Coefficient or temp. drop coefficient : ψ

* It express the work capacity of a stage

$$\psi = \frac{C_p \, \Delta T_{os}}{U^2/2} = \frac{2 C_p \, \Delta T_{os}}{U^2}$$

$$\psi = \frac{2 C_a}{U} \left(\tan \beta_2 + \tan \beta_3 \right)$$

③ Flow Coefficient : ϕ

$$\phi = \frac{C_a}{U}$$

④ Degree of Reaction : X

$$X = \frac{\text{static enthalpy change due to rotor}}{\text{static enthalpy change due to stage}}$$

$$X = \frac{C_p (T_2 - T_3)}{C_p (T_1 - T_3)}$$

$$C_p (T_1 - T_3) = U C_a (\tan \beta_3 + \tan \beta_2)$$

* There is no work done by the gas relative to the rotor ;

$$\therefore \quad 0 = \frac{1}{2}\left(V_{r3}^2 - V_{r2}^2\right) + C_p\left(T_3 - T_2\right)$$

$$\therefore \quad C_p\left(T_2 - T_3\right) = \frac{1}{2}\left(V_{r3}^2 - V_{r2}^2\right)$$

$$= \frac{1}{2} C_a^2 \left(\sec^2\beta_3 - \sec^2\beta_2\right)$$

$$= \frac{1}{2} C_a^2 \left(\tan^2\beta_3 - \tan^2\beta_2\right)$$

$$\therefore \quad X = \frac{C_a}{2U}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$X = \frac{\phi}{2}\left(\tan\beta_3 - \tan\beta_2\right)$$

Example: Refer to problem 7.1, Page-324 - of Gas Turbine theory.

- Mean diameter design, $\dot{m} = 20 \ \frac{kg}{s}$,

$T_{01} = 1000$ K, $P_{01} = 4$ bar, $C_a = 260 \ \frac{m}{s}$,

$U_m = 360$ m/s, $\alpha_2 = 65^\circ$, $\alpha_3 = 10^\circ$

Solution:

$$\tan\alpha_2 - \tan\beta_2 = \frac{U}{C_a}$$

$$\tan\beta_2 = \tan 65 - \frac{360}{260} = 2.1445 - 1.385$$

$$\tan\beta_2 = 0.7595 \Rightarrow \beta_2 = 37^\circ$$

$$\tan\beta_3 - \tan\alpha_3 = \frac{360}{260}$$

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$$\tan\beta_3 = 1.385 + 0.1763 = 1.5613$$

$$\Rightarrow \beta_3 = 57^\circ$$

$$X = \frac{C_a}{2U}\left(\tan\beta_3 - \tan\beta_2\right)$$

$$= \frac{260}{2\times 360}\left(1.5613 - 0.7595\right)$$

$$\therefore X = 0.289$$

$$\Delta T_{os} = \frac{U C_a}{C_p}\left(\tan\beta_3 + \tan\beta_2\right)$$

$$\therefore \Delta T_{os} = \frac{360\times 260}{1147}\left(1.5613 + 0.7595\right) =$$

$$\Delta T_{os} = 189.38\ K$$

$$\psi = \frac{2C_a}{U}\left(\tan\beta_3 + \tan\beta_2\right)$$

$$= \frac{2\times 260}{360}\left(1.56 + 0.72\right)$$

$$\psi = 3.35$$

$$\text{power} = \dot{m}\, C_p\, \Delta T_{os} = 20\times 1147 \times 189.38$$

$$= 4344.4\ kW$$

$$\lambda_N = \frac{T_2 - T_2'}{C_2^2/2C_p}$$

$$\dot{m} = \rho_2 A_2 C_a$$

$$A_2 = \frac{\dot{m}}{\rho_2 C_a}$$

$$\rho_2 = \frac{P_2}{RT_2}$$

$$T_{o2} = T_2 + \frac{C_2^2}{2C_p}$$

$$\cos\alpha_2 = \frac{C_a}{C_2}$$

$$\therefore C_2 = \frac{260}{\cos 65} = 615.2\,\frac{m}{s}$$

$$\therefore T_2 = 1000 - \frac{615.2^2}{2\times 1147}$$

$$= 835\,K$$

$$\frac{P_{o1}}{P_2} = \left(\frac{T_{o1}}{T_2'}\right)^{\frac{K}{K-1}}$$

$$T_2' = 0.05 \times \frac{615.2}{2\times 1147}$$

$$T_2' = 826.75\,K$$

$$\therefore P_2 = 1.869\text{ bar}$$

$$\therefore \rho_2 = \frac{1.869\times 10^5}{287\times 826.75}$$

$$= 0.788$$

$$A_{annulus} = \frac{\dot{m}}{\rho_2 C_a}$$

$$= \frac{20}{0.788 * 260}$$

$$A_{Annulus} = 0.0412\,m^2$$