FLUID MECHANICS II

Week	عدد الساعات	Subject	الموضوع	ت
1	3	Basics of compressible flow	اساسيات الجريان الأنضغاطي	1
2	4	Isentropic flow	الجريان الأيزنتروبي	2
3-4	6	Isentropic flow through variable area duct	الجريان الأيزنتروبي في مقطع متغير المساحة	
5	3	Normal shock wave	الموجة الصدمية العمودية	3
7	3	Oblique shock wave	الموجة الصدمية المائلة	4
8	3	Prandtl Meyer Function	دالة براندل- ماير	5
9	4	Adiabatic flow (Fanno flow)	الجريان الأديباتيكي (جريان فانو)	6
10	3	Jet propulsion- Rocket engine	الدفع النفاث –المحرك الصاروخي	7
10-11	6	Turbojet engine	المحرك التوربيني النفاث	
12-13	6	Twin spool turbojet engine	المحرك التوربيني النفاث مزدوج التيار	
14-15	3	Water turbines -Impulse type- Pelton wheel	التوربينات المائية- النوع الدفعي (توربين بلتون)	8
16-17	4	Reaction type- Francis Turbine	النوع التفاعلي- توربين فرانسس	
18	4	Reaction type- Kaplan Turbine	النوع التفاعلي- توربين كابلان	
19	4	Homologous rules	علاقات التشابه الهندسي	9
20-21	6	Centrifugal pumps and fans	المضخات و المراوح المركزية	10
22	3	Cavitations in pumps	ظاهرة التكهف في المضخات	
23-24	6	Axial compressor	الضواغط المحورية	11
25-26	3	Centrifugal compressor	الضواغط المركزية	12
27-28	6	Axial gas turbine	التوربينات الغازية المحورية	13
29-30	3	Centrifugal gas turbine	التوربينات الغازية المركزية	14

Books:

1-" Fluid Mechanics" Victor L. Streeter
2-"Fundamentals of Fluid mechanics" Brunce Munson
4- "Gas Turbine Theory" H. Cohen
5-"ميكانيك الموائع" د. كامل الشماع

Compressible Flow

Compressible flow is the study of fluids flowing at speeds comparable to the local speed of sound. This occurs when fluid speeds are about 30% or more of the local acoustic velocity. Then, the fluid density no longer remains constant throughout the flow field. This typically does not occur with fluids but can easily occur in flowing gases. Two important and distinctive effects that occur in compressible flows are (1) *choking* where the flow is limited by the sonic condition that occurs when the flow velocity becomes equal to the local acoustic velocity and (2) *shock waves* that introduce discontinuities in the fluid properties and are highly irreversible. Since the density of the fluid is no longer constant in compressible flows, there are now four dependent variables to be determined throughout the flow field. These are pressure, temperature, density, and flow velocity. Two new variables, temperature and density, have been introduced and two additional equations are required for a complete solution. These are the *energy equation* and the fluid *equation of state*. These must be solved simultaneously with the *continuity* and *momentum* equations to determine all the flow field variables.

We need to do review for some thermodynamic relations:

Equation of state $P=\rho R T, \quad \rho = \frac{1}{v}$

$Cv = \left(\frac{\partial u}{\partial T}\right)_v \qquad Cp = \left(\frac{\partial h}{\partial T}\right)_p$ Specific heats in constant volume and pressure respectively.

$h = u + pv = u + RT$

$dh = du + R\,dT$

$Cp\,dT = Cv\,dT + R\,dT \quad \rightarrow \quad Cp = Cv + R$

$$k = \frac{C_p}{C_v} \quad \rightarrow \quad C_p = \frac{kR}{k-1} \quad , C_v = \frac{R}{k-1}$$

<u>The first law of thermodynamic:</u>

$$du = \left(\frac{\partial u}{\partial s}\right)_v ds + \left(\frac{\partial u}{\partial v}\right)_s dv = Tds - pdv$$

$$dh = \left(\frac{\partial h}{\partial s}\right)_p ds + \left(\frac{\partial h}{\partial p}\right)_s dp = Tds + vdp$$

$$T\,ds = du + p\,dv$$

$$dS = \frac{du}{T} + \frac{p}{T}dv$$

$$dS = \frac{C_v dT}{T} + \frac{R}{v}dv$$

$$\Delta S = C_v \ln\frac{T_2}{T_1} + R\ln\frac{\rho_1}{\rho_2}$$

$$\Delta S = C_v \ln\frac{T_2}{T_1} + C_v(k-1)\ln\frac{\rho_1}{\rho_2}$$

$$\Delta S = C_v \ln\left[\frac{T_2}{T_1}\left(\frac{\rho_1}{\rho_2}\right)^{k-1}\right]$$

For reversible, adiabatic process (Isentropic), $\Delta S = 0.0 \rightarrow$

$$\frac{T_2}{T_1} = \left(\frac{\rho_2}{\rho_1}\right)^{k-1} \quad \text{and} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}$$

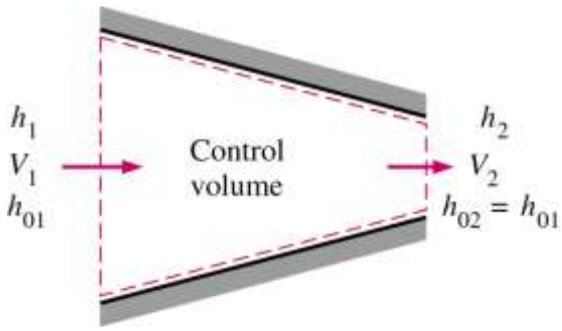
Stagnation Condition

Recall definition of enthalpy

$$h = u + P/\rho$$

which is the sum of internal energy u and flow energy P/ρ
For high-speed flows, enthalpy and kinetic energy are combined into **stagnation enthalpy h_o**

$$h_0 = h + \frac{V^2}{2}$$



Steady adiabatic flow through duct with no shaft/electrical work and no change in elevation and potential energy

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_{0,1} = h_{0,2}$$

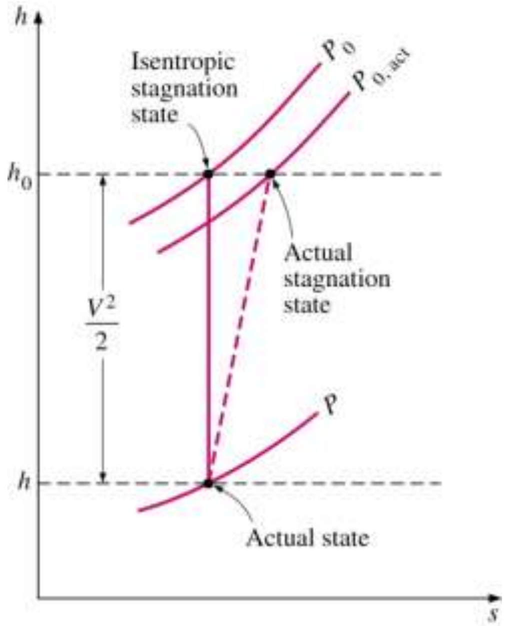
Therefore, stagnation enthalpy remains constant during steady-flow process

- If a fluid were brought to a complete stop ($V_2 = 0$) → $h_1 + \frac{V_1^2}{2} = h_2 = h_{0,2}$
- Therefore, h_0 represents the enthalpy of a fluid when it is brought to rest adiabatically.
- During a stagnation process, kinetic energy is converted to enthalpy.
- Properties at this point are called **stagnation properties** (which are identified by subscript o)

- Stagnation enthalpy is the same for isentropic and actual stagnation states

- Actual stagnation pressure $P_{o,act}$ is lower than P_o due to increase in entropy s as a result of fluid friction.

-Nonetheless, stagnation processes are often approximated to be isentropic, and isentropic properties are referred to as stagnation properties



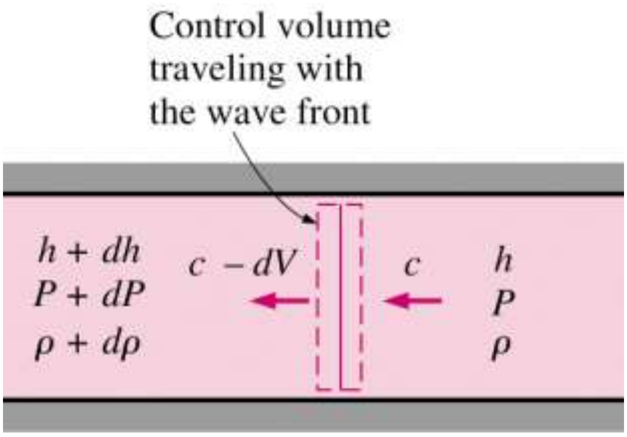
- For an ideal gas, $h = C_p T$, which allows the *h0* to be rewritten

$$c_p T_0 = c_p T + \frac{V^2}{2} \Longrightarrow T_0 = T + \frac{V^2}{2c_p}$$

  - T_o is the stagnation temperature. It represents *the temperature an ideal gas attains when it is brought to rest adiabatically.*
  - $V^2/2C_p$ corresponds to the temperature rise, and is called the **dynamic temperature**.

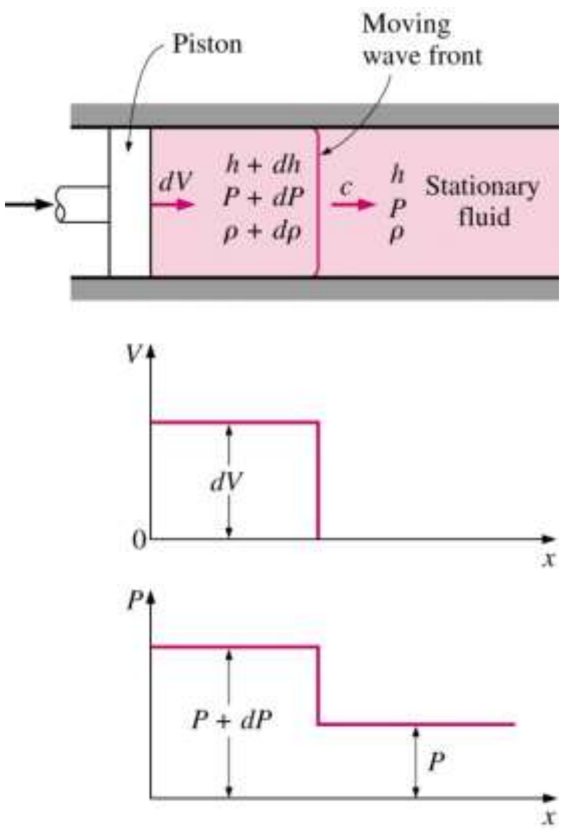
Speed of sound

- **Consider a duct with a moving piston**
  - **Creates a sonic wave moving to the right**
  - **Fluid to left of wave front experiences incremental change in properties**
  - **Fluid to right of wave front maintains original properties**
  - **Construct CV that encloses wave front and moves with it**



<u>Continuity equation (Mass Balance)</u>

$$\dot{m}_{right} = \dot{m}_{left}$$
$$\rho A c = (\rho + d\rho) A (c - dV)$$
$$\rho A c = A(\rho c - \rho\, dV + c\, d\rho - d\rho\, dV)$$
$$c\, d\rho - \rho\, dV = 0$$



Energy Equation (Energy balance)

E_{in} = E_{out}

$$h+\frac{c^2}{2}=h+dh+\frac{(c-dV)^2}{2}$$

$$h+\frac{c^2}{2}=h+dh+\frac{c^2-2c\,dV+dV^2}{2}$$

$$dh-c\,dV=0$$

From thermodynamic relations:

Tds=dh-vdp

ds=0

→ dh=dp/ρ

Combining this with mass and energy equations gives:

$$c^2=k\left(\frac{\partial P}{\partial \rho}\right)_T$$

Speed of sound for Gases

From ideal gas relation (equation of state $p = \rho RT$)

$$c^2=k\left(\frac{\partial P}{\partial \rho}\right)_T=k\left[\frac{\partial(\rho RT)}{\partial \rho}\right]_T=kRT$$

$$\boxed{c=\sqrt{kRT}}$$

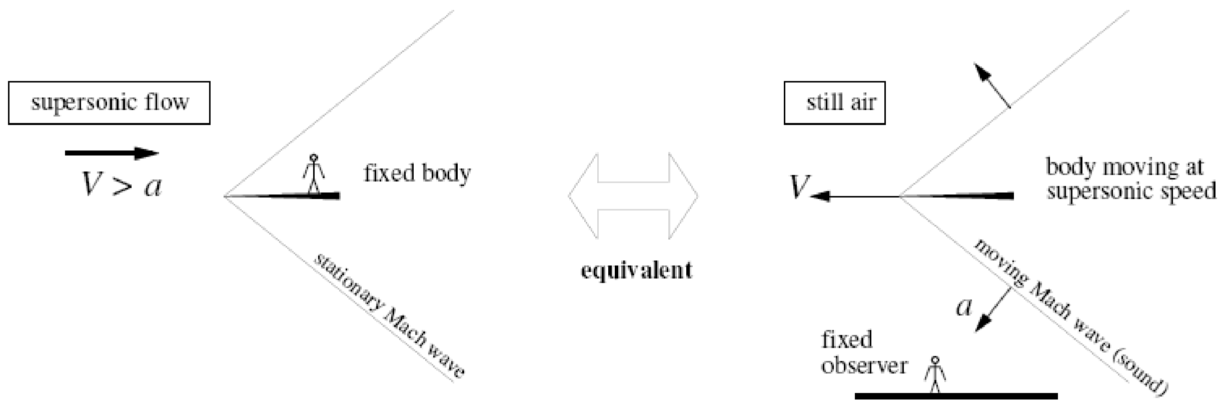
Speed of sound for Liquids

$E=\rho\dfrac{dp}{d\rho}$ Bulk modulus of compression

$$\boxed{c=\sqrt{\frac{E}{\rho}}}$$

Mach waves (M = V/C)

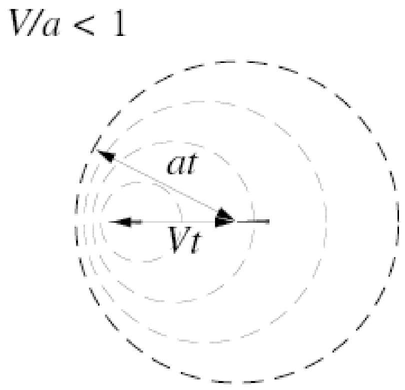
Small disturbances created by a slender body in a supersonic flow will propagate diagonally away as Mach waves. These consist of small isentropic variations in ρ, V, p, and h, and are loosely analogous to the water waves sent out by a speedboat. Mach waves appear stationary with respect to the object generating them, but when viewed relative to the still air, they are in fact indistinguishable from sound waves, and their normal-direction speed of propagation is equal to a, the speed of sound.

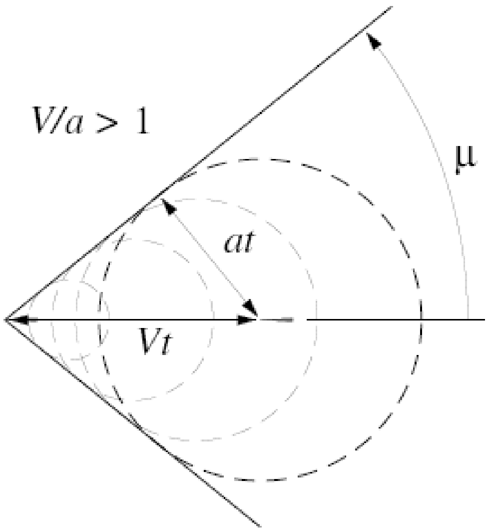


The angle μ of a Mach wave relative to the flow direction is called the Mach angle. It can be determined by considering the wave to be the superposition of many pulses emitted by the body, each one producing a disturbance circle (in 2-D) or sphere (in 3-D) which expands at the speed of sound a. At some time interval t after the pulse is emitted, the radius of the circle will be at, while the body will travel a distance Vt. The Mach angle is then seen to be

$$\mu = tan^{-1}\frac{ct}{Vt} = tan^{-1}\frac{1}{M}$$

which can be defined at any point in the flow. In the subsonic flow case where M = V/a < 1 the expanding circles do not coalesce into a wave front, and the Mach angle is not defined.





ISENTROPIC FLOW

When the flow of an ideal gas is such that there is no heat transfer (i.e., adiabatic) or irreversible effects (e.g., friction, etc.), the flow is isentropic. The steady-flow energy equation applied between two points in the flow field becomes

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = constant$$

From thermodynamic relation

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{k/(k-1)} \qquad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{1/(k-1)}$$

and $h = C_p\, T$, $ho = C_p T_o$

$$\frac{P_o}{P} = \left[1 + \frac{k-1}{k} M^2\right]^{\frac{k}{k-1}}$$

$$\frac{\rho_o}{\rho} = \left[1 + \frac{k-1}{k} M^2\right]^{\frac{1}{k-1}}$$

$$\frac{c_o}{c} = \left[1 + \frac{k-1}{k} M^2\right]^{\frac{1}{2}}$$

Example: Air is flowing isentropically through a duct is supplied from a large supply tank in which the pressure is 500 kPa and temperature 400 K. What are the Mach number, the temperature, density and fluid velocity v at a location in this duct where the pressure is 430 kPa

P_o=500 kPa, T_o=400 K (Stagnation conditions)

$$\frac{P_o}{P} = \left[1 + \frac{k-1}{2} M^2\right]^{\frac{k}{k-1}}$$

$$\frac{500}{430} = \left[1 + \frac{1.4-1}{2} M^2\right]^{\frac{1.4}{1.4-1}}$$

M=0.469

$$\frac{400}{T} = \left[1 + \frac{1.4-1}{2} 0.469^2\right]$$

T=383K

$$\rho = \frac{P}{RT} = \frac{430{,}000}{(287)(383)} = 3.91 kg/m^3$$

$$V = M\sqrt{kRT} = 0.469\sqrt{1.4 * 287 * 383} = 184\, m/s$$

Critical (Sonic) Condition

The values of the ideal gas properties when the Mach number is 1 (i.e., sonic flow) are known as the *critical or sonic properties* and are given by:

$$\frac{T_o}{T^*} = 1 + \frac{k-1}{2}$$

$$\frac{P_o}{P^*} = \left(\frac{T_o}{T^*}\right)^{\frac{k}{k-1}} = \left(1 + \frac{k-1}{2}\right)^{\frac{k}{k-1}}$$

$$\frac{\rho_o}{\rho^*} = \left(\frac{T_o}{T^*}\right)^{\frac{1}{k-1}} = \left(1 + \frac{k-1}{2}\right)^{\frac{1}{k-1}}$$

$$\frac{c_o}{c^*} = \left(\frac{T_o}{T^*}\right)^{\frac{1}{2}} = \left(1 + \frac{k-1}{2}\right)^{\frac{1}{2}}$$

Both the critical (sonic, Ma = 1) and stagnation values of properties are useful in compressible flow analyses.

Flow Through Varying Area Duct

Such flow occurs through nozzles, diffusers, and turbine blade passages, where flow quantities vary primarily in the flow direction. This flow can be approximated as 1D isentropic flow.

Continuity

$$\dot{m} = \rho A V = \text{constant}$$

$$\frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dV}{V} = 0$$

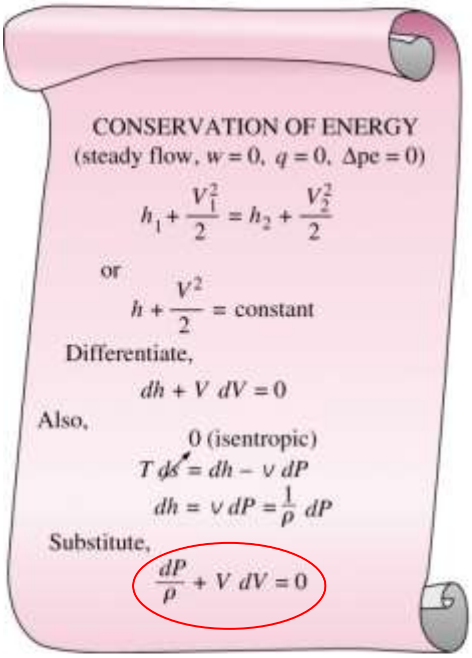
Derived relation (on image at right) is the differential form of Bernoulli's equation (Euler equation) and combining this with speed of sound gives:

$$C^2 \frac{d\rho}{\rho} = -VdV$$

$$\frac{d\rho}{\rho} = -\frac{V}{C^2} dV$$

Substitute the result in continuity equation:

$$-\frac{V}{C2} dV + \frac{dA}{A} + \frac{dV}{V} = 0$$

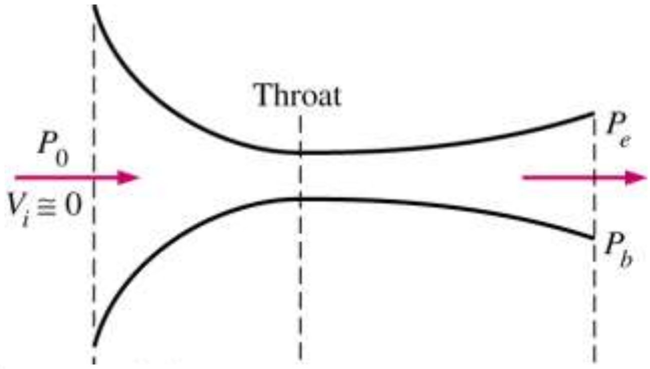


$$\frac{dA}{A} = \frac{dV}{V}\left(\frac{V^2}{C^2} - 1\right)$$

$$\boxed{\frac{dA}{dV} = \frac{A}{V}\left(M^2 - 1\right)}$$

For subsonic flow (M < 1) *dA/dV < 0*
For supersonic flow (M > 1) *dA/dV > 0*
For sonic flow (M = 1) *dA/dV = 0*

Application: Converging or converging-diverging nozzles are found in many engineering applications
Steam and gas turbines, aircraft and spacecraft propulsion, industrial blast nozzles, torch nozzles

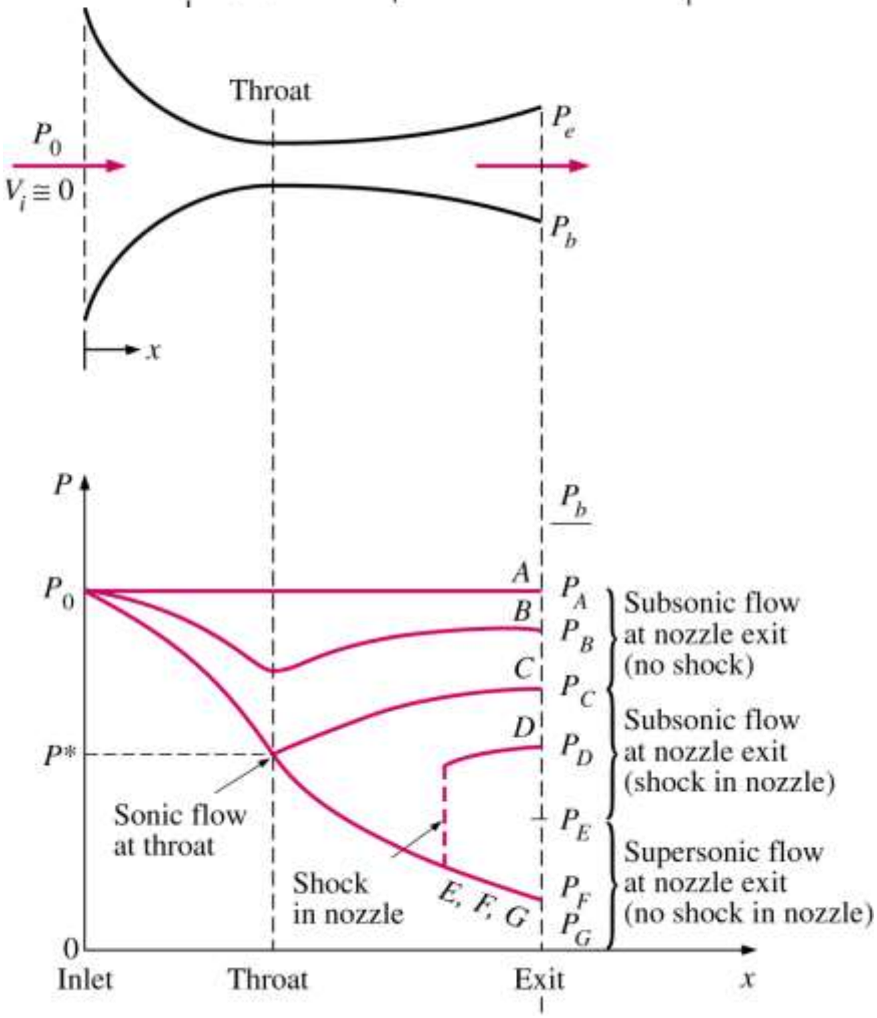


Flow Cases in Converging Diverging Nozzle

1-: $P_o > Pe > Pc$
Flow remains subsonic, and mass flow is less than for choked flow. Diverging section acts as diffuser

2-: $P_e = Pc$
Sonic flow achieved at throat. Diverging section acts as diffuser. Subsonic flow at exit. Further decrease in Pb has no effect on flow in converging portion of nozzle

3-:Pc > Pe > PE
Fluid is accelerated to supersonic velocities in the diverging section as the pressure decreases. However, acceleration stops at location of **normal shock.** Fluid decelerates and is subsonic at outlet. As *Pe* is decreased, shock approaches nozzle exit.



PE > Pe > 0

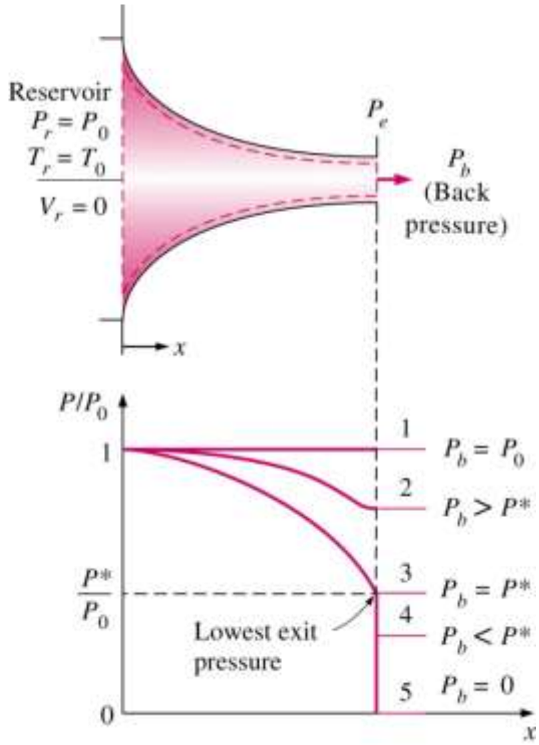
Flow in diverging section is supersonic with no shock forming in the nozzle. Without shock, flow in nozzle can be treated as isentropic.

Flow Cases in Converging (Truncated) Nozzle

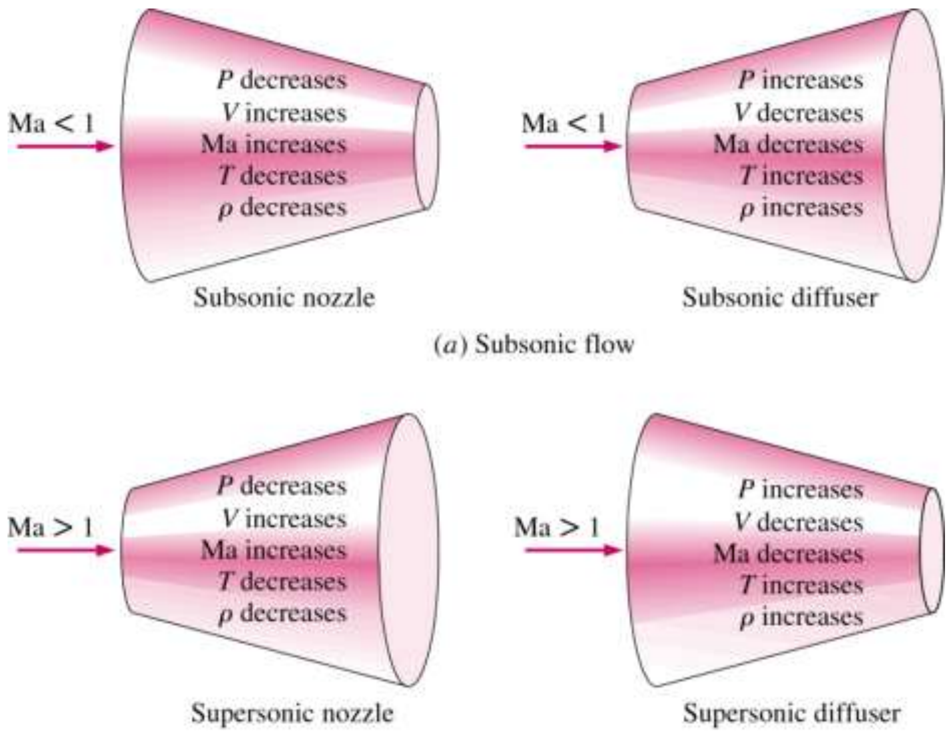
The highest velocity in a converging nozzle is limited to the sonic velocity (M = 1), which occurs at the exit plane (throat) of the nozzle

Accelerating a fluid to supersonic velocities (M > 1) requires a diverging flow section

Forcing fluid through a C-D nozzle does not guarantee supersonic velocity, It requires proper back (exit) pressure P_e



- ✓ State 1: $P_b = P_o$, there is no flow, and pressure is constant.
- ✓ State 2: $P_b < P0$, pressure along nozzle decreases.
- ✓ State 3: $P_b = P^*$, flow at exit is sonic, creating maximum flow rate called **choked flow.**
- ✓ State 4: $P_b < P_b$, there is no change in flow or pressure distribution in comparison to state 3
- ✓ State 5: $P_b = 0$, same as state 4.



(*a*) Subsonic flow

(*b*) Supersonic flow

To Find the Critical Area:

$$m = \rho A V = \rho^* A^* V^*$$

$$\frac{A}{A^*} = \frac{\rho^*}{\rho}\frac{V^*}{V}$$

$$\frac{\rho^*}{\rho} = \frac{\rho^*}{\rho_o}\frac{\rho_o}{\rho} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\left(1+\frac{k-1}{2}M^2\right)^{\frac{1}{k-1}}$$

$$\frac{V^*}{V} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M^2\right)\right]^{\frac{1}{2}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}\left(1+\frac{k-1}{2}M^2\right)^{\frac{k+1}{2(k-1)}}\right]$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1+\frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{2(k-1)}}$$

$$\boxed{\frac{A}{A^*} = \frac{1}{M}\left[\frac{2+(K-1)M^2}{k+1}\right]^{\frac{k+1}{2(k-1)}}}$$

The Maximum Mass Flow Rate

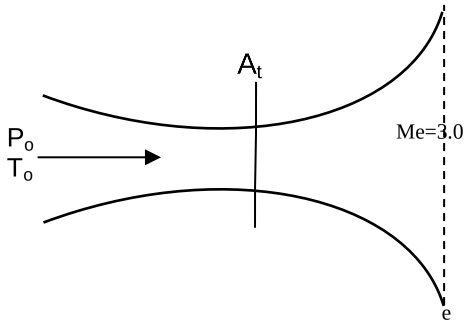
If the sonic condition does occur in the duct, it will occur at the duct minimum area. If the sonic condition occurs, the flow is said to be choked since the mass flow rate is maximum ($\dot{m}_{max}$) which is defined as the maximum mass flow rate the duct can accommodate without a modification of the duct geometry.

$$\dot{m}_{max} = \rho^* A^* V^* = \rho o\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} A^* \left(\frac{2k}{k+1}RT\right)$$

$$and \quad for \quad gases \quad having \quad k = 1.4:$$

$$\boxed{\dot{m}_{max} = \frac{0.686\, P_o A^*}{\sqrt{RT_o}}}$$

Ex: The reservoir conditions of air entering a converging-diverging nozzle are 100 kPa and 300 K. Mach number at exit equals to 3.0 and the mass flow rate is 1.0 kg/s. Determine: (a) throat area (b) the exit area and (c) the air conditions at exit section.



$$\dot{m}_{max} = \frac{0.686\, P_o A^*}{\sqrt{RT_o}}$$

$$\dot{m}_{max} = \frac{0.686\, A^*(100{,}1000)}{\sqrt{287(300)}}$$

$= 0.00428m^2$

$$\frac{A_e}{A^*} = \frac{1}{M}\left[\frac{2+(K-1)M_e^{\,2}}{k+1}\right]^{\frac{k+1}{2(k-1)}}$$

=4.234

Ae=0.0181m^2

From tables at M=3.0 , (Pe/Po)=0.027, (Te/To)=0.357

➔Pe=2.7 kPa, Te=107.1K

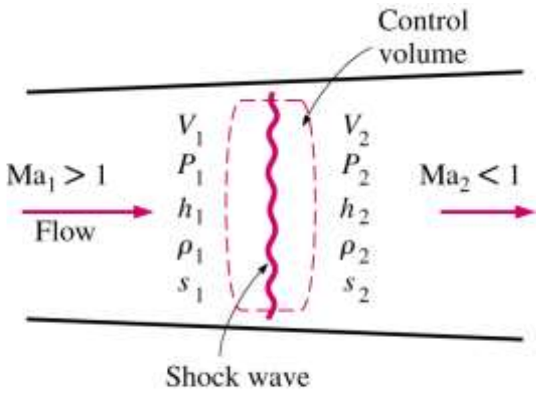
$P_e=P_e/(RT_e)$=0.088 kg/m^3

Shock waves

Under the appropriate conditions, very thin, highly irreversible discontinuities can occur in otherwise isentropic compressible flows. These discontinuities are known as *shock waves*. Flow process through the shock wave is highly irreversible and *cannot* be approximated as being isentropic. Shocks that occur in a plane normal to the direction of flow are called **normal shock waves**, some are inclined to the flow direction, and are called **oblique shocks**.

Normal Shock Wave

Developing relationships for flow properties before and after the shock using conservation of mass, momentum, and energy:



Conservation of mass

$$\rho_1 A V_1 = \rho_2 A V_2 \longrightarrow \rho_1 V_1 = \rho_2 V_2$$

------------(1)

Conservation of momentum

$$A(P_1 - P_2) = \dot{m}(V_2 - V_1)$$ -------------(2)

Conservation of energy

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \longrightarrow h_{01} = h_{02}$$

$$C_p T_1 + \frac{V_1^2}{2} = C_p T_2 + \frac{V_2^2}{2}$$ -----------(3)

Now, from equ(2), $P_2=P_1+\rho_1 V_1^2 - \rho_2 V_2^2$ -----(4)

Equ(1) into (4) : $P_2=P_1+\rho_1 V_1^2 - \rho_2 V_1 V_2$ -----(5)

Substitute on ρ_2 from equ(1) into equ(2)

$$\frac{V_1^2}{2} + \frac{k}{k-1}\frac{P_1}{\rho_1} = \frac{V_1^2}{2} + \frac{k}{k-1}\frac{P_1}{\rho_1}\frac{V_2}{V1}$$ -------(6)

By substituting V2 from equ (5) into equ(6), we obtain:

Normal Shock Relations

$$Ma_2^2 = \frac{(k-1)Ma_1^2 + 2}{2\,k\,Ma_1^2 - (k-1)}\,, \quad Ma_1 > 1$$

$$\frac{P_2}{P_1} = \frac{1 + k\,Ma_1^2}{1 + k\,Ma_2^2}$$

$$\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(k+1)\,Ma_1^2}{(k-1)\,Ma_1^2 + 2}$$

$$T_{o1} = T_{o2}$$

$$\frac{T_2}{T_1} = \left[2 + (k-1)Ma_1^2\right]\frac{2\,k\,Ma_1^2 - (k-1)}{(k+1)^2\,Ma_1^2}$$

$$\frac{P_{o2}}{P_{o1}} = \frac{\rho_{o2}}{\rho_{o1}} = \left[\frac{(k+1)\,Ma_1^2}{2 + (k-1)Ma_1^2}\right]^{k/(k-1)}\left[\frac{k+1}{2\,k\,Ma_1^2 - (k-1)}\right]^{1/(k-1)}$$

$$\frac{A_2^*}{A_1^*} = \frac{Ma_2}{Ma_1}\left[\frac{2 + (k-1)Ma_1^2}{2 + (k-1)Ma_2^2}\right]^{(k+1)/[2(k-1)]}$$

When using these equations to relate conditions upstream and downstream of a normal shock wave, keep the following points in mind:

1. Upstream Mach numbers are always supersonic while downstream Mach numbers are subsonic.
2. Stagnation pressures and densities decrease as one moves downstream across a normal shock wave while the stagnation temperature remains constant (a consequence of the adiabatic flow condition).
3. Pressures increase greatly while temperature and density increase moderately across a shock wave in the downstream direction.
4. The critical/sonic throat area changes across a normal shock wave in the downstream direction and $A_2^* > A_1^*$
.
5. Shock waves are very irreversible causing the specific entropy downstream of the shock wave to be greater than the specific entropy upstream of the shock wave.

k = 1.4

M_1	M_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_1/P_{02}
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	0.5283
1.01	0.9901	1.0235	1.0167	1.0066	1.0000	0.5221

Normal Shock Tables

k = 1.4

M_1	M_2	P_2/P_1	ρ_2/ρ_1	T_2/T_1	P_{02}/P_{01}	P_1/P_{02}
1.00	1.0000	1.0000	1.0000	1.0000	1.0000	0.5283
1.01	0.9901	1.0235	1.0167	1.0066	1.0000	0.5221
1.02	0.9805	1.0471	1.0334	1.0132	1.0000	0.5160
1.03	0.9712	1.0711	1.0502	1.0198	1.0000	0.5100
1.04	0.9620	1.0952	1.0671	1.0263	0.9999	0.5039
1.05	0.9531	1.1196	1.0840	1.0328	0.9999	0.4979
1.06	0.9444	1.1442	1.1009	1.0393	0.9998	0.4920
1.07	0.9360	1.1691	1.1179	1.0458	0.9996	0.4861
1.08	0.9277	1.1941	1.1349	1.0522	0.9994	0.4803
1.09	0.9196	1.2195	1.1520	1.0586	0.9992	0.4746
1.10	0.9118	1.2450	1.1691	1.0649	0.9989	0.4689
1.11	0.9041	1.2708	1.1862	1.0713	0.9986	0.4632
1.12	0.8966	1.2968	1.2034	1.0776	0.9982	0.4576
1.13	0.8892	1.3231	1.2206	1.0840	0.9978	0.4521
1.14	0.8820	1.3495	1.2378	1.0903	0.9973	0.4467
1.15	0.8750	1.3763	1.2550	1.0966	0.9967	0.4413
1.16	0.8682	1.4032	1.2723	1.1029	0.9961	0.4360
1.17	0.8615	1.4304	1.2896	1.1092	0.9953	0.4307
1.18	0.8549	1.4578	1.3069	1.1154	0.9946	0.4255
1.19	0.8485	1.4855	1.3243	1.1217	0.9937	0.4204
1.20	0.8422	1.5133	1.3416	1.1280	0.9928	0.4154
1.21	0.8360	1.5415	1.3590	1.1343	0.9918	0.4104
1.22	0.8300	1.5698	1.3764	1.1405	0.9907	0.4055
1.23	0.8241	1.5984	1.3938	1.1468	0.9896	0.4006
1.24	0.8183	1.6272	1.4112	1.1531	0.9884	0.3958
1.25	0.8126	1.6563	1.4286	1.1594	0.9871	0.3911
1.26	0.8071	1.6855	1.4460	1.1657	0.9857	0.3865
1.27	0.8016	1.7151	1.4634	1.1720	0.9842	0.3819
1.28	0.7963	1.7448	1.4808	1.1783	0.9827	0.3774
1.29	0.7911	1.7748	1.4983	1.1846	0.9811	0.3729
1.30	0.7860	1.8050	1.5157	1.1909	0.9794	0.3685
1.31	0.7809	1.8355	1.5331	1.1972	0.9776	0.3642
1.32	0.7760	1.8661	1.5505	1.2035	0.9758	0.3599
1.33	0.7712	1.8971	1.5680	1.2099	0.9738	0.3557
1.34	0.7664	1.9282	1.5854	1.2162	0.9718	0.3516

EX:

A normal shock wave exists in a air flow with upatream M=2.0 and a pressure of 20 kPa and temperature of 15°C. Find the Mach number, preeure, stagnation pressure, temperature, stagnation temperature and air velocity downstream of the shock wave

From Shock wave table: M2=0.577, $(P_2/P_1)=4.5$, $(T_2/T_1)=1.688$
$(P_{o2}/P_{o1})=0.721$

P2=4.5*20=90 kPa
T2=1.688*(273+15)=486 K
$M_2=V_2/C_2$ $V_2=0.577[1.4*287*486]^{1/2}$ =255 m/s

To find P_{02} and T_{o2}
From Isentropic table
At $M_1=2.0$, $(P_1/P_{o1})=0.128$, $(T_1/T_{o1})=0.556$
$P_{o1}=20/0.128$ =156.25 kPa
$P_{o2}=0.721*156.25=112.6$ kPa
$T_{o1}=288/0.556=518$ K
$T_{o2}=T_{o1}=518$ K

H.W.

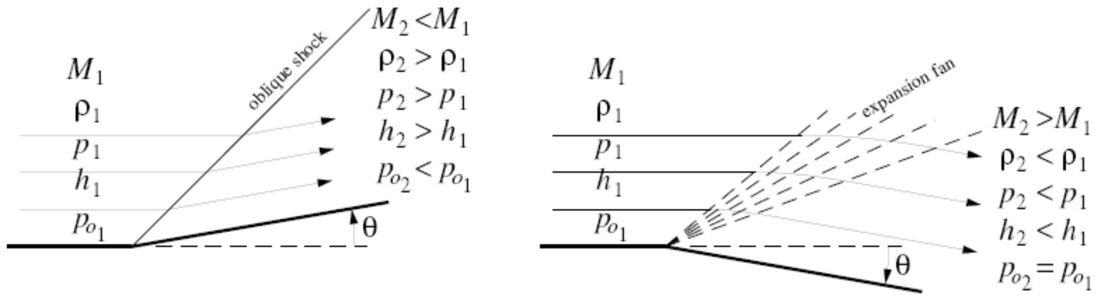
Air is supplied to the converging-diverging nozzle shown here from a large tank where P = 2 MPa and T = 400 K. A normal shock wave in the diverging section of this nozzle forms at a point where the upstream Mach number is 1.4. The ratio of the nozzle exit area to the throat area is 1.6. Determine (a) the Mach number downstream of the shock wave, (b) the Mach number at the nozzle exit, and (c) the pressure and temperature at the nozzle exit.

Oblique shock and expansion waves

Mach waves can be either compression waves (p2 >p1) or expansion waves (p2 <p1), but in either case their strength is by definition very small (|p2 −p1|<<p1). A body of finite thickness, however, will generate oblique waves of finite strength, and now we must distinguish between compression and expansion types. The simplest body shape for generating such waves is

– a concave corner, which generates an oblique shock (compression), or

– a convex corner, which generates an expansion fan. The flow quantity changes across an oblique shock are in the same direction as across a normal shock, and across an expansion fan, they are in the opposite direction. One important difference is that po decreases across the shock, while the fan is isentropic, so that it has no loss of total pressure, and hence $po_2 = po_1$

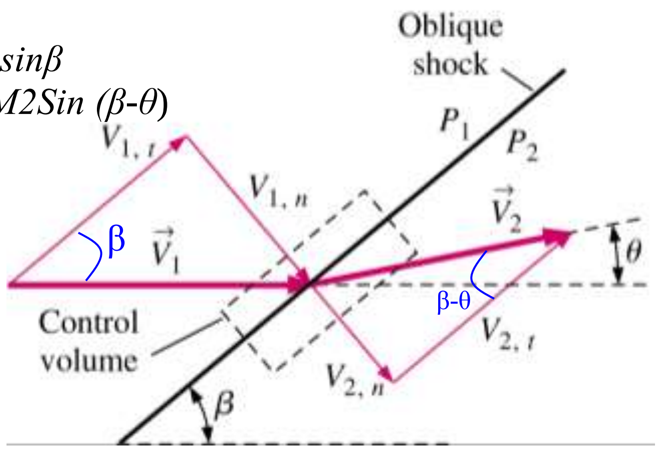


At leading edge, flow is deflected through an angle θ called the **turning angle**
Result is a straight oblique shock wave aligned at **shock angle** β relative to the flow direction

Due to the displacement thickness, θ is slightly greater than the wedge half-angle δ.

- Like normal shocks, Ma decreases across the oblique shock, and are only possible if upstream flow is supersonic
- However, unlike normal shocks in which the downstream Ma is always subsonic, Ma2 of an oblique shock can be subsonic, sonic, or supersonic depending upon Ma1 and θ.

- All equations and shock tables for normal shocks apply to oblique shocks as well, provided that we use only the **normal components** of the Mach number
    - *M1,n = V1,n/c1=V1sinβ/C1=M1sinβ*
    - *M2,n=V2,n/c2=V2sin(β-θ)/C2=M2Sin (β-θ)*



tanβ=V1,n/V1,t
tan(β-θ)= V2,n/V2,t

But *V2,t=V1,t* (there is no pressure change in the tangential direction)
Hence:

$$\frac{\tan\beta}{\tan(\beta-\theta)}=\frac{V_{1,n}}{V_{2,n}}$$

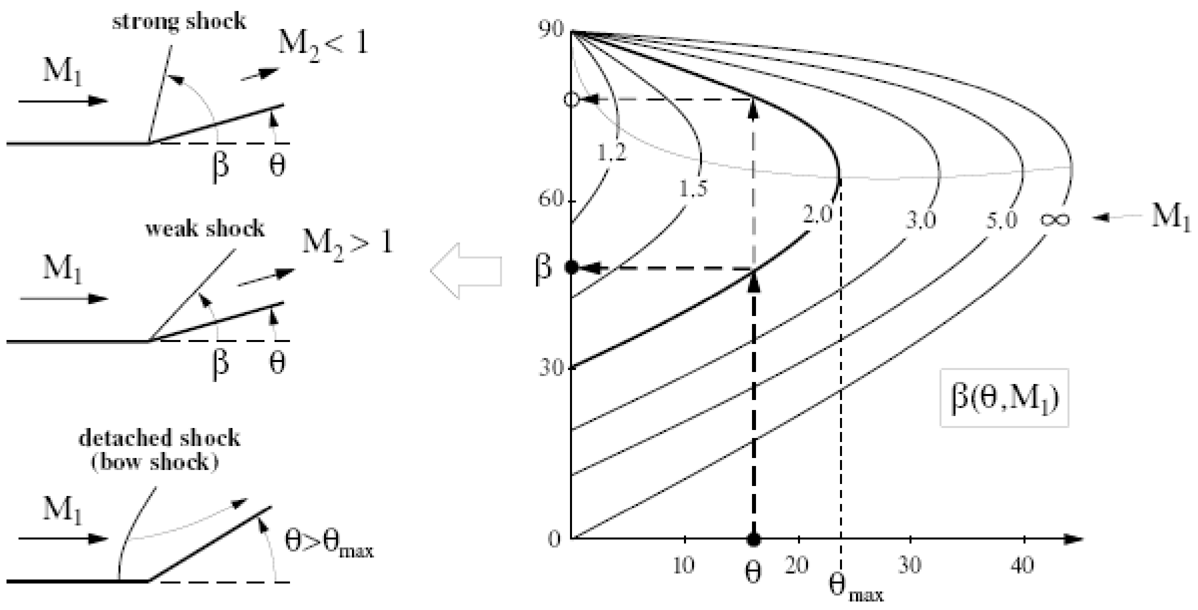
$$\frac{V_{1,n}}{V_{2,n}}=\frac{\rho_2}{\rho_1}=\frac{(k+1)M_1^2Sin^2\beta}{2+(k-1)M_1^2Sin^2\beta}$$

Hence:

$$\frac{\tan\beta}{\tan(\beta-\theta)}=\frac{(k+1)M_1^2Sin^2\beta}{2+(k-1)M_1^2Sin^2\beta}$$

Solving the above relation for θ:

$$\boxed{\tan\theta=\frac{2\cot\beta(Ma_1^2\sin^2\beta-1)}{Ma_1^2(k+\cos 2\beta)+2}}$$



The oblique shock chart above reveals a number of important features:

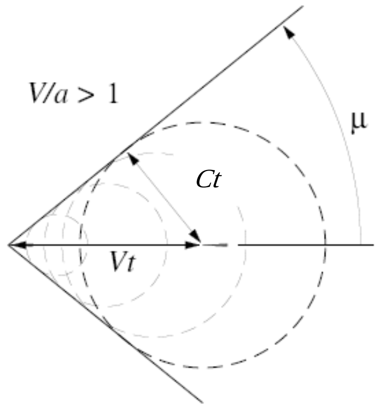
1-There is a maximum turning angle θ_{max} for any given upstream Mach number M1. If the wall angle exceeds this, or $\theta > \theta_{max}$, no oblique shock is possible. Instead, a detached shock forms ahead of the concave corner. Such a detached shock is in fact the same as a bow shock discussed earlier.

2-If $\theta < \theta_{max}$, two distinct oblique shocks with two different β angles are physically possible. The smaller β case is called a weak shock, and is the one most likely to occur in a typical supersonic flow. The larger β case is called a strong shock, and is unlikely to form over a straight-wall wedge. The strong shock has a subsonic flow behind it.

3-The strong-shock case in the limit $\theta \to 0$ and $\theta \to 90^o$, in the upper-left corner of the oblique shock chart, corresponds to the normal-shock case.

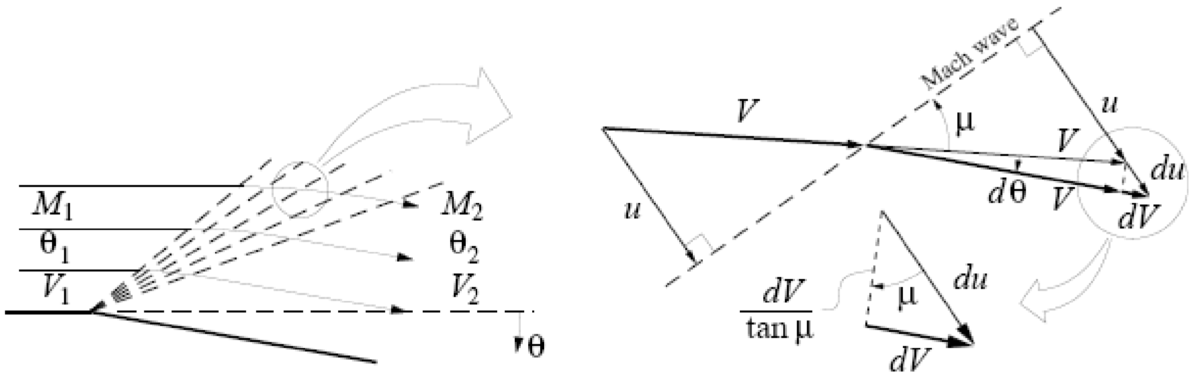
Prandtle-Meyer Waves

An expansion fan, sometimes also called a Prandtl-Meyer expansion wave, can be considered as a continuous sequence of infinitesimal Mach expansion waves. To understand the analysis clearly, we shall back to explain Mach cone or Mach wave.



*sin*μ=Ct/Vt=1/M
or $\mu=\sin^{-1}(1/M)$

To analyze this continuous change, we will now consider the flow angle θ to be a flow field variable, like M or V .
Across each Mach wave of the fan, the flow direction changes by d_, while the speed changes by dV . Oblique-shock analysis dictates that only the normal velocity component u can change across any wave, so that dV must be entirely due to the normal-velocity change du.



From the u-V and du-dV velocity triangles, it is evident that $d\theta$ and dV are related by

$$d\theta = \frac{dV}{\tan\mu}\,\frac{1}{V}$$

assuming $d\theta$ is a small angle. With $\sin\mu = 1/M$, we have

$$\frac{1}{\tan\mu} = \frac{\cos\mu}{\sin\mu} = \frac{\sqrt{1-\sin^2\mu}}{\sin\mu} = \frac{\sqrt{1-1/M^2}}{1/M} = \sqrt{M^2-1}$$

and so the flow relation above becomes

$$d\theta = \sqrt{M^2-1}\,\frac{dV}{V} \qquad (1)$$

This is a differential equation which relates a change $d\theta$ in the flow angle to a change dV in the flow speed throughout the expansion fan.

Prandtle-Meyer Function

The differential equation (1) can be integrated if we first express V in term of M.

$$V = MC = MC_o\left(1+\frac{k-1}{2}M^2\right)^{-\frac{1}{2}}$$

$$\ln V = \ln M + \ln C_o - \frac{1}{2}\left(\mathbf{1}+\frac{\mathbf{k-1}}{\mathbf{2}}M^2\right)$$

Differentiation the above relation:

$$\frac{dV}{V} = \frac{dM}{M} - \frac{1}{2}\left(1+\frac{k-1}{2}M^2\right)^{-1}\frac{k-1}{2}2M\,dM$$

$$\frac{dV}{V} = \frac{1}{1+\frac{k-1}{2}M^2}\frac{dM}{M}$$

Equation (1) then becomes:

$$d\theta = \frac{\sqrt{M^2-1}}{1+\frac{k-1}{2}M^2}\frac{dM}{M} \qquad (2)$$

which can now be integrated from point 1 to any point 2 in the Prandtle-Meyer wave

$$\int_{\theta_1}^{\theta_2} d\theta = \int_{M_1}^{M_2}\frac{\sqrt{M^2-1}}{1+\frac{k-1}{2}M^2}\frac{dM}{M}$$

$$\theta_2 - \theta_1 = \upsilon(M_2) - \upsilon(M_1) = \theta \quad (3)$$

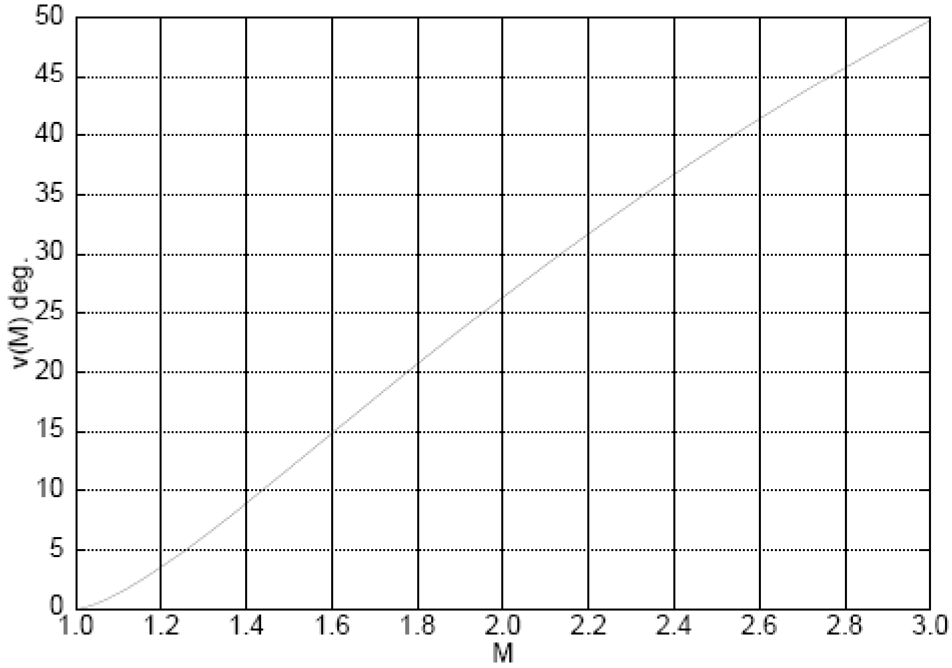
Where $$\upsilon(M) = \sqrt{\frac{k+1}{k-1}}\tan^{-1}\sqrt{\frac{k-1}{k+1}\left(M^2-1\right)} - \tan^{-1}\sqrt{M^2-1} \qquad (4)$$

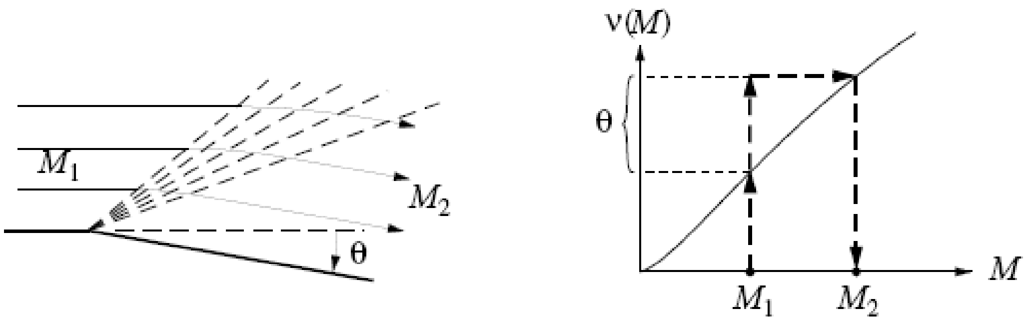
θ: is the total turning of the corner

Here ν(M) is called the Prandtle-Meyer function, and is shown plotted for k=1.4

Equation (3) can be applied to any two points within an expansion fan, but the most common use is to relate the two flow conditions before and after the fan. Reverting to our previous notation where θ is the total turning of the corner, the relation between θ and the upstream and downstream Mach number is

$$\theta = \nu(M_2) - \nu(M_1) \qquad (5)$$



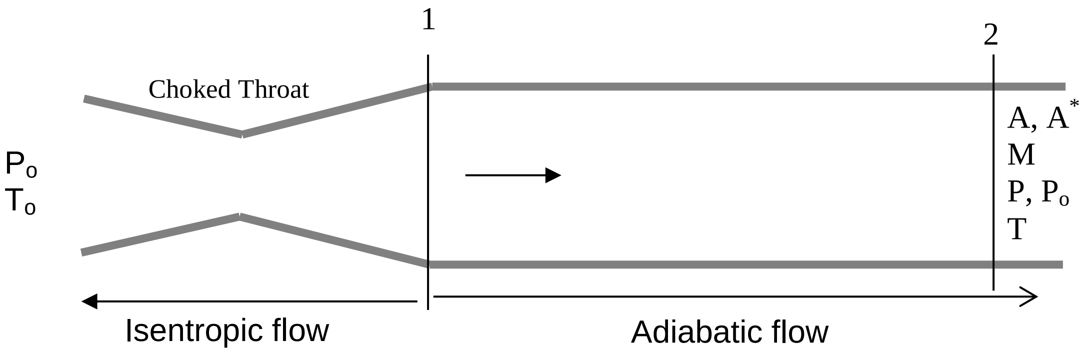


This can be considered an implicit definition of $M_2(M_1, \theta)$, which can be evaluated graphically using the $\nu(M)$ function plot, as shown in the figure.

Fan angle=μ_1-[μ_2-θ]

Adiabatic Flow

All the isentropic flow relations can be used through the adiabatic flow, in any section, provided that all variables of the relation refer to the same section.



Find A^*_1, and A^*_2 :

$$\dot{m}_{max} = \frac{0.686\ P_o A^*}{\sqrt{RT_o}}$$

Hence

$$A_1^* = \frac{\dot{m}_{max}\sqrt{RT_o}}{0.686\ P_{o1}} \tag{1}$$

$$A_2^* = \frac{\dot{m}_{max}\sqrt{RT_o}}{0.686\ P_{o2}} \tag{2}$$

divide (1) by (2)

We got:

$A^*_1 P_{o1} = A^*_2 P_{o2}$

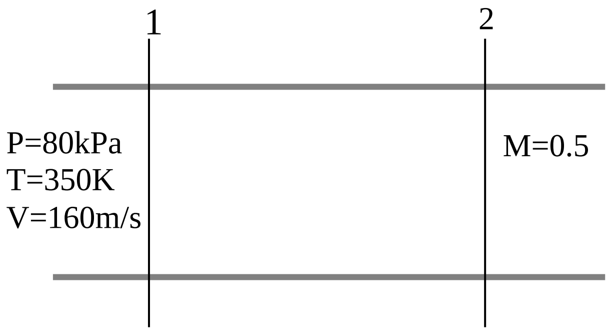
Generally

Through adiabatic flow:

$$\boxed{A^*_1 P_{o1} = \text{constant}}$$

Ex:

A constant area adiabatic duct has the following conditions of air flow: At section 1, the pressure P=0.8 bar, T=350K, air velocity=160 m/s. At section 2, Mach number=0.5, Find P, T, V at section 2.



<u>Sol.</u>

$A^*_1 P_{o1} = A^*_2 P_{o2}$

$$M_1 = \frac{V_1}{\sqrt{kRT}} = \frac{160}{\sqrt{1.4*287*350}} = 0.426$$

$$\frac{P_{o1}}{P_1} = \left[1 + \frac{k-1}{2} M_1^2\right]^{\frac{k}{k-1}}$$

$P_{o1}=80(1+0.2(0.426)^2)^{3.5}=90.632$

$$\frac{A_1}{A_1^*} = \frac{1}{M_1}\left[\frac{2+(K-1)M_1^{\ 2}}{k+1}\right]^{\frac{k+1}{2(k-1)}} = 1.511$$

$A2/A^*_2=1.34$

Hence: $(A1/1.511)*90.632=(A2/1.34)*P_{o2}$

Po2=80.3 kPa

$$\frac{P_{o2}}{P_2} = \left[1 + \frac{k-1}{2} M_2^2\right]^{\frac{k}{k-1}}$$

P2=80.3/1.1863=67.7

$T_{o2}=T_{o1}=T_1(1+0.2(0.426)^2)=362.7K$

T2=345.4K

V2=186.3m/s