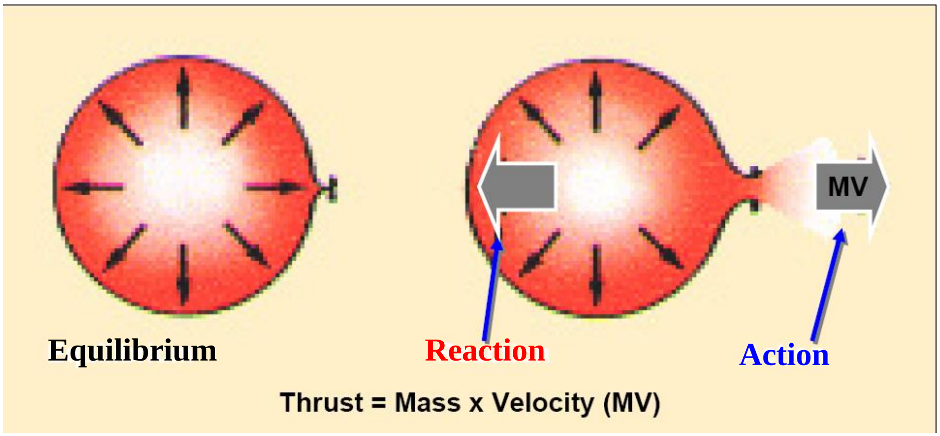
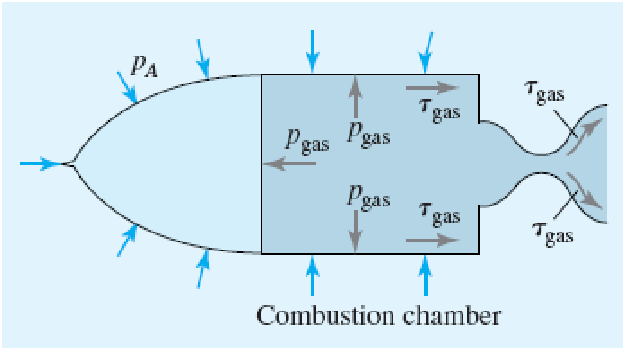
Jet Propulsion



1- Rocket Engine



$F_{th}=mV_e+P_eA_e-PaA_e$

$\quad =m[V_e+A_e/m(P_e-Pa)]$

$=mV_{ef}$

$V_{ef}= V_e+A_e/m(P_e-Pa)$

V_{ef}: is the effective jet velocity (resulting from the sum of exit gas velocity together with the velocity resulting from pressure difference)

$C_oT_o=C_pT+V_e^2/2$

$$\dot{m}_{\max} = \frac{A^*Po}{\sqrt{To}}\sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}}$$

$$F_{th} = \sqrt{2Cp(To-Te)}\,\frac{A^*Po}{\sqrt{To}}\sqrt{\frac{k}{R}\left(\frac{2}{k+1}\right)^{(k+1)/(k-1)}} + Ae(Pe-Pa)$$

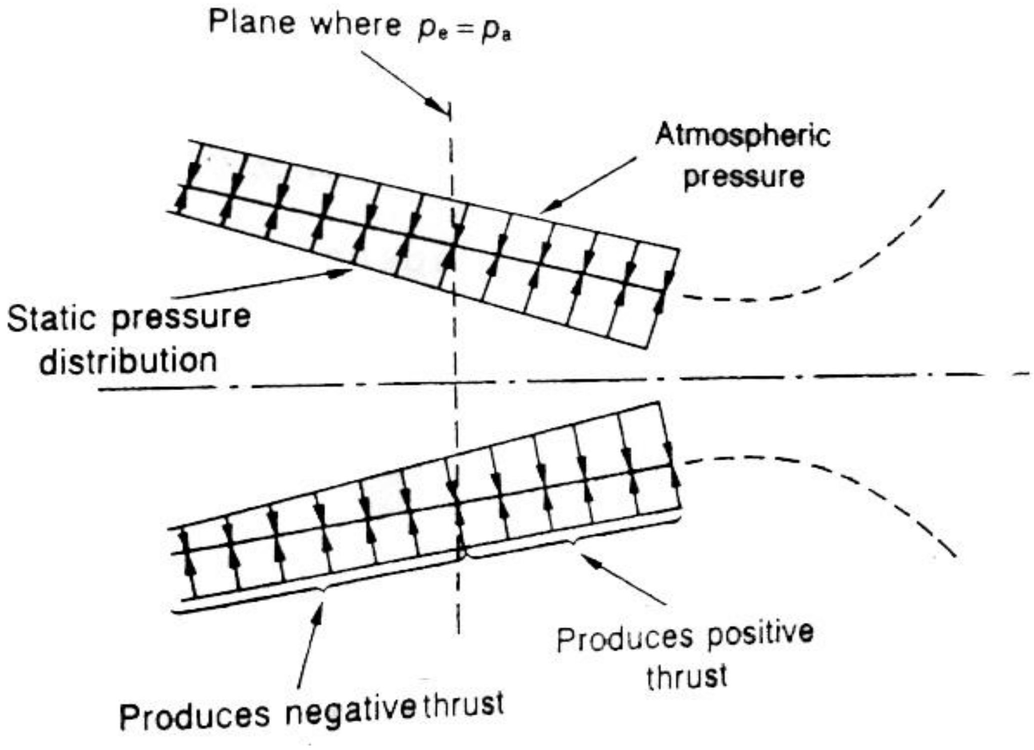
$$\frac{F_{th}}{A^*Po} = \sqrt{\frac{2kR}{k-1}(1-\frac{Te}{To})\frac{k}{R}(\frac{2}{k+1})^{(k+1)/k-1}} + \frac{Ae}{A*}(\frac{Pe}{Po}-\frac{Pa}{Po})$$

$$\frac{F_{th}}{A^*Po} = k\sqrt{\frac{2}{k-1}(1-(\frac{Pe}{Po})^{\frac{k-1}{k}})(\frac{2}{k+1})^{(k+1)/k-1}} + \frac{Ae}{A*}(\frac{Pe}{Po}-\frac{Pa}{Po})$$

$$\frac{F_{th}\max}{A^*Po} = k\sqrt{\frac{2}{k-1}(1-(\frac{Pe}{Po})^{\frac{k-1}{k}})(\frac{2}{k+1})^{(k+1)/k-1}}$$

To find Maximum thrust : Put $(dF_{th}/dA_e)=0$ ➔Pe=Pa

Hence, $\frac{F_{th}\max}{A^{*}Po} = k\sqrt{\frac{2}{k-1}(\frac{2}{k+1})^{\frac{k+1}{k-1}}[1-(\frac{Pa}{Po})^{\frac{k-1}{k}}]}$



Propulsion Efficiency

It is define as the thrust power to the total thrust power

$$\zeta_{pr} = \frac{F_{th}V}{FthV + \frac{\dot{m}}{2}(Vef - V)^2}$$

$$= \frac{\dot{m}V_{ef}.V}{\dot{m}V_{ef}.V + \frac{\dot{m}}{2}\left(V_{ef}^2 - 2VV_{ef} + V^2\right)} = \frac{V_{ef}.V}{\frac{1}{2}\left(V_{ef}^2 + V^2\right)}$$

$$\zeta_{pr} = \frac{2(V/V_{ef})}{1+(V/V_{ef})^2}$$

Ex:

The max thrust generated from a rocket is 2300 kN when its operating at an altitude where the ambient pressure is 5.1kPa and the combustion temperature is 2200 K. If the rocket uses 120 kg/s of gases, determine the stagnation pressure in the combustion chamber and the nozzle throat and exit areas. (Assume choked nozzle) (k=1.333, Cp=1147 J/kh.K, R=287J/kg)

Sol.

Fthmax=230 kN, Pe=Pa

To=2200 K

m_{max}=120 kg/s

A_t=A*

$Fth_{max}=mVe$ ➔ $Ve=230x10^3/120=1916.67$ m/s

$CpTo=CpTe+Ve^2/2$

$Te=To-(Ve^2/2Cp)=2200-(1916.67/2x1147)$

Te=598.6K

$$\frac{Po}{Pe}=\left(\frac{To}{Te}\right)^{\frac{k}{k-1}}$$ ➔ $Po=5.1(2200/598.6)^4=930.3$ kPa

$m=\rho_e AeVe$

$\rho_e=Pe/RTe=5.1x103/(287x598.6)=0.0296kg/m^3$

$Ae=120/(0.0296*1916.67)=2.11\ m^2$

$$\frac{Fth\,max}{A^* Po}=k\sqrt{\frac{2}{k-1}(1-(\frac{Pa}{Po})^{\frac{k-1}{k}})(\frac{2}{k+1})^{\frac{(k+1)}{k-1}}}$$

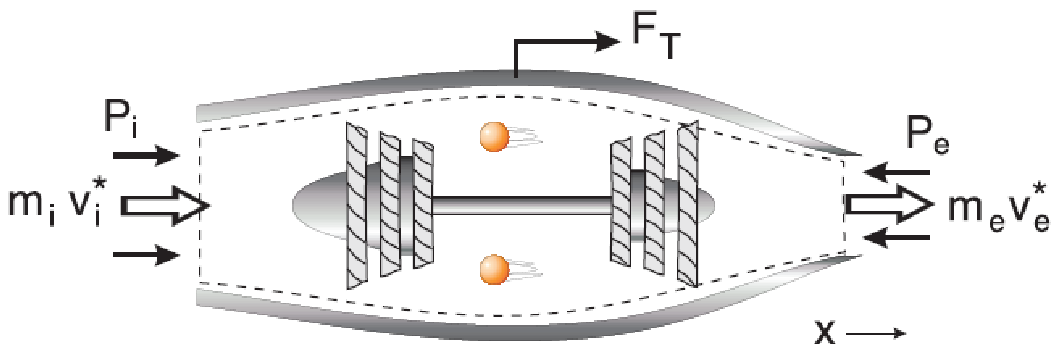
$$\frac{230}{930.3A^*}=1.333\sqrt{\frac{2}{1.333-1}(1-(\frac{5.1}{930})^{0.25})(\frac{2}{2.333})^{\frac{2.333}{0.333}}}=1.625$$

$A^*=0.152m^2$

Turbojet engine

Turbojet engine use gas turbine as main part in the engine. Aircraft gas turbine differs from shaft power cycles in:

1- The useful output power of aircraft engines is produced wholly or in part as a result of expansion nozzle.
2- In aircraft engine, the forward speed and altitude effect on the performance of the engine.
3- The (power/Weight) ratio must be considered in aircraft engine.
4- Gases are expanded in the turbine to a pressure where the turbine work is just equal to the compressor work plus some auxiliary power for pumps and generators i.e. the net work output is zero
5- Typically operate at higher pressure ratios, often in the range of 10 to 25



Momentum balance:

$$\frac{d(Mom)_{x,cv}}{dt} = (\dot{M}om)_{x,in} - (\dot{M}om)_{x,out} + \sum F_x$$

For steady flow;

$0=m_o Ve+PeAe-(P_a A_e+m_i V)+\Sigma F_x$

$m_o=m_i+m_f$

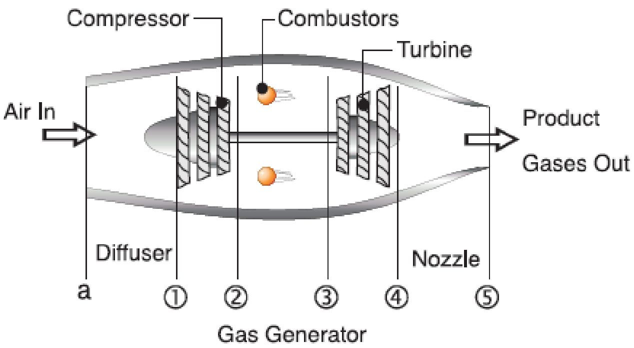
It is always preferred to use excess air for cooling process, hence the fuel to air ratio mostly 1/50, so that

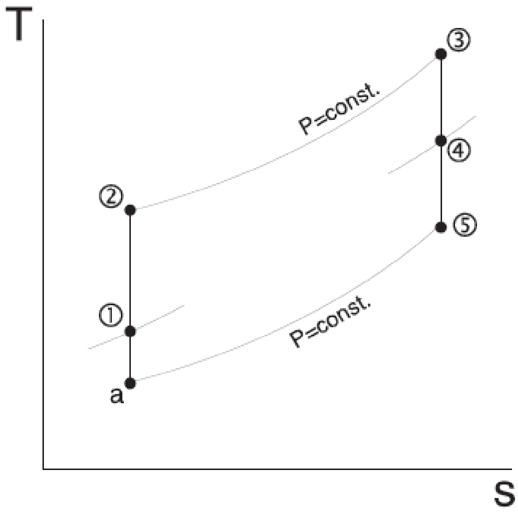
$m_f<<m_i$

Hence:

$$\boxed{F_{th}=m_o(V_e-V)+A_e(P_e-P_a)}$$

Simple Turbojet engine Cycle:

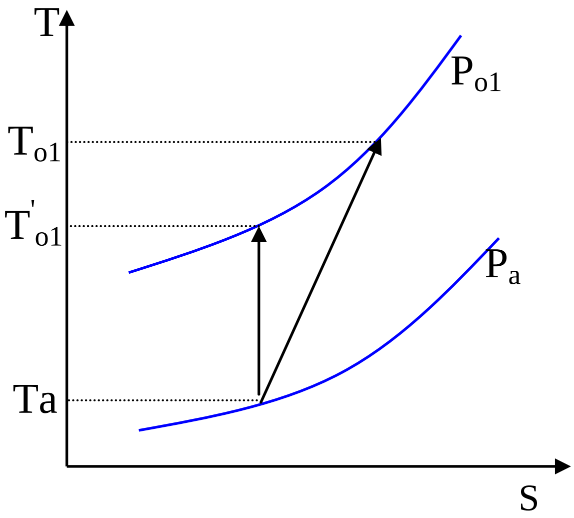




Diffuser

$$\zeta i = \frac{T'_{o1} - T_a}{T_{o1} - T_a} :$$

is the intake usentrioc efficiency defined in term of temperature



$$\frac{T'_{o1}}{T_{o1}} = \left(\frac{P_{o1}}{P_a}\right)^{k-1/k}$$

$$\left(\frac{P_{o1}}{P_a}\right) = \left(\frac{T'_{o1}}{T_a}\right)^{k/k-1}$$

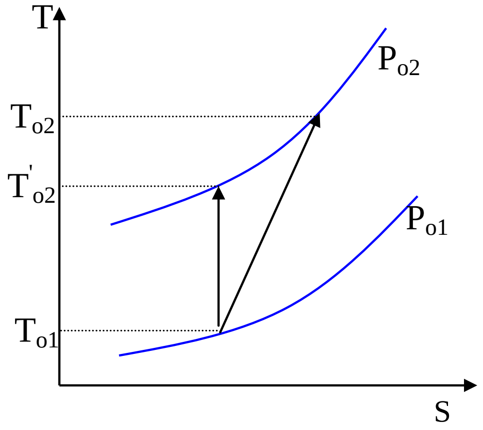
$T'_{o1}=T_a+(T_{o1}-T_a)\zeta_i$

$$\left(\frac{P_{o1}}{P_a}\right) = \left(\frac{T_a + (T_{o1} - T_a)\zeta_i}{T_a}\right)^{k/k-1}$$

$$\left(\frac{P_{o1}}{P_a}\right) = \left(1 + \frac{(T_{o1} - T_a)\zeta_i}{T_a}\right)^{k/k-1}$$

$T_{o1}=T_a+V_2/2Cp$

Compressor



$$\zeta_c = \frac{T'_{o2} - T_{01}}{T_{o2} - T_{01}}$$

ζc: is the compressor isentropic efficiency

$$\left(\frac{P_{o2}}{P_{o1}}\right) = \left(\frac{T'_{o2}}{T_{o1}}\right)^{k/k-1}$$

$$\left(\frac{P_{o2}}{P_{o1}}\right) = \left(1 + \frac{(T_{o2} - T_{o1})\zeta_c}{T_{o1}}\right)^{k/k-1}$$

P_{o2}/P_{o1} is the compressor pressure ratio

Polytropic efficiency of the compressor

$$\left(\frac{P_{o2}}{P_{o1}}\right) = \left(\frac{T_{o2}}{T_{o1}}\right)^{k.\zeta c\infty/k-1}$$

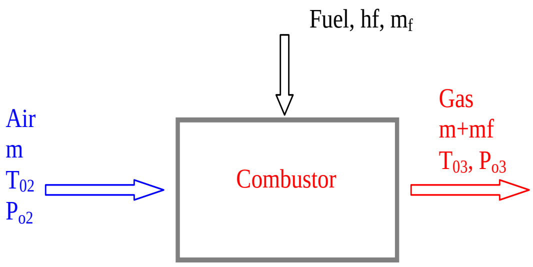
Where $\zeta_{c\infty}$ is the compressor polytropic efficiency which is should be discussed later in the subject of the *axial flow compressor*.

Combustion chamber

Energy Balance

298=Room temp.(25°C+273)

$mC_{pa}(T_{o2}-298)+m_f hf=m(1+f_t)C_{pg}(T_{o3}-298)$



$f_t=m_f/m$ is the theoretical fuel to air ratio

$$f_t = \frac{C_{pg}(T_{o3} - 298) - C_{pa}(T_{o2} - 298)}{hf - C_{pg}(T_{o3} - 298)}$$

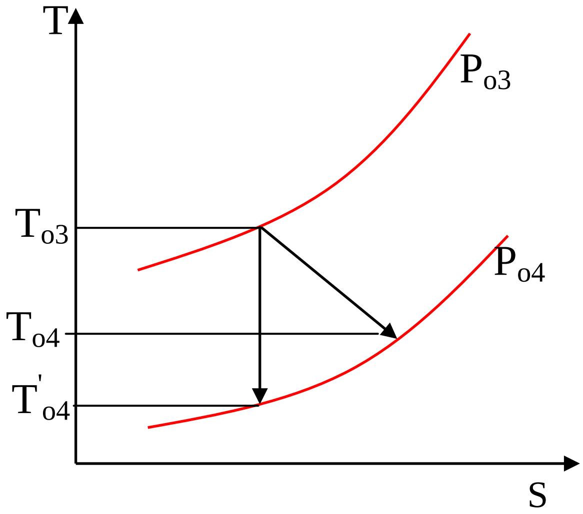
hf: Enthalpy of reaction (depends on fluid type), for most aircraft fuels its value is 43100 kJ/kg.K

Combustion efficiency $\zeta_b=f_t/f_a$

f_a : is the actual fuel to air ratio

Combustion pressure loss $\Delta_{Pb} = P_{o2} - P_o$

<u>Turbine</u>



$$\zeta_t = \frac{T_{o4} - T_{o3}}{T'_{o4} - T_{o3}}$$

ζ_t: is the isentropic turbine efficiency

$$\left(\frac{P_{o4}}{P_{o3}}\right) = \left(\frac{T'_{o4}}{T_{o3}}\right)^{k/k-1}$$

$$\left(\frac{P_{o4}}{P_{o3}}\right) = \left(1 + \frac{(T_{o4} - T_{o3})}{\zeta_t T_{o3}}\right)^{k/k-1}$$

Polytropic efficiency of the turbine

$$\left(\frac{P_{o4}}{P_{o3}}\right) = \left(\frac{T_{o4}}{T_{o3}}\right)^{k/\zeta_{t\infty}(k-1)}$$

Where $\zeta_{c\infty}$ is the turbine polytropic efficiency which is should be discussed later in the subject of the *axial flow turbine*.

<u>Mechanical transmission</u>

$$\zeta_m = \frac{Compressor \quad power}{Turbine \quad power} = \frac{mC_{pa}(T_{o2} - T_{o1})}{mC_{pg}(T_{o3} - T_{o4})}$$

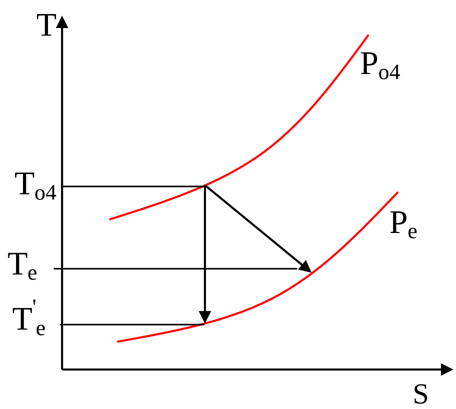
$$\zeta_m = \frac{C_{pa}(T_{o2} - T_{o1})}{C_{pg}(T_{o3} - T_{o4})}$$

ζ_m : is the mechanical transmission efficiency

<u>Nozzle</u>

$$\zeta_j = \frac{T_e - T_{o4}}{T'_e - T_{o4}}$$

ζ_j: is the nozzle isentropic efficiency



$$\left(\frac{P_e}{P_{o4}}\right)=\left(1+\frac{(T_e-T_{o4})}{\zeta_j T_{o4}}\right)^{k/k-1}$$

Specific thrust is the net thrust per unit mass flow of air (e.g. Ns/kg)

$Fs=F_{th}/m$

Specific fuel consumption:

S.f.c.=m_f/F_{th} (kg/s/N)

Example

A single-spool turbojet engine is cruising with Mach number of 0.8 with ambient conditions of 55 kPa and 258 K. The mass flow rate interring the engine is 10 kg/s. The compressor pr. ratio is 9.0 and the maximum temp. in the engine is 1200 K. The intake isentropic efficiency is 90% and the compressor isentropic efficiency is 87%, the turbine polytropic efficiency is 86%. The mechanical efficiency is 98% and the combustion pr. loss is 3 percent of compressor delivery pressure. Calculate the generated thrust force.

Solution

$V=MC=0.8(1.4*287*258)^{1/2}= 257.6$ m/s

$T_{o1}=T_a+V^2/2C_p=258+(257.6)^2/2010= 291K$

$$\left(\frac{P_{o1}}{P_a}\right)=\left(1+\frac{(T_{o1}-T_a)\zeta_i}{T_a}\right)^{k/k-1}$$

$P_{o1}=81.34$ kPa

$P_{o2}=9*81.34=732$ kPa

$$\left(\frac{P_{o2}}{P_{o1}}\right)=\left(1+\frac{(T_{o2}-T_{o1})\zeta_c}{T_{o1}}\right)^{k/k-1} \quad \rightarrow T_{o2}=583\text{ K}$$

To3=1200 K (given)

$P_{o3}=p_{o2}-0.03*P_{o2}=0.97*732=710$ kPa

$C_{pa}(T_{o2}-T_{o1})= \zeta_m C_{pg}(T_{o3}-T_{04})$

$T_{o4}=T_{o3}-(C_{pa}/C_{pg}*\zeta_m)(T_{o2}-T_{o1})=993$ K

$$\left(\frac{P_{o4}}{P_{o3}}\right)=\left(\frac{T_{o4}}{T_{o3}}\right)^{k/\zeta_{t\infty}(k-1)} \quad \rightarrow \text{Po4}=227\text{ kPa}$$

Nozzle

Assume Me=1

$P^{*}=(2/(k+1))^{4} * Po4= 122.6$ kPa

$P^{*}>Pa \rightarrow$ choked nozzle

$T^{*}=(2/2.333)T_{o4} =805$ K

$V^{*}=(kRT^{*})^{1/2}=555$m/s

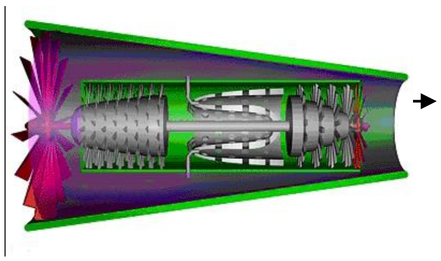
$m=\rho_e A_e V_e$

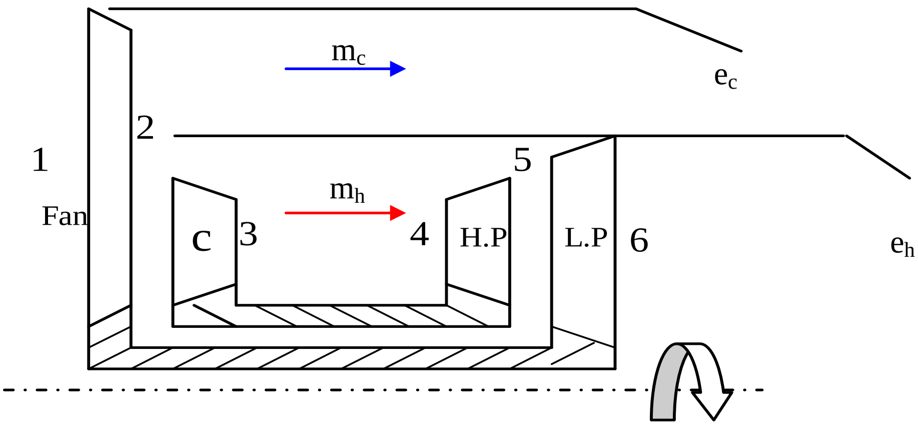
$\rho_e=P^{*}/RT^{*}=122.6x10^{3}/(287x805)=0.53kg/m^{3}$

$A_e=0.034m^{2}$

$Fth=m(Ve-V)+Ae(Pe-Pa)=10(555-257.6)+0.034(122.6-54)x10^{3}=5306.4$ N

Twin Spool-Turbofan Engine



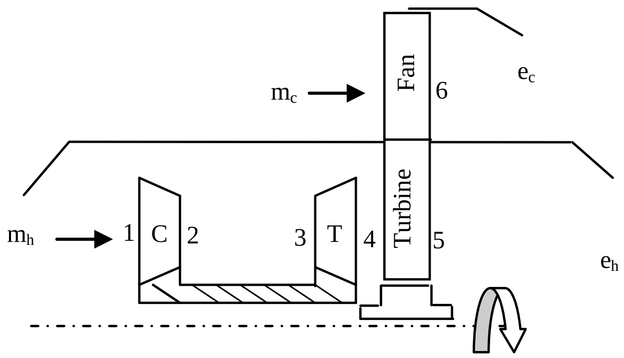


- ✓ The turbofan engine was developed as a method of improving the propulsive efficiency of the jet engine by reducing the mean jet velocity.
- ✓ But reducing the jet velocity has a considerable effect on jet noise.
- ✓ The thrust is made up of two componenets, the cold stream thrust and the hot stream thrust.
- ✓ The by-pass ratio ($B=m_c/m_h$) is usualy more than unity

$m=m_h+m_c \quad m=m_h(1+B)$

$$F_{th}=(m_cV_{ec}+m_hV_{eh})-mV+A_{eh}(P_{eh}-P_a)+A_{ec}(P_{ec}-P_a)$$

Aft-Fan Engine



The turbofan engine was develpoed from existing turbojet engine to 'aft fan' configutation.

A combined turbine-fan wa mounted downstream of the gas turbine. Two major problems arise with this configuration:

- ✓ The blading of the turbine-fan unit must be desinged to give turbine bladesection for the hot stream and compressor blade section for the cold stream, and this leads to high cost.
- ✓ The problem of sealing between the two streams?

Improoving the Engine Performance

The improvement and development of the jet propulsion engine is to be in:
1- Increasing fuel consumption (*s.f.c.*)
2- Reducing weight
3- Increasnig thrust

1-Iprooving *s.f.c.*

The *s.f.c.* could be improoved by:

- Higher turbine inlet tempearture
- Higher compressor pressure ratio
- Improoving the efficiencies of intake, combustion chamber, turbine and exahust nozzle.

It is found that:

→ 1% increase in compressor efficeiency can reduce the *s.f.c.* to 0.3% - 0.5%

→ 1% increase in turbine efficienecy can reduce the *s.f.c.* to 0.6-0.8%

2-Reducing Weight

The present researches is to reduce the turbine compressor length (hence, weight) i.e . reducing the number of stages. This require a special design of blades.
The engines of this decade operate at much higher temps. and prs. and higher RPMs than the engine of earlier decades with lighter and stonger structure components. This improvent have reduced the engine weight to half of the engine weight of 50s and 60s.

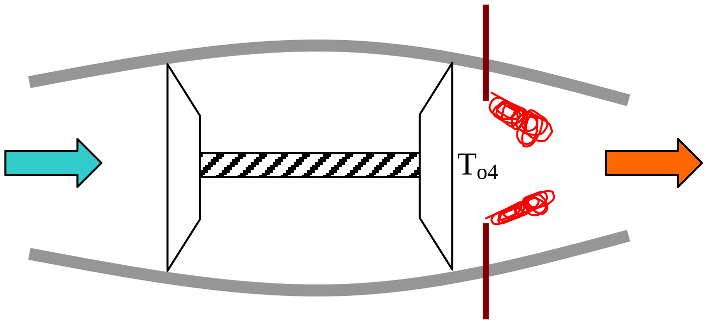
3-Incresing Thrust

It is necessary to increase thrust for short periods of time like, take-off, climb or combat performance. Two methods are followed for this purpose

1-Afterburning

The gases leaving the turbine are reheated in order to increase the velocity of the gas stream at the engine exit. Afterburner have been used in both turbojet and turbofan engines.

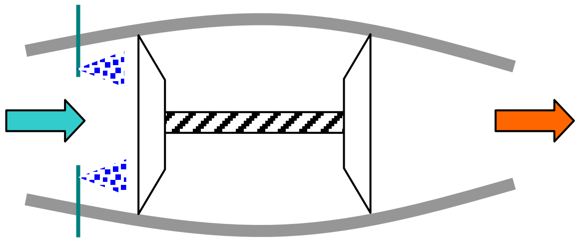
$$Ve=(2C_p(T_{o4}-T_e))^{1/2}$$



2-Water injection

(A) Water injection into the inlet of the compressor

- ✓ Reduces the compressor inlet temperature which increase the mass flow rate of the engine.
- ✓ Reduces the work needed fot the compressor.



(B) Water injection into the inlet of the combustion chamber

- ✓ Increases the mass flowrate through the turbine for a given compressor flow rate.