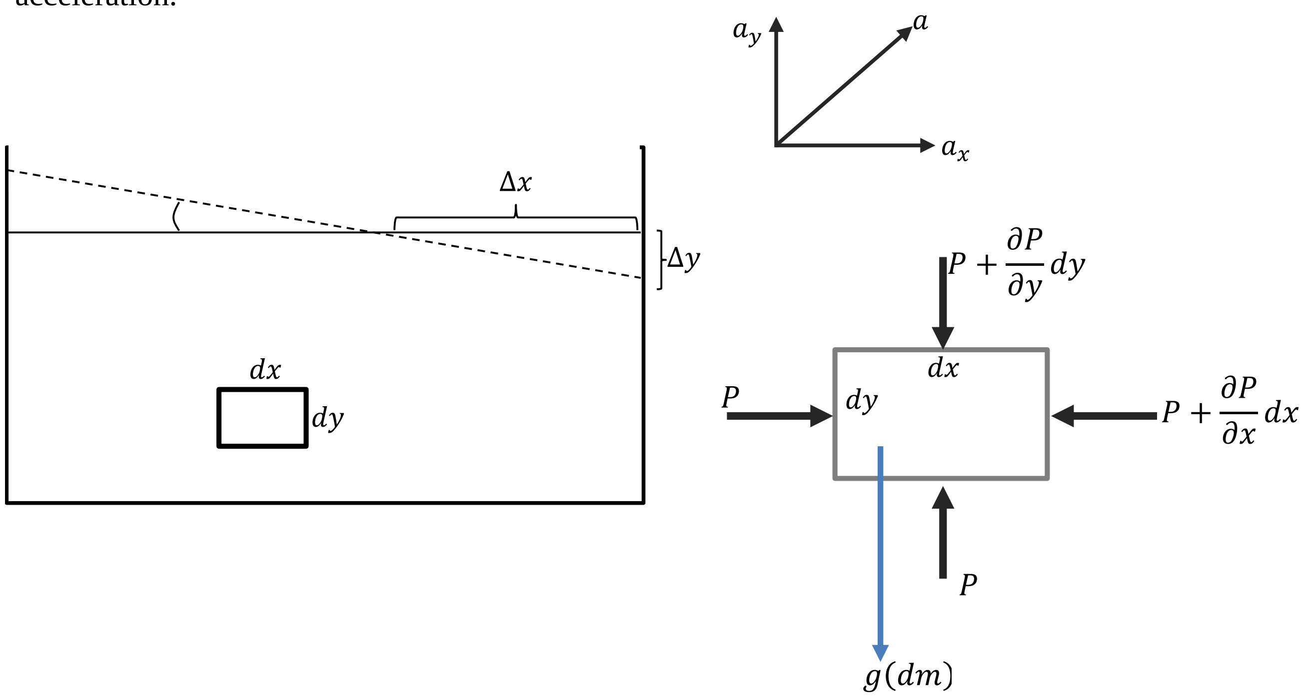
Chapter-5 Accelerated Fluid

When a fluid mass is moving with constant acceleration, we assume no relative motion between the fluid layers, i.e. no shear stress.

1- <u>Linear motion with constant acceleration.</u>

Assume a fluid in a vessel (of unit width), the vessel is moving with constant acceleration.



<u>Equation of Newton 2nd law in x-direction</u>

$$ma_x = \sum F_x$$

$$d_m a_x = P d_y - \left(P + \frac{\partial P}{\partial x} dx\right) dy$$

$$d_m a_x = -\frac{\partial P}{\partial x} dx dy$$

$$\therefore\ a_x = -\frac{1}{\rho}\frac{\partial P}{\partial x} \qquad (1)$$

<u>Equation of Newton 2nd law in y-direction</u>

$ma_y = \sum F_y$

$d_m a_y = Pd_x - \left(P + \frac{\partial P}{\partial y}dy\right)dx - gdm$

$$d_m a_y = -\frac{\partial P}{\partial y}dxdy - gdm$$

$dm = \rho(dx * dy * 1)$

$$\therefore a_y = -g - \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad (2)$$

Note: if $a_y = 0$, the pressure along y direction will vary hydrostatically i.e. P = γh.

But, $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$

Hence, from equations (1) and (2),

$$\boldsymbol{dP = -\rho a_x d_x - (\rho g + \rho a_y)dy} \qquad (3)$$

The line of constant pressure, can be found from the above equation, by setting $dP = 0$

⇨ $\rho a_x d_x = -\rho(g + a_y)dy$

$$\therefore \boldsymbol{\frac{dy}{dx} = -\frac{a_x}{g + a_y}} \quad \text{(negative slope).}$$

The line of constant pressure is free surface itself.

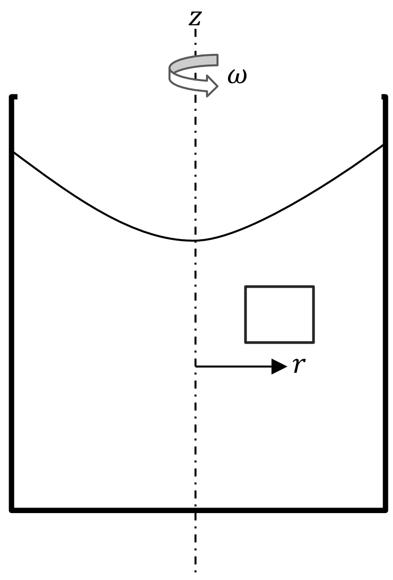
2- <u>Rotation with constant acceleration</u>

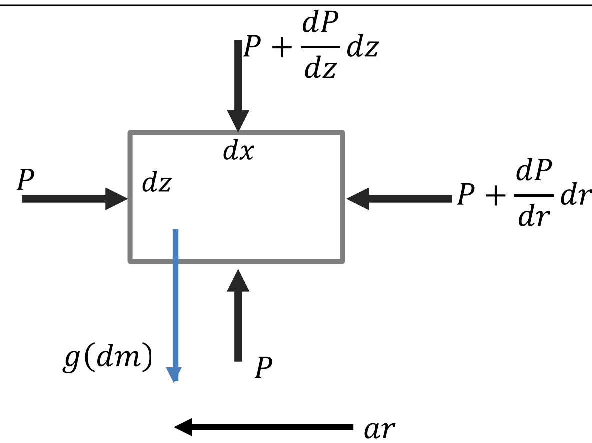
Assumptions:

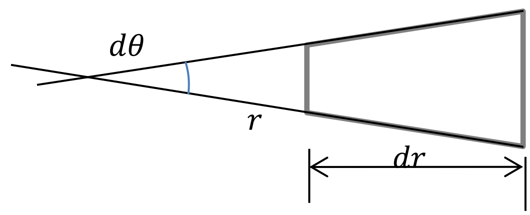
- No pressure variation with θ direction
- The horizontal rotation will not alter the pressure distribution in the vertical direction (i.e. the pressure equals to $P = \gamma h$).

Applying Newton's 2nd low in r-direction:

$-ma_r = \sum F_r$







$$-dm\, a_r = \sum F_r$$

$$-\rho d\theta dr dz\, a_r = P r d\theta dz - \left(P + \frac{\partial P}{\partial r} dr\right) dz\, r\, d\theta$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, a_r$$

$$a_r = r\omega^2$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, r\omega^2 \qquad (1)$$

$$-ma_z = \sum F_z = 0\,, \quad a_z = 0$$

$$P\, r d\theta\, dr - \left(P + \frac{\partial P}{\partial z} dz\right) r\, dr\, d\theta - \rho\, r\, dr\, d\theta\, dz\, g = 0$$

$$\therefore \frac{\partial P}{\partial z} = -\rho\, g \qquad (2)$$

But, $dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$

$$\boldsymbol{dP = \rho\, r\, \omega^2 dr - \rho g\, dz} \qquad (3)$$

on the free surface, dP = 0 .

$$\omega^2 \left(\frac{r_2^2}{2} - \frac{r_1^2}{2}\right) = g(z_2 - z_1)$$

If we put point 1 at the z-axis so that $r_1 = 0$

$$\omega^2 \frac{r_2^2}{2} = g(z_2 - z_1)$$ Equation of Parabola.

<u>*Example 5.1*</u>

The tank shown in Fig. 1a is accelerated to the right. Calculate the acceleration ax needed to cause the free surface shown in Fig. 1b to touch point A. Calculate also the pressure at point

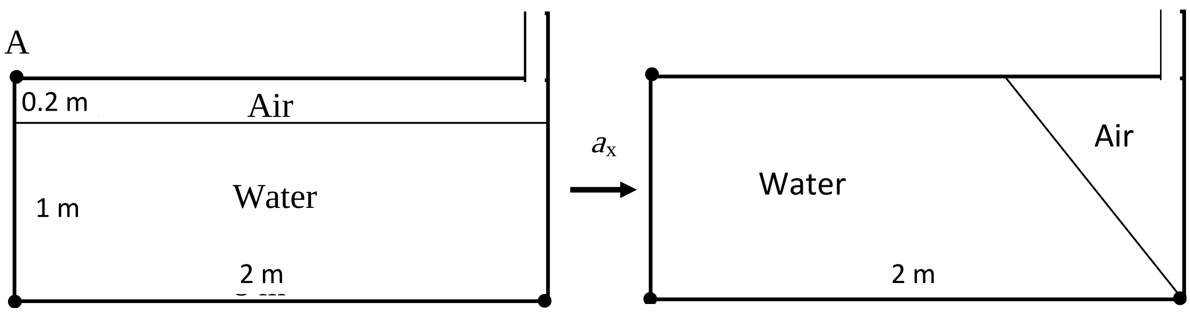
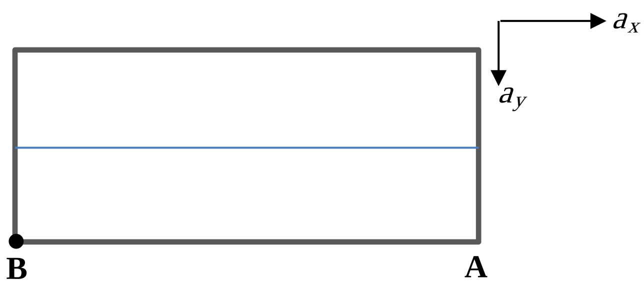


Fig. 1a Fig. 1b

Example 5.2

A closed box with horizontal base of 6x6 m and height of 2 m is half filled with water. It is given a_x =g/2 and a_y=-g/4. Find the pressure at point b as shown.



<u>*Example 5.3*</u>

A water-filled cylinder is rotating about its center line. Calculate the rotational speed that is necessary for the water to just touch the origin and the pressures at A and B.