Turbomachines

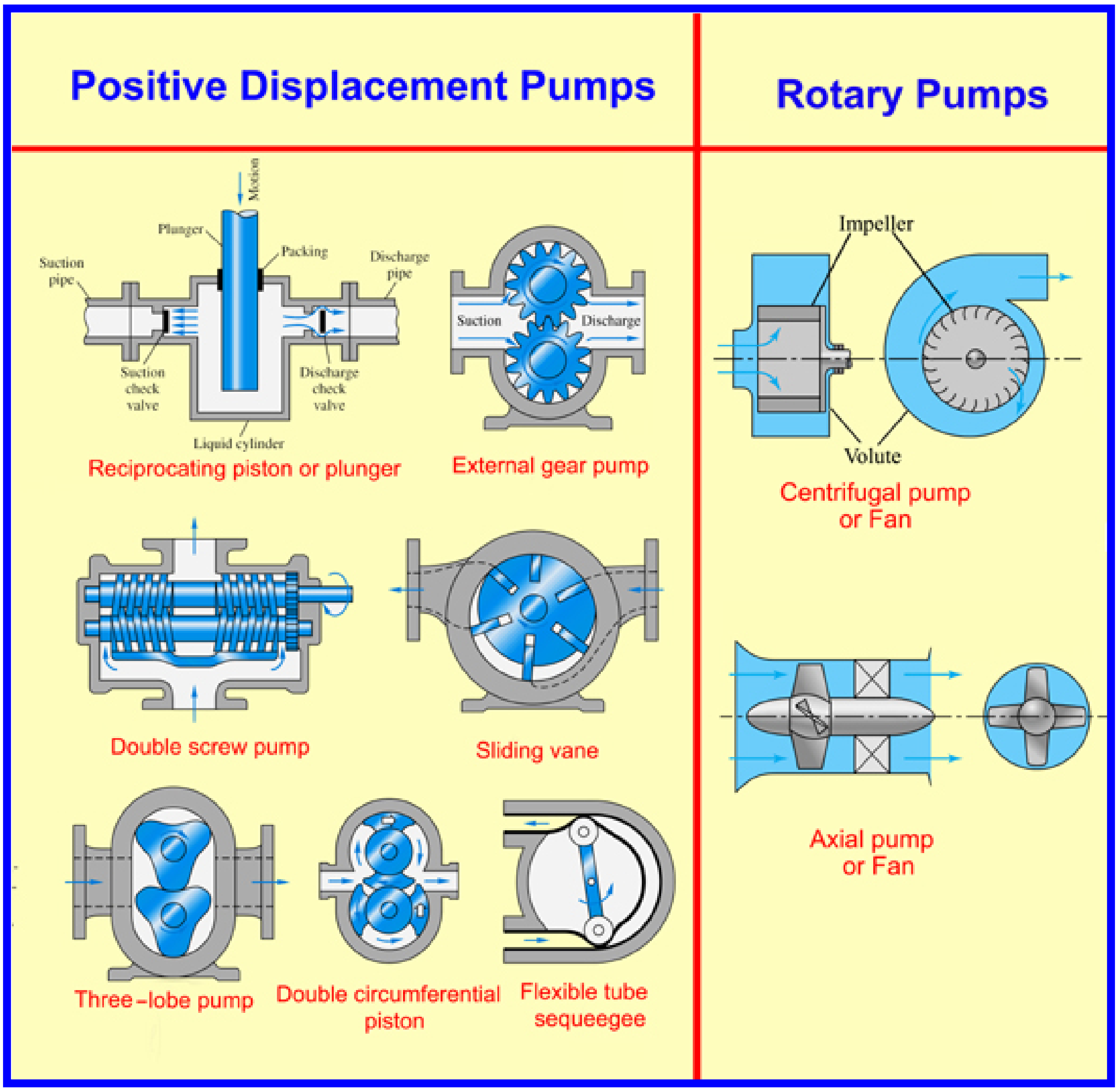
Pumps

A device which adds energy such us the pump. The pump is used when the common requirement of all applications is to move liquid from one point to another. In general the selection of any pump can be characterized according to the followings:

- Capacity range of liquid to be moved
- Differential head required
- NPSH
- Shape of head capacity curve
- Pump speed
- Liquid characteristics
- Construction

1.1 Classification of pumps

Several types of pumps are exist in industrial and a classified as shown in figure below:



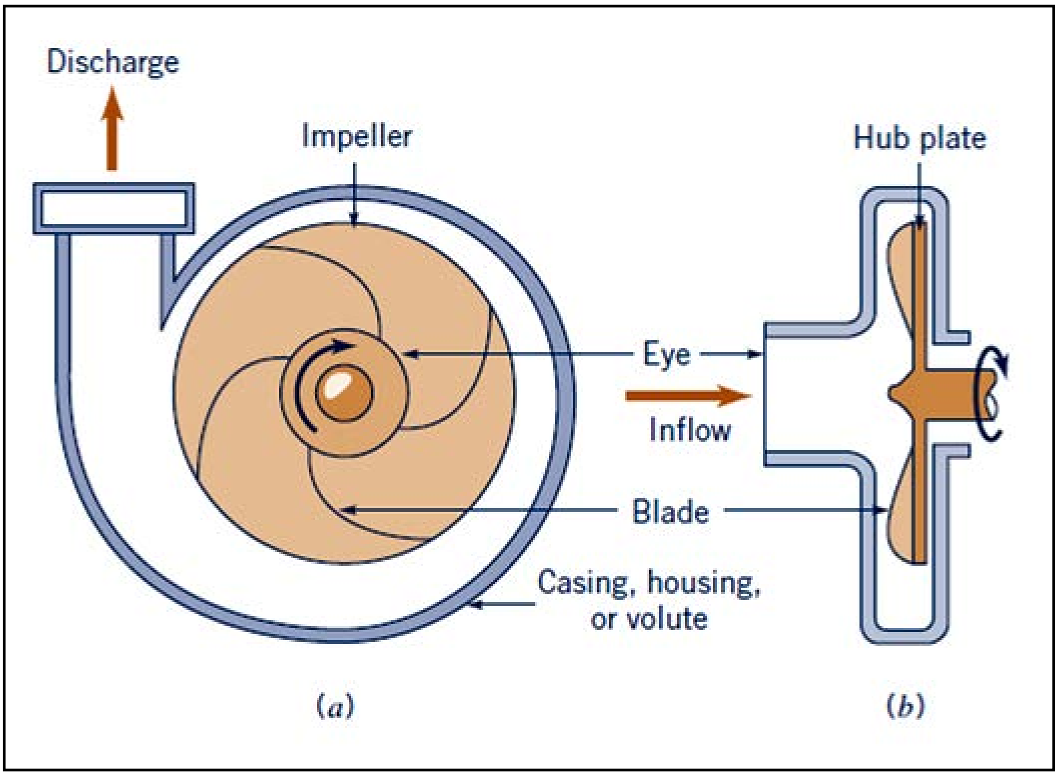
In general the positive displacement pumps posses high head and low flow rates like oil pump in cars, fuel pump in cars. They can be manufactured as single or double acting (to minimize the pulsating flow).

Rotary pumps**,** steady discharge flow (no pulsating flow), on the other hand, can be classified to:

1- Centrifugal (Radial) pumps [high head, low flow rate]
2- Axial [high flow rate, low head]
3- Mixed flow [Moderate head and flow rate]

1.2 Centrifugal pumps

Most common turbomachine used in industry. Includes the general categories of (a) liquid pumps (b) fans, (c) blowers, etc. They are generally designed so that the angular momentum entering the impeller is zero. Typical schematic shown as:



α_1: Inlet fluid angle

β_1: blade angle at inlet

α_2: Exit fluid angle

β_2: blade angle at exit

V_1: absolute velocity at inlet

V_2: absolute velocity at exit

W_1: relative velocity at inlet

W_2: relative velocity at exit

U_1: blade tip speed at inlet

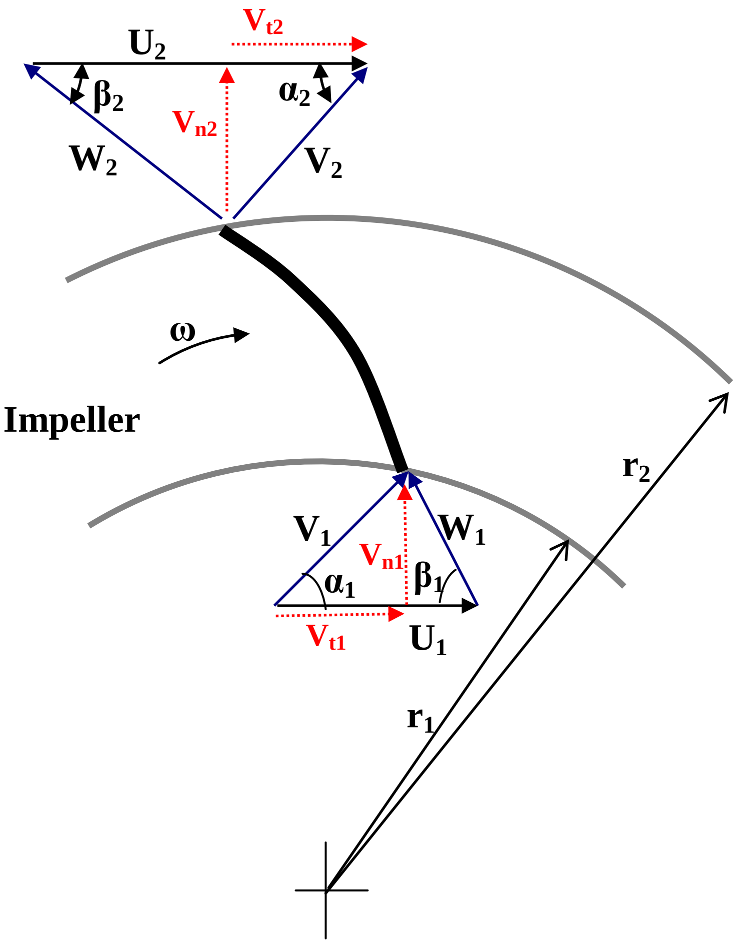
U_2: blade tip speed at exit

V_{n1} : normal component of V_1

V_{n2} : normal component of V_2

V_{t1} : tangential component of V_1

V_{t2} : tangential component of V_2



Momentum Balance

$$T = \rho Q\left(V_{t2} \times r_2 - V_{t1} \times r_1\right)$$

$$W_P = T.\omega = \left(\frac{U}{r}\right)\rho Q\left(V_{t1} \times r_1 - V_{t2} \times r_2\right)$$

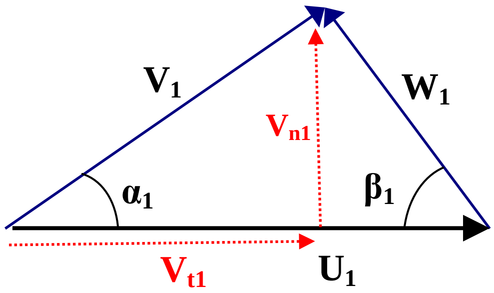
$$\therefore W_P = \rho Q\left(V_{t2} U_2 - V_{t1} U_1\right)$$

Pump Efficiency

$$\eta_p = \frac{\gamma Q h}{W_P}$$

$$Q = A_1 V_{n1} = 2\pi r_1 \times b_1 \times V_{n1} = A_2 V_{n2} = 2\pi r_2 \times b_2 \times V_{n2}$$

For the inlet velocity diagram



$$U_1 = r_1.\omega$$

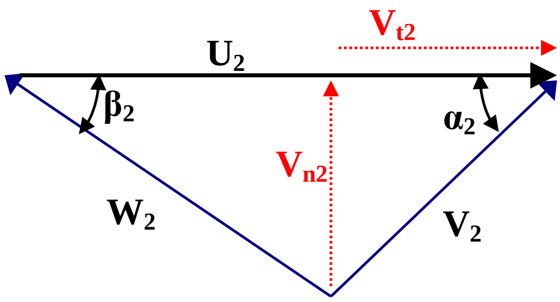
$$V_{n1} = V_1 \sin\alpha_1 = W_1 \sin\beta_1$$

$$V_{t1} = V_1 \cos\alpha_1 = U_1 - W_1 \cos\beta_1$$

For ideal pump design $\alpha_1 = 90^o$

$$V_{n1} = V_1 \quad , \quad V_{t1} = 0$$

For the outlet velocity diagram



$$U_2 = r_2.\omega$$

$$V_{n2} = V_2 \sin\alpha_2 = W_2 \sin\beta_2 = \frac{Q}{2\pi r_2 b_2}$$

$$V_{t2} = V_2 \cos\alpha_2 = U_2 - W_2 \cos\beta_2$$

$$or\ \ V_{t2} = U_2 - \frac{V_{n2}}{\tan\beta_2} = U_2 - \frac{Q}{2\pi\,\mathrm{r_2 b_2}\tan\beta_2}$$

(i) For ideal pump design (Theoretical design)

Assumptions: (Efficiency=100% , and α_1=90^o)

$$\because V_{t1} = 0$$

$$\because W_P = \rho Q\left(V_{t2}U_2 - \cancel{V_{t1}U_1}\right) \quad \Rightarrow \quad \therefore W_P = \rho Q V_{t2} U_2 \quad \text{ideal power}$$

$$\because \eta_p = \frac{\gamma Q h}{W_P} \quad \rightarrow \quad W_P = \rho g Q h \quad \Rightarrow \quad \cancel{\rho Q} V_{t2} U_2 = \cancel{\rho}\, g\, \cancel{Q}\, h$$

$$\therefore h = \frac{V_{t2}U_2}{g}$$

From outlet velocity diagram, we have;

$$\because V_{t2} = U_2 - \frac{Q}{2\pi\,\mathrm{r_2 b_2}\tan\beta_2}$$

$$\therefore h = \frac{U_2^2}{g} - \frac{\omega Q}{2\pi\,\mathrm{b_2 g}\tan\beta_2} \qquad \text{where, } \omega = \frac{U_2}{r_2}$$

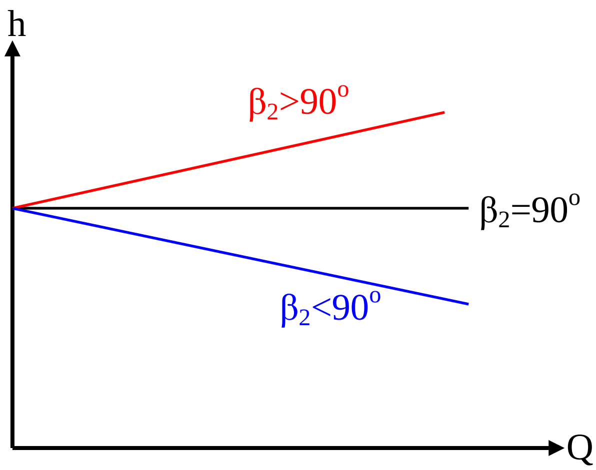
$$\boxed{h = K_1 - K_2 Q}$$

$$\text{where, } K_1 = \frac{U_2^2}{g} \quad , \quad K_2 = \frac{\omega}{2\pi\,\mathrm{b_2 g}\tan\beta_2}$$

$\beta_2 > 90^o$ forward facing blades

$\beta_2 = 90^o$ radial blades

$\beta_2 < 90^o$ backward facing blades



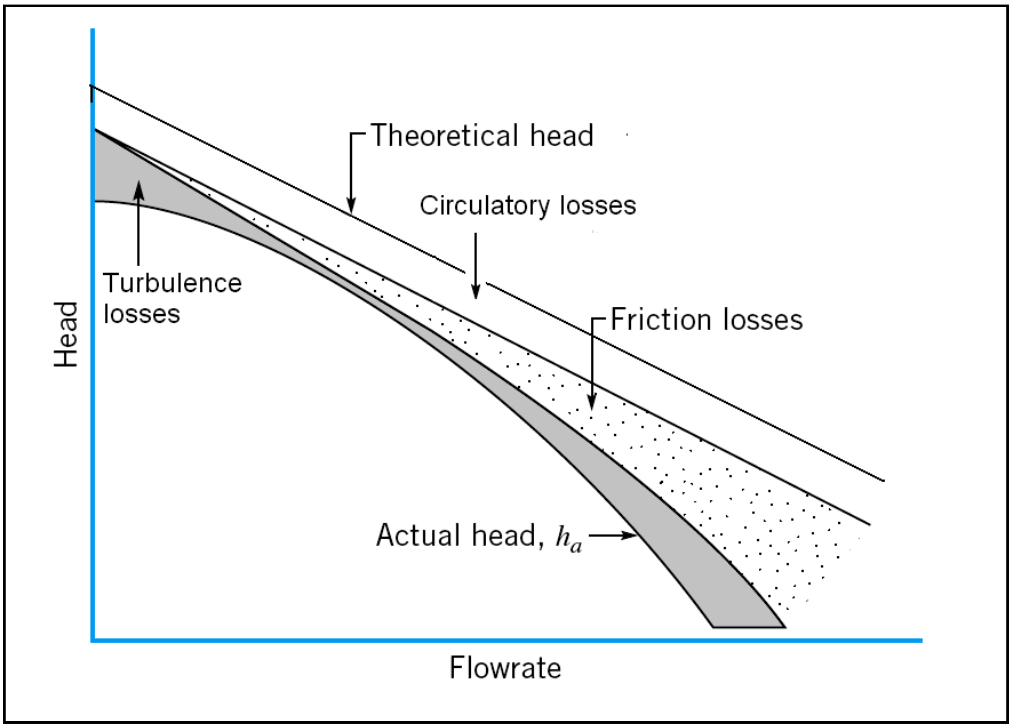
Actual head discharge curve

The most important losses that occur in pumps are:

1- Circulatory flow losses: liquid leaves the blades with an angle less than the blade angle.

2- Friction losses: due to friction of liquid with vanes and with the passages. This losses proportional with the square of velocity and hence with Q.

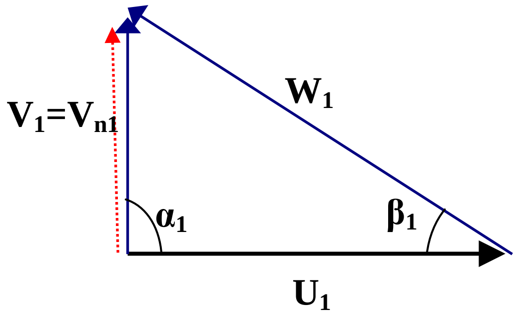
3- Turbulence losses: due to operation the pump under or over the designing speed



Ex. A centrifugal water pump operates at the following conditions: N = 1440 rpm, r_1=0.1 m, r_2= 0.177 m, β_1=30°, β_2=20°, b_1=b_2 = 0.044 m. Assuming the inlet flow enters normal to the impeller (zero absolute tangential velocity): find, flow rate, Torque (T), Power pump (W_P), head (h), and ΔP,

sol.

$$\alpha_1 = 90^o \quad , \; V_{t1} = 0 \quad \text{ideal design} \;\; \eta_p = 100\%$$



inlet velocity diagram

$$U_1 = \omega r_1 = \frac{2\pi N}{60} r_1 = 15.08\, m/s$$

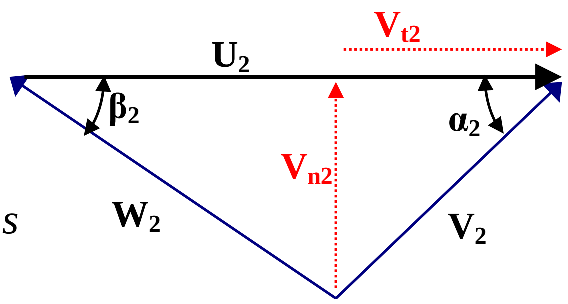
$$U_2 = \omega r_2 = \frac{2\pi N}{60} r_2 = 26.69\, m/s$$

$$\because Q = 2\pi r_1 b_1 V_{n1}$$

$$V_{n1} = U_1 \tan 30 = 8.71\, m/s$$

$$\therefore Q = 2\pi r_1 b_1 V_{n1} = 2\pi * 0.1 * 0.044 * 0.885 = 0.241\; m^3/s$$

$$\because T = \rho Q V_{t2} r_2 \qquad \textbf{for ideal design}$$



$$\because V_{t2} = U_2 - \frac{V_{n2}}{\tan\beta_2}$$

$$\because V_{n2} = \frac{Q}{2\pi r_2 b_2} = \frac{0.241}{2\pi * 0.177 * 0.044} = 5\, m/s$$

$$\therefore V_{t2} = U_2 - \frac{V_{n2}}{\tan\beta_2} = 26.69 - \frac{5}{\tan 20} = 13\, m/s$$

$$\because T = \rho Q V_{t2} r_2 = 1000 * 0.241 * 13 * 0.177 = 554.54\, N.m$$

$$\because W_P = T.\omega = 554.54 * \frac{2\pi N}{60} = 83.622\; kW$$

For ideal design efficiency 100%

$$\because \eta_p = \frac{\gamma Q h}{W_P} \quad \rightarrow \quad W_P = \rho g Q h \quad \Rightarrow \; h = \frac{W_P}{\rho g Q} = 35.37\; m$$

$$\Delta P = \rho g h = 9810 * 35.37 = 346.98\, kPa$$

1.3 Pump Performance Curves

Typical centrifugal pump performance curves at fixed pump speed and diameter as shown in the below figure (1). These curves are observed to have the following characteristics:

1. Head, $H_p \approx$ constant at low flow rate, Q.
2. Brake horsepower, BHP increases with increase of Q.
3. Efficiency, $\eta_p = 0$ at Q=0.
4. Maximum pump efficiency occurs at approximately $Q^* \approx 0.6 Q_{max}$

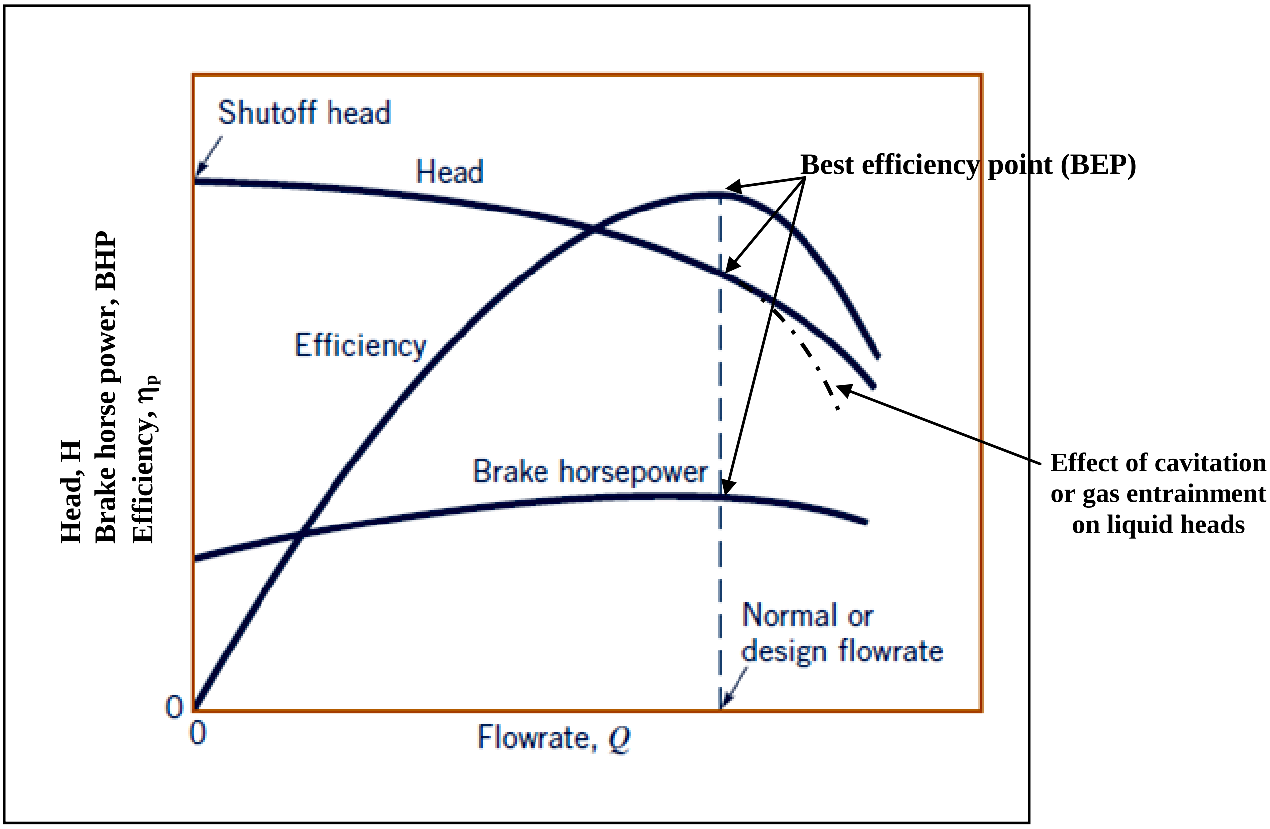


Figure (1) Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed.

Pump performance characteristics are also presented in charts of the type shown in Figure (1). Since impellers with different diameters may be used in a given casing, performance characteristics for several impeller diameters can be provided with corresponding lines of constant efficiency and brake horsepower. It is to be noted that an additional curve is given in Figure (1), labeled which stands for ***required net positive suction head*** ($NPSH_R$). As discussed in the following section, the significance of this curve is related to conditions on the suction side of the pump, which must also be carefully considered when selecting and positioning a pump.

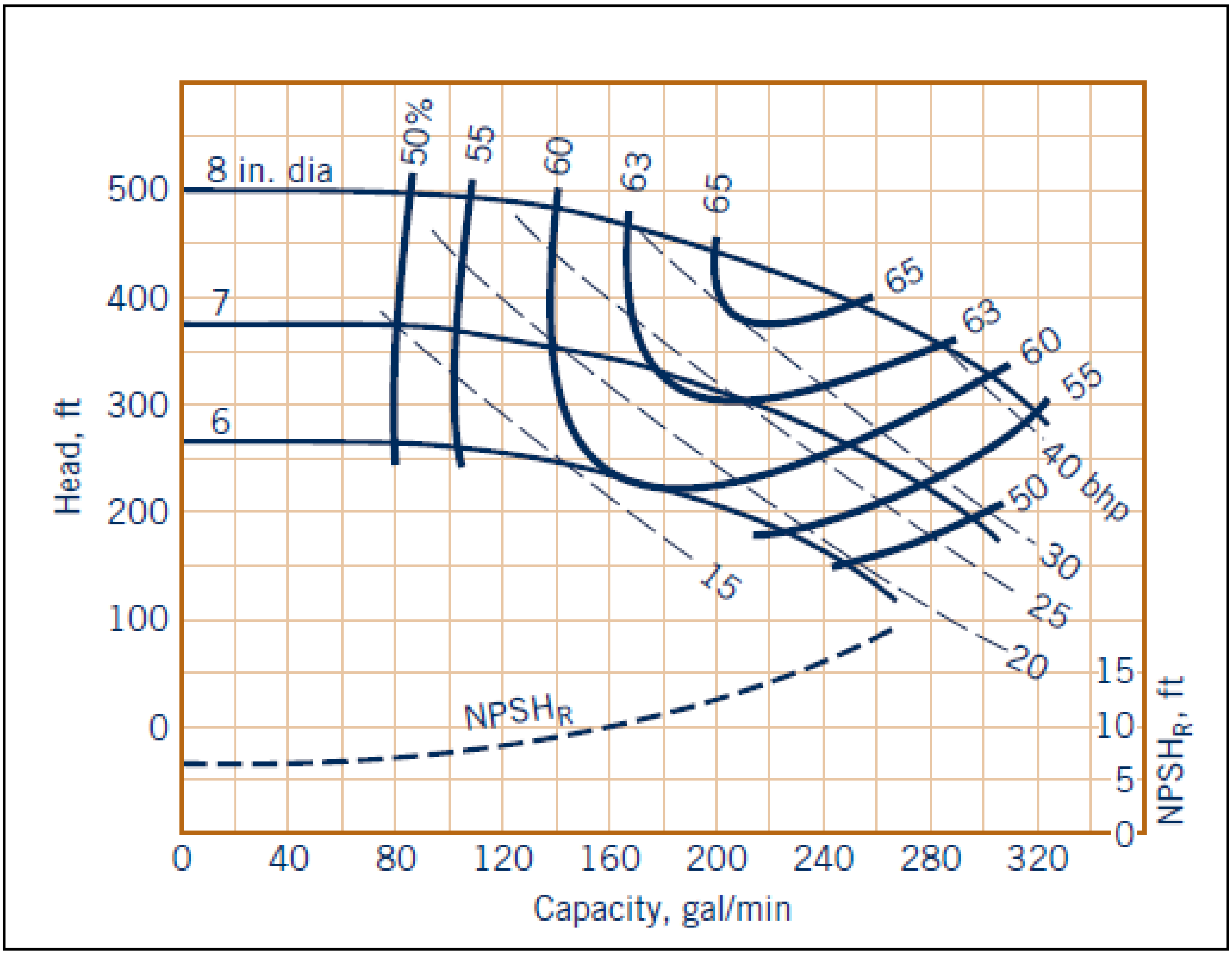


Figure (2) Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameter.

Ex. for Q = 200 gpm, D = 7 in, N=3500

Sol. H = 310 ft, BHP = 25 bhp, η_p = 63.2%, $NPSH_R$ = 12.5 ft

1.4 Cavitation

Cavitation is one such phenomenon that can be very damaging to pumps, is a direct result of improper operating conditions.

When water enters a pump, its velocity increases causing a reduction in pressure within the pumping unit. If this pressure falls too low of the vapor pressure of the liquid, some of the water will vaporize, forming bubbles entrained in the liquid. These bubbles collapse violently (rapidly) as they move to areas of higher pressure creating the noise and vibration from the pump.

The main problems of cavitations are:

(1) Noise, (2) Vibration, and (3) Lowering of the h-Q and efficiency curves

(4) Weaken the solid surfaces,

(5) After repeated collapse cycles, pitting and further erosion and fatigue of surfaces can occurs.

Net Positive Suction Head - NPSH

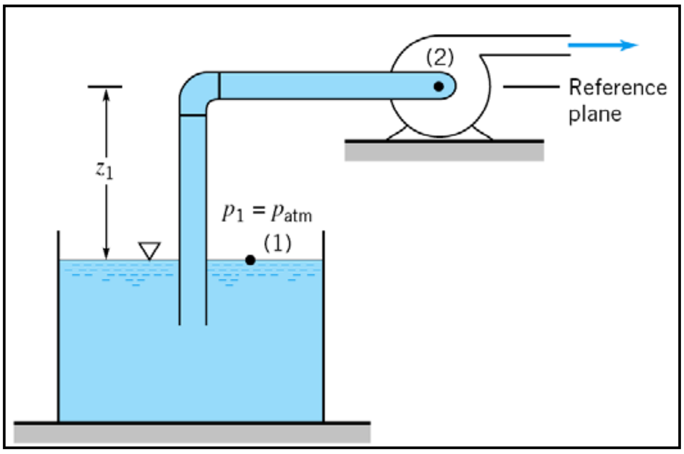
To characterize the potential for cavitation, the difference between the total head on the suction side, near the pump impeller inlet $(P_s/\gamma + V_s^2/2g)$, and the liquid vapor pressure head $(P_v/\rho g)$, is used. The position reference for the elevation head passes through the centerline of the pump impeller inlet. This difference is called the ***net positive suction head*** (NPSH), so that

$$\text{NPSH} = \frac{P_s}{\rho g} + \frac{V_s^2}{2g} - \frac{P_v}{\rho g} \qquad (1)$$

There are actually two values of NPSH of interest.

- The pump manufacturers specify a required NPSH, or denoted $\mathbf{NPSH_R}$. Similar to the pump operating characteristics it is a curve dependent on the flow rate.
- From the system layout and pump placement an available NPSH, or denoted $\mathbf{NPSH_A}$, can be determined. This is the head present at pump inlet for a given operating point.
  - For proper pump operation it is necessary that, $\mathbf{NPSH_A \geq NPSH_R}$.

A typical flow system is shown in figure (3)



Apply Bernoulli equation between the free liquid surface (point 1), and a point on the suction side of the pump near the impeller inlet yields (point 2).

$$\frac{P_{atm}}{\rho g}+\frac{V_1^2}{2g}=\frac{P_s}{\rho g}+\frac{V_s^2}{2g}+Z_1-h_L$$

where $h_{f,a-s}$, head losses between the free surface and the pump impeller inlet.

$$\frac{P_s}{\rho g}+\frac{V_s^2}{2g}=\frac{P_{atm}}{\rho g}-Z_1-h_L \qquad (2)$$

Subs. Eq. (2) in Eq. (1), we have;

$$NPSH_A=\frac{P_{atm}}{\rho g}-Z_1-h_L-\frac{P_v}{\rho g}=\frac{P_{atm}-P_v}{\rho g}-Z_1-h_L$$

Note: All pressures in absolute value.

Example. A centrifugal pump is to placed above a large open water tank, and the required net positive suction head $NPSH_R$, is 7.5 m. Determine the maximum elevation that the pump can be located above the water surface without cavitation. Water temperature is 15ºC. Neglect pipe losses, P_{atm} = 101 kPa.

Sol. From table of water properties P_v = 1666 Pa, at t = 15ºC, $h_L = 0$

the $NPSH_A$ is given by;

$$NPSH_A=\frac{P_{atm}-P_v}{\rho g}-Z_1-h_L$$

At the maximum value for Z_1, well occur when $NPSH_A$=$NPSH_R$, thus;

$$Z_{1,\max}=\frac{P_{atm}-P_v}{\rho g}-NPSH_R=\frac{101000-1666}{9810}-7.5=2.626\,m$$

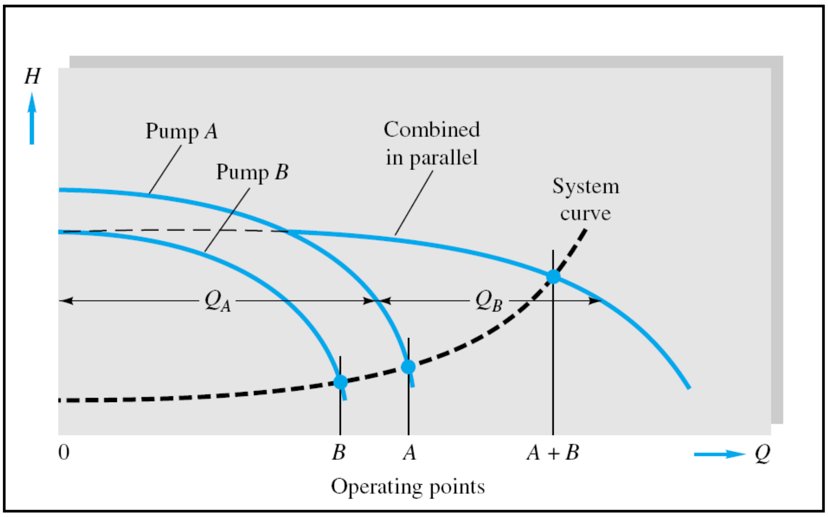
1.5 Pumps Connection

(i) Parallel connection

Where large flow quantities are required, two or more pumps are placed in ***parallel*** configuration. The overall efficiency of pumps in ***parallel*** is:

$$\eta = \frac{\gamma H \sum_{i=1}^{n} Q}{\sum_{i=1}^{n} W_p}$$

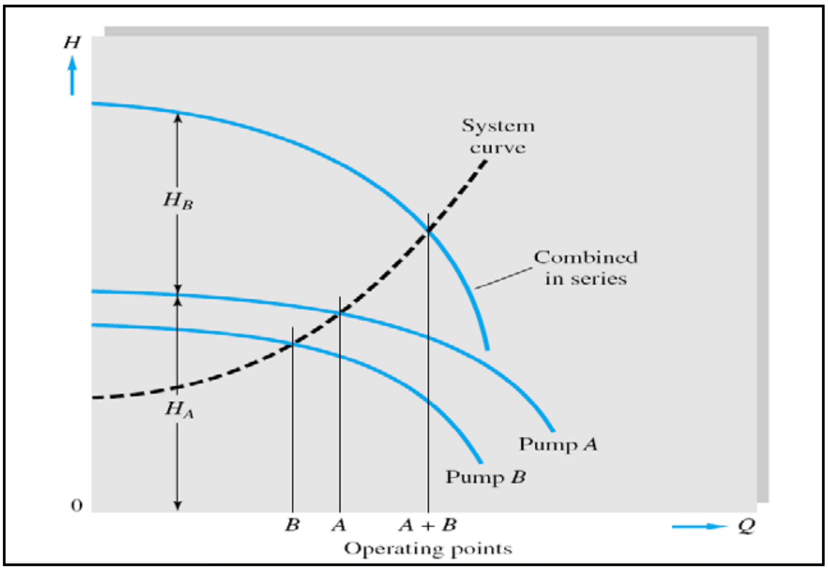
$$Q_t = Q_A + Q_B + \ldots$$



(ii) Series connection

For high head demands, pumps placed in series:

$$\eta = \frac{\gamma Q \sum_{i=1}^{n} H}{\sum_{i=1}^{n} W_p}$$



1.6 Similarity Laws for Pumps

Application of the dimensional analysis procedures of (Turbine water) will yield the following three dimensional performance parameters.

$$\frac{Q}{ND^3} = \text{constant} \qquad \text{(Dimensional flow coefficient)}$$

$$\frac{H}{N^2D^2} = \text{constant} \qquad \text{(Dimensional head coefficient)}$$

$$\frac{Power}{\rho N^3D^5} = \text{constant} \qquad \text{(Dimensional power coefficient)}$$

Relations 1 to 3 are called the **Affinity law**, are used to:-

1-Design or select a pump from a family of geometrically similar units.
2- Examine the effects of changing speed, fluid or size on a given unit.
3- To design a pump to deliver flow on the moon or on a space station!

1.7 Specific Speed for Pumps

$$\left[\frac{H}{N^2D^2} = const.\right]^{3/2} \rightarrow \frac{H^{3/2}}{N^3D^3} = const.$$

$$\left[\frac{H^{3/2}}{N^2Q} = const.\right]^{1/2} \rightarrow \frac{H^{3/4}}{NQ^{1/2}} = const.$$

$$\frac{N\sqrt{Q}}{H^{3/4}} = const. = \text{Ns}$$

Where:
N: in rpm
Q: in m^3/2
H: in m

Example:

Two similar pumps A and B, the angular speed of pump A is 1000 rpm and the head developed is 12.2 m of water with a flow rate of 0.0151 m^3/s. Pump B is has a diameter twice that of Pump A. Find the angular speed and the head developed by pump B when its flow rate is 0.0453 m^3/s. What is the specific speed of this type of pump?.

Sol. Using Affinity law (where using index for pump A (1) and pump B (2))

$$\frac{Q_1}{N_1D_1^3} = \frac{Q_2}{N_2D_2^3} \rightarrow \frac{0.0151}{1000 * D_1^3} = \frac{0.0453}{N_2(2D_1)^3} \Rightarrow \therefore N_2 = 375\,rpm$$

$$\frac{H_1}{N_1^2D_1^2} = \frac{H_2}{N_2^2D_2^2} \rightarrow \frac{12.2}{(1000)^2 * D_1^2} = \frac{H_2}{(375)^2(2D_1)^2} \Rightarrow \therefore H_2 = 6.86\ m$$

$$\text{Ns} = \frac{\text{N}\sqrt{Q}}{H^{3/4}} \rightarrow \text{Ns} = \frac{375\sqrt{0.0453}}{6.86^{3/4}} =$$

<u>Example</u>

It is desired to modify the operating conditions for the 38 in diameter impeller pump (form performance curve at N= 710 rpm, we read the best efficiency point (BEP) values as; Q = 20000 gpm, H = 225 ft, BHP= 1250 hp) to a new pump speed of 900 rpm and a larger impeller diameter of 40 in. Determine the new pump head and power for the new pump speed.

<u>Sol.</u>

Assume the index pump (1) at N = 710 rpm and D = 38 in

new pump (2) at N = 900 rpm and D = 40 in

Using Affinity law

$$\frac{Q_1}{N_1 D_1^3} = \frac{Q_2}{N_2 D_2^3} \quad \rightarrow \quad \frac{Q_2}{Q_1} = \frac{N_2}{N_1}\left(\frac{D_2}{D_1}\right)^3 = \frac{900}{710}\left(\frac{40}{38}\right)^3 = 1.478$$

$$\therefore Q_2 = Q_1 * 1.478 = 20000 * 1.478 = 29570\, gpm$$

$$\frac{H_1}{N_1^2 D_1^2} = \frac{H_2}{N_2^2 D_2^2} \quad \rightarrow \quad \frac{H_2}{H_1} = \left(\frac{N_2}{N_1}\right)^2 \left(\frac{D_2}{D_1}\right) = \left(\frac{900}{710}\right)^2 \left(\frac{40}{38}\right)^2 = 1.78$$

$$\therefore H_2 = H_1 * 1.78 = 225 * 1.78 = 400.5 \text{ ft}$$

$$\frac{P_1}{\rho_1 N_1^3 D_1^5} = \frac{P_2}{\rho_2 N_2^3 D_2^5} \quad \rightarrow \quad \frac{P_2}{P_1} = \left(\frac{N_2}{N_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5 = 2.632$$

$$\therefore P_2 = 3290 \;\; hp$$