## Chapter 4- Buoyancy and Stability

Buoyancy: resultant force exerted on a body by static fluid which is submerged or floating. It always acts vertically upward.


- The buoyancy force acts through the centroid of the displaced liquid volume.
- It can be proven that the Buoyancy force equals the weight of the displaced liquid.
- For equilibrium, $F_{\mathrm{B}}=W, F_{\mathrm{B}}=\gamma V_{\text {displaced liquid }}$

Example: A 0.2 m cube is floating as shown, find the density of the cube material.


Hydrometer: an instrument used to measure the specific gravity of liquids. It consists of bulb and constant area stem. When placed in pure water the specific gravity is marked to read 1.0. The force balance is

$$
W=\gamma_{\text {water }} V_{\text {displaced }}
$$

Where W is the weight of the hydrometer and V is the submerged volume below the $S=1.0$ line. In an unknown liquid of specific gravity, $\gamma_{x}$, a force balance would be:

$$
W=\gamma_{x}(V-A \Delta h)
$$

Where A is the cross-sectional area of the stem. Equating the two equations above gives

$$
\Delta h=\frac{V}{A}\left(1-\frac{1}{S_{x}}\right)
$$

Where $\operatorname{Sx}=\gamma_{x} / \gamma_{\text {water }}$


Hydrometer: (a) in water, (b) in unknown liquid

## Stability

Stability becomes an important consideration when floating bodies such as a boat or ferry is designed. It is an obvious requirement that a floating body such as a boat does not topple when slightly disturbed. We say that a body is in stable equilibrium if it is able to return to its position when slightly disturbed. Failure to do so denotes unstable equilibrium

## Stability of submerged bodies



Stability of floating bodies: in this case, the stability is more complicated to deal with.
When the body is slightly rotated about O ,
1- The center of gravity remains unchanged.
2- The center of buoyancy is changed to $\mathrm{C}^{\prime}$
The center of the buoyancy ( C , the centroid of the displaced volume of fluid) of a floating body depends on the shape of the body and on the position in which it is floating.

Extending a line from C' vertically. It will intercept with a line extended from the point O (axis of rotation) at a point M . This point M is called the Metacenter.

Now:
If M is above G , the body is stable, otherwise, it is unstable and according to the
 following relation:

$$
\overline{G M}=\frac{I_{o}}{V_{\text {displac }}}-\overline{C G}
$$

Where $\overline{G M}$ : distance between G and M
$\overline{C G}$ distance between C and G


$$
I_{o}=\frac{b L^{3}}{12}
$$

b: length into the paper
$I_{0}$ : second moment area of the waterline area about an axis passing through the Origin O .
$\mathrm{V}_{\text {displac }}$ : Volume of displaced liquid or (submerged body)

Example: A 0.25 long cylinder with 0.25 m diameter composed of material with density of $\rho=815 \mathrm{~kg} / \mathrm{m}^{3}$. Will it float on water on its base?


