Theory of Turbo machines

In turbo machine theory:

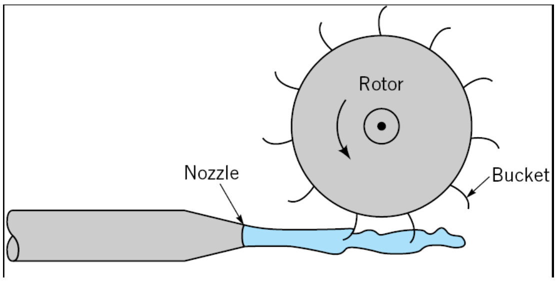
1- Friction is neglected
2- The fluid is assumed to have perfect guidance through the machine, i.e., an infinite number of thin vanes.

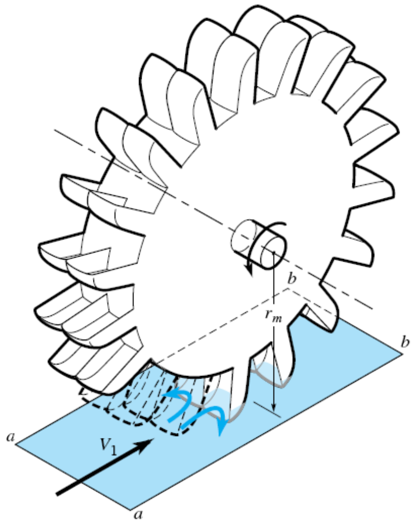
Water Turbines

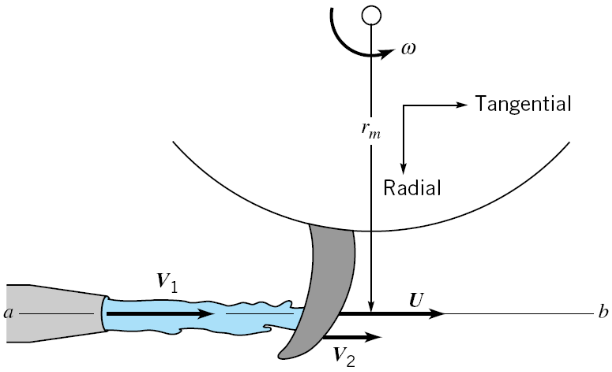
1- Impulse Turbine

All the available energy of the flow is converted by a nozzle into kinetic energy at atmospheric pressure before the fluid contacts the moving blades. It is suitable for high heads.

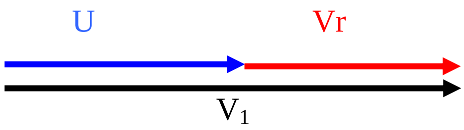
The most common type of these turbines is Pelton Wheel which is composed of:
1-One or more stationary inlet nozzle(s).
2-Runner attached to it a number of buckets
3- Casing





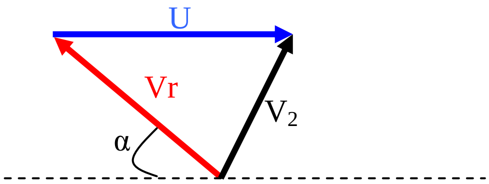


Velocity Diagram at Inlet



$V_1 = U + V_r$

Velocity Diagram at Exit



$\mathbf{V_1 = U + V_r}$

V : Liquid absolute velocity
U: blade speed
Vr: Liquid relative velocity
α: Bucket angle

Notes:

1- Ignoring the friction between the liquid and the blades and since the inlet and exit ports are in at atmospheric pressure, hence Vr is the same at inlet and exit.
2- The absolute velocity of liquid at exit V_2 represents waste energy.

In order to calculate the inlet absolute velocity (jet velocity), apply Bernoulli equation:

$Pa + 0 + 1/2\ \rho gh = Pa + 1/2\ \rho V_1^2$

$V_1 = \sqrt{2gh}$

And taking into account the nozzle opening loss, then:

$V_1 = Cv\sqrt{2gh}$

Where Cv is the nozzle coefficient.

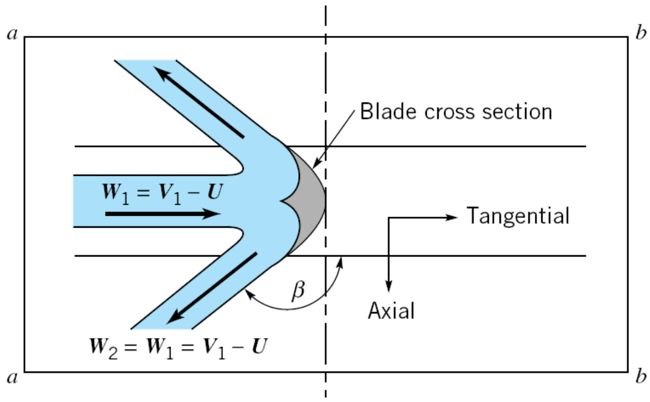
Momentum balance

$\Sigma F = M_{out} - M_{in}$

$F = -\rho\, Q\, V_r \cos\alpha - \rho\, Q\, V_r$ (of liquid)

$F = -\rho\, Q\, (V_1 - U)(1 + \cos\alpha)$ (of liquid)

$F = \rho\, Q\, (V_1 - U)\, (1 + \cos\alpha)$ (on blades)



Power of Turbine (W_T)

$W_T = \omega\, T$

$\omega = U/r$

$T = F \times r$

Hence:

$$\boxed{W_T = \rho\, Q\, U\, (V_1 - U)\, (1 + \cos\alpha)}$$

Q is the total flow rate of water
$Q = A\, V_1$
A is the area of the nozzle
$A = \pi/4(dj^2)$
dj is the nozzle diameter

Speed Factor

$$\phi = \frac{U}{\sqrt{2gh}}$$

From the output power equation above, it could be concluded at first sight that the maximum power is attained when the value of α equals to zero. But this is not attainable practically as the exiting liquid must stay free of the tailing buckets. To obtain the condition of the maximum power we derive the power relation with respect to U:

$dW_T/dU = \rho\, Q\,\, (V_1 - 2U)\, (1 + Cos\alpha) = 0$

hence $U = V_1/2$ or ($\Phi = Cv/2$) is the condition of the maximum power

$W_{Tmax} = \rho\, Q\, U^2\, (1 + Cos\alpha)$

Turbine Efficiency

$$\zeta_T = \frac{W_T}{\gamma Q h}$$

$$\zeta_T = \frac{\rho Q U (V_1 - U)(1 + \cos\alpha)}{\rho g Q h}$$

$$\boxed{\zeta_T = 2\Phi\, (Cv - \Phi)(1 + \cos\alpha)}$$

Example

A Pelton wheel is supplied with $0.035 m^3/s$ of water under a head of 92 m. The wheel rotates at 725 rpm and the velocity coefficient of the nozzle is 0.95. The efficiency of the wheel is 82% and the speed factor is 0.45. Determine the following:
1. Speed of the wheel
2. Wheel to jet diameter ratio
3. The developed power
4. The bucket angle

Solution

U=ω r   Or $\phi = \dfrac{U}{\sqrt{2gh}}$ hence, U=0.45 x (2 x 9.81 x 92)$^{1/2}$= 19.12 m/s

Wheel diameter D

ω=U/r   Or r=U/ ω   hence, r=19.12/(2π x 725/60)= 0.252 m

Hence D=2r = 0.504 m

Jet diameter d

Q=A_1 V_1 = π/4 d^2 x V_1

V1=Cv (2gh)$^{1/2}$ = 0.95 (2 x 9.81 x 92)$^{1/2}$ =40.36 m/s

0.035= π/4 d^2 x 40.36 → d= 0.033 m

Hence the ratio D/d= 0.504/0.033 =15.12

$$\zeta_T = \frac{W_T}{\gamma Q h}$$

Hence WT=0.82 x 9810 x 0.035 x 92 = 25902 Watts

ζ_T=2Φ (Cv - Φ)(1 + cos α)

0.82 = 0.9 (0.95-0.45)(1+cos α)

cos α*=0.822*

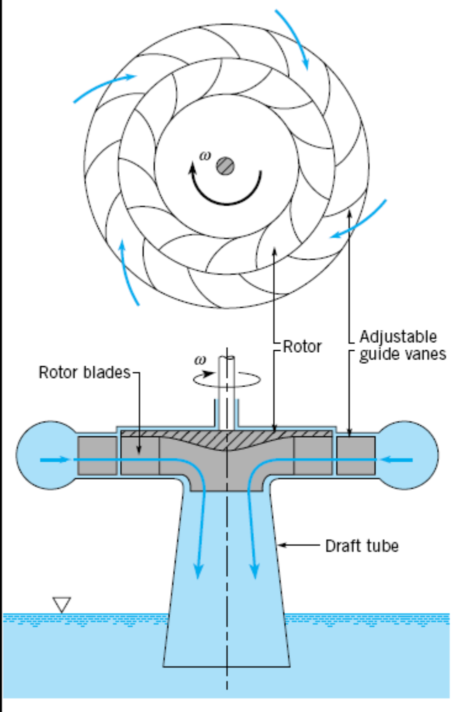
Hence α*=34.7^o*

Reaction Turbine

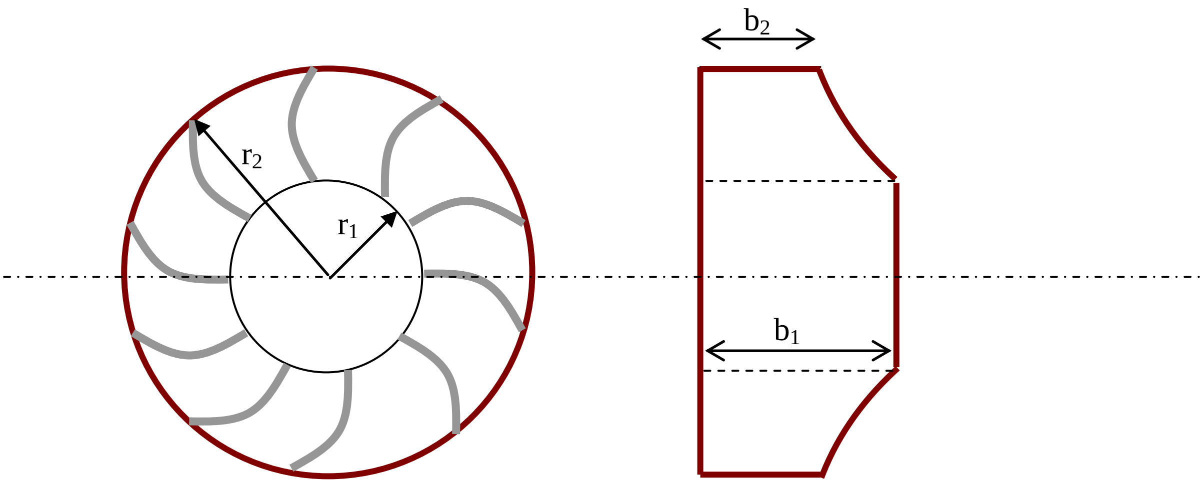
In the reaction turbine a portion of the energy of the fluid is converted into kinetic energy by the fluid's passing through fixed vanes before entering the runner, and the remainder of conversion takes place through the runner.

1- Francis Turbine

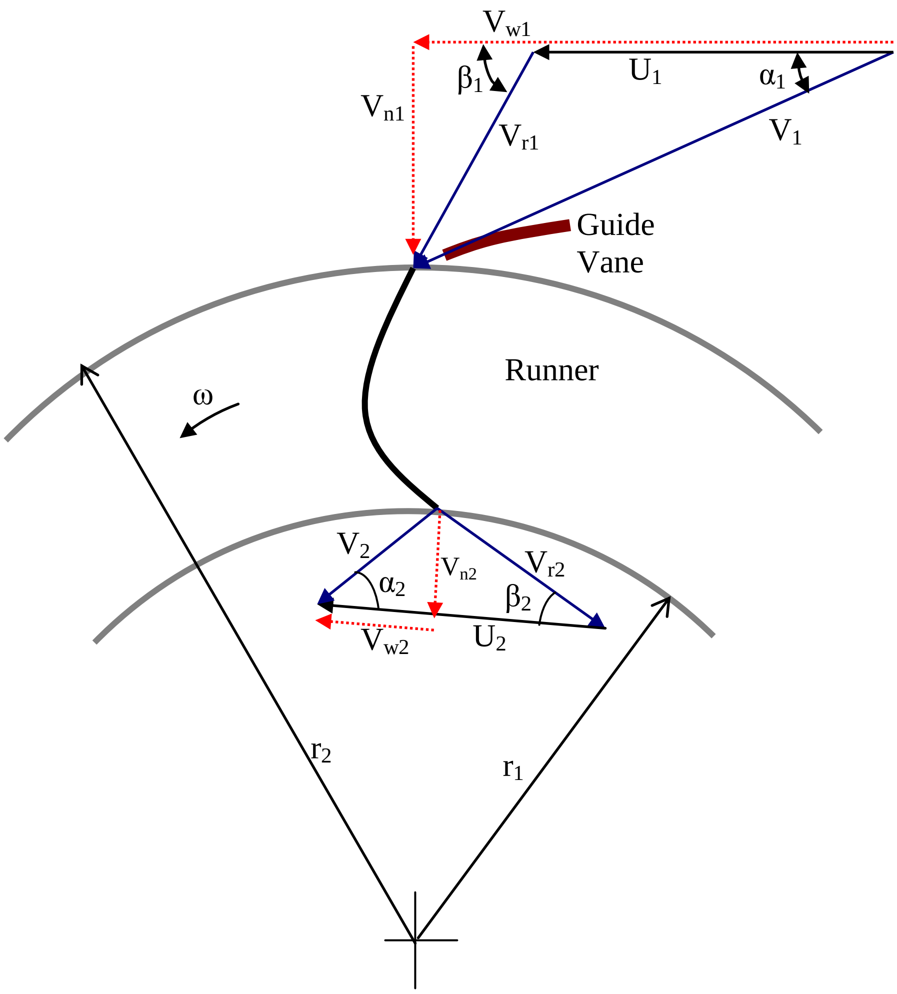
- ✓ The function of the guide vane is to control the tangential component of velocity at the runner inlet.
- ✓ The function of the draft tube is to convert the kinetic energy remaining in the liquid into flow energy.
- ✓ The pressure at the runner exit is below atmospheric.



(a)



b2
r2
r1
b1



Vw1
Vn1
β1
U1
α1
Vr1
V1
Guide
Vane
Runner
ω
V2
Vn2
Vr2
α2
β2
Vw2
U2
r2
r1

α_1: guide vane angle
β_1: blade angle at inlet
α_2: Exit fluid angle
β_2: blade angle at exit
V_1: absolute velocity at inlet
V_2: absolute velocity at exit
V_{r1}: relative velocity at inlet
V_{r2}: relative velocity at exit
U_1: Blade or runner speed at inlet
U_2: Blade or runner speed at exit
$V_{n1}=V_1 \sin \alpha_1$: normal component of V_1
$V_{n2}=V_2 \sin \alpha_2$: normal component of V_2
$V_{w1}=V_1 \cos \alpha_1$: tangential component of V_1
$V_{w2}=V_2 \cos \alpha_2$: tangential component of V_2

Momentum Balance

$T=\rho Q(V_{w1} \times r_1 - V_{w2} \times r_2)$

$W_T=\omega T$
$W_T=U/r\ \rho Q(V_{w1} \times r_1 - V_{w2} \times r_2)$

$$\boxed{W_T= \rho Q(V_{w1}U_1 - V_{w2}U_2)}$$

$$\zeta_T = \frac{W_T}{\gamma Q h}$$

$Q= A\ Vn = 2\pi \times r_1 \times b_1 \times V_{n1} = 2\pi \times r_2 \times b_2 \times V_{n2}$

Guide Vane Angle Relation
Referring to the inlet velocity diagram:
$V_{w1}=U_1+ V_{n1}/\tan\beta_1$
$\tan\alpha_1=V_{n1}/V_{w1}$

and from flow rate relation $V_{n1}=Q/(2\pi\ r_1\ b_1)$

Hence,
$\cot \alpha_1\ Q/(2\pi\ r_1\ b_1) =\omega\ r_1 + \cot \beta_1\ Q/(2\pi\ r_1\ b_1)$

$$cot\,\alpha_1 = \frac{2\pi\, r_1 b_1}{Q}\,\omega + cot\,\beta_1$$

$$\alpha_1 = cot^{-1}\left[\frac{2\pi\, r_1 b_1}{Q}\,\omega + cot\,\beta_1\right]$$

This relation discover the importance of controlling the angular speed by varying the guide vane angle to components any changes in the flow rate.

Notes:

1- Radial Discharge: $\alpha_2 = 90^o \rightarrow V_{w2}=0 \rightarrow W_T = \rho Q V_{w1} U_1$

2- Radial inlet blades: $\beta_1 = 90^o \rightarrow V_{w1} = U_1$

<u>Example</u>: A Francis turbine has radii of r_1=0.2 m and r_2=0.1 m, operates under the following conditions: Q=0.06 m^3/s, rotational speed = 240 rpm, guide vane angle=30^o $\alpha_2=80^o$, absolute velocity at inlet and outlet, respectively, 6 m/s and 3 m/s. Take the turbine efficiency as 90%. Find (1) the developed power (2) the applied head (3) inlet blade angle and (3) the thickness of the inlet impeller.

Solution:

$V_{w1}=V_1 \cos\alpha_1 = 6 \cos 30 = 5.2$ m/s

$V_{w2}=V_2 \cos\alpha_2 = 3 \cos 80 = 0.521$ m/s

$U_1=\omega r_1 = 2\pi \times 240/60 * 0.2 = 5.03$m/s

$U_2=\omega r_2 = 2\pi \times 240/60 * 0.1 = 2.51$ m/s

W_T=1000 x 0.06(5.2 x 5.03 – 0.521 x 2.51)=<u>1490.9 Watts</u>

$$\zeta_T = \frac{W_T}{\gamma Q h}$$

h=1490.9/(0.9 x 9810 x 0.06) =<u>2.814 m</u>

$\tan\beta_1 = V_{n1}/(V_{w1}-U_1)$

$\tan\alpha_1 = V_{n1}/V_{w1}$

V_{n1}=tan 30 x 5.2= 3 m/s

$\beta_1 = \tan^{-1}(3/(5.2-5.03))$= <u>86.7</u>

$Q=2\pi\, r_1\, b_1\, V_{n1}$

Hence,

b_1= 0.06(2π x 0.2 x 3)=<u>0.016 m</u>

2-Kaplan Turbine

The *Kaplan turbine*, named after Victor Kaplan, a German professor, is an efficient axial-flow hydraulic turbine with adjustable blades. It is the most efficient type at very low heads.

-To satisfy large power demands, very large volume flow rates necessary in the Kaplan turbine i.e. the product Qh is large.

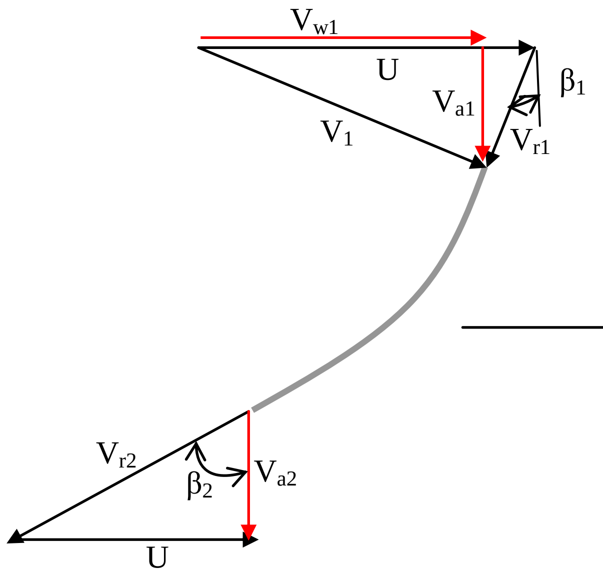
-The flow enters from a volute into the inlet guide vanes which impart a degree of swirl to the flow.

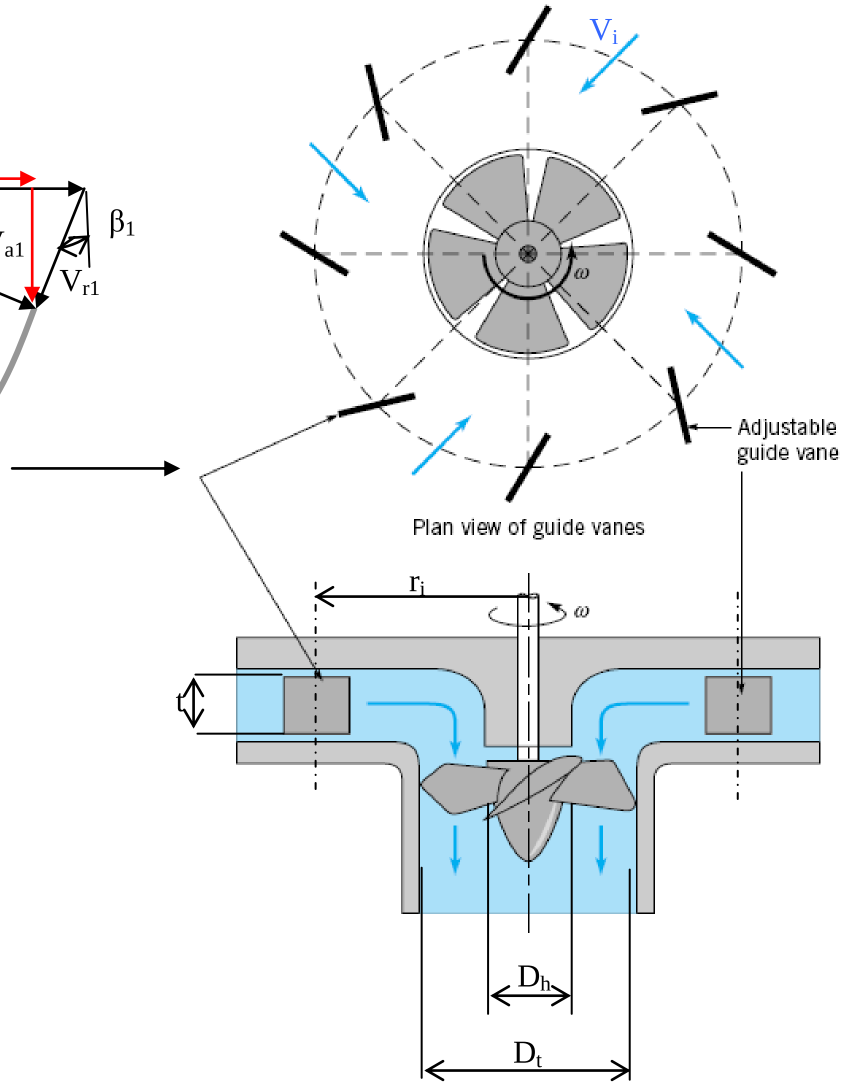
-The overall flow configuration is from radial to axial

-The number of blades is small, usually 4,5 or 6

-The blades are designed with a twist suitable for free-vortex flow at entry and axial flow at outlet.

-The Kaplan turbine incorporates one essential feature not found in other turbines that is setting of the blades angle can be controlled.





D_h: hub diameter
D_t: tip diameter
r_i:

$V_{n2}=V_{n1}=V_n$

$Q=2\pi\, r_i \times t \times V_i = \pi/4 \times (D_t^2-D_h^2)\, V_a$

$$\boxed{W_T = \rho Q U V_{w1}}$$

Turbine efficiency

$$\zeta_T = \frac{W_T}{\gamma Q h} = \frac{\rho Q U V w_1}{\rho g Q h} = \frac{U V w_1}{g h}$$

Free Vortex Principle

$$\boxed{V_w \times r = \text{constant}}$$

Example: A Kaplan turbine develops 9 MW and rotates at 145 rpm under a head of 20 m. The tip and hub diameters are 4m and 1.75 m respectively. Calculate the inlet and outlet blade angle measured at mean radius if turbine efficiency is 90%.

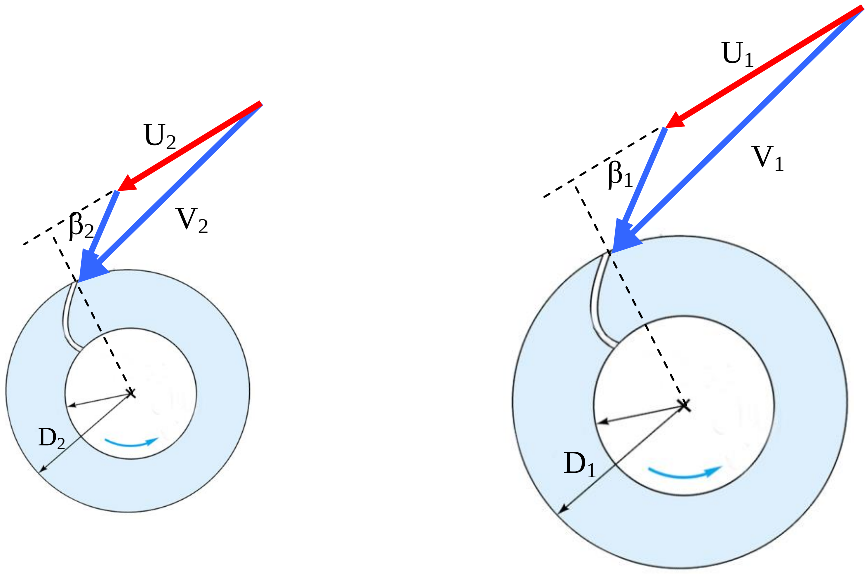
$D_m=(4+1.75)/2 =2.875$ m
$U=\omega r_m=(2\pi N/60) \times 2.875/2=21.83$ m/s
$\tan^{-1}(90-\beta)=Va/(U-V_{w1})$

$$\zeta_T = \frac{U V w_1}{g h}$$

$\rightarrow V_{w1}= 0.9 \times 9.81 \times 20/21.83 = 8.088$ m/s
$Q=\pi/4 \times (D_t^2-D_h^2)\, V_a$

$W_T= \rho Q U V_{w1}$
$Q=9x103/1000x8.088 \times 21.83=50.97$
$50.97= \pi/4 \times (4^2-1.75^2)\, V_a$
$\rightarrow Va=5.02$ m/s
$90-\beta_1=5.02/(21.83-8.088)=20^o$
$\beta 1=70^o$
$\beta 2=\tan^{-1} U/Va=21.33/5.02=77^o$

Homologous Machines



Assumptions

1- The machines are geometrically similar ($\beta_{i1} = \beta_{i2}$)
2- Similar stream lines i.e. the Reynolds number is the same in the similar machines

This results in a similar velocity triangle in the inlet.

$V_1/U_1 = V_2/U_2$
Or $V/U = const.$
$U = \omega r$
$\omega \alpha N$
$r \alpha D$
$U \alpha ND$
$V \alpha U \alpha ND$
$Q = AVn,$
$A \alpha D^2$
$Vn \alpha V$
$Q \alpha ND^3$

$$\boxed{\frac{Q}{ND^3} = const. \qquad ....(1)}$$

$Q = CdA\,(2gh)^{1/2}$
$Q \alpha D^2 H^{1/2}$
$ND^3 \alpha\, D^2 H^{1/2}$

ND α $H^{1/2}$

$$\boxed{\frac{H}{N^2 D_2} = const. \qquad ....(2)}$$

Wt=γQh

Wt αND^{3}N^{2}D^2

$$\boxed{\frac{W_T}{N^3 D^5} = const. \qquad ....(3)}$$

Relations 1 to 3 are called the turbo-machinery Similarity rules, are used to:

1-Design or select a turbo machine from a family of geometrically similar units

2- Examine the effects of changing speed, fluid or size on a given unit.

3- To design a pump to deliver flow on the moon or on a space station!

Specific speed

1) Specific Speed for Turbines

$$\frac{Q}{ND^3} = const. \qquad ....(1)$$

$$[\frac{H}{N^2 D_2} = const. \qquad ....(2)]^{3/2} \rightarrow H^{3/2}/N^3\,D^3=\text{cons.}$$

$H^{3/2}/N^2Q$=const.

$H^{3/2}H/N^2W_T$=const.

$[H^{5/2}/N^2\,W_T$ =const.$]^{1/2}$

$$\boxed{\frac{N\sqrt{\mathbf{W_T}}}{\mathbf{H}^{5/4}} = cons. = Ns}$$

Where:
N: in rpm
WT: in kW
H: in m

2) Specific Speed for Pumps

$$\left[\frac{H}{N^2 D_2} = const. \quad ....(2)\right]^{3/2} \rightarrow H^{3/2}/N^3 D^3 = \text{cons.}$$

$$[H^{3/2}/N^2Q = \text{const}]^{1/2} \rightarrow H^{3/4}/NQ^{1/2} = \text{const}$$

$$\boxed{\frac{N\sqrt{\mathbf{Q}}}{\mathbf{H}^{3/4}} = cons. = Ns}$$

Where:
N: in rpm
Q: in m3/s
H: in m

Example:

Two similar pumps A and B, the angular speed of pump A is 1000 rpm and the head developed is 12.2 m of water with a flow rate of 0.0151 m^3/s. Pump B is has a diameter twice that of Pump A. Find the angular speed and the head developed by pump B when its flow rate is 0.0453 m^3/s. What is the specific speed of this type of pump?

$Q_1/N_1D^3{}_1 = Q_2/N_2D^3{}_2 \rightarrow 0.0151/1000 \times D^3{}_1 = 0.0453/N_2(2D_1)^3$
$N_2 = 375$ rpm

$H_1/(N^2{}_1D^2{}_1) = H_2/(N^2{}_2D_2{}^2) \rightarrow 12.2/(1000)^2D_1{}^2 = H^2/(375)^2(2D_1)^2$
$H_2 = 6.86$ m

$$Ns = \frac{N\sqrt{\mathbf{Q}}}{\mathbf{H}^{3/4}} \rightarrow Ns = \frac{1000\sqrt{\mathbf{0.0151}}}{12.2^{3/4}} = 18.82$$