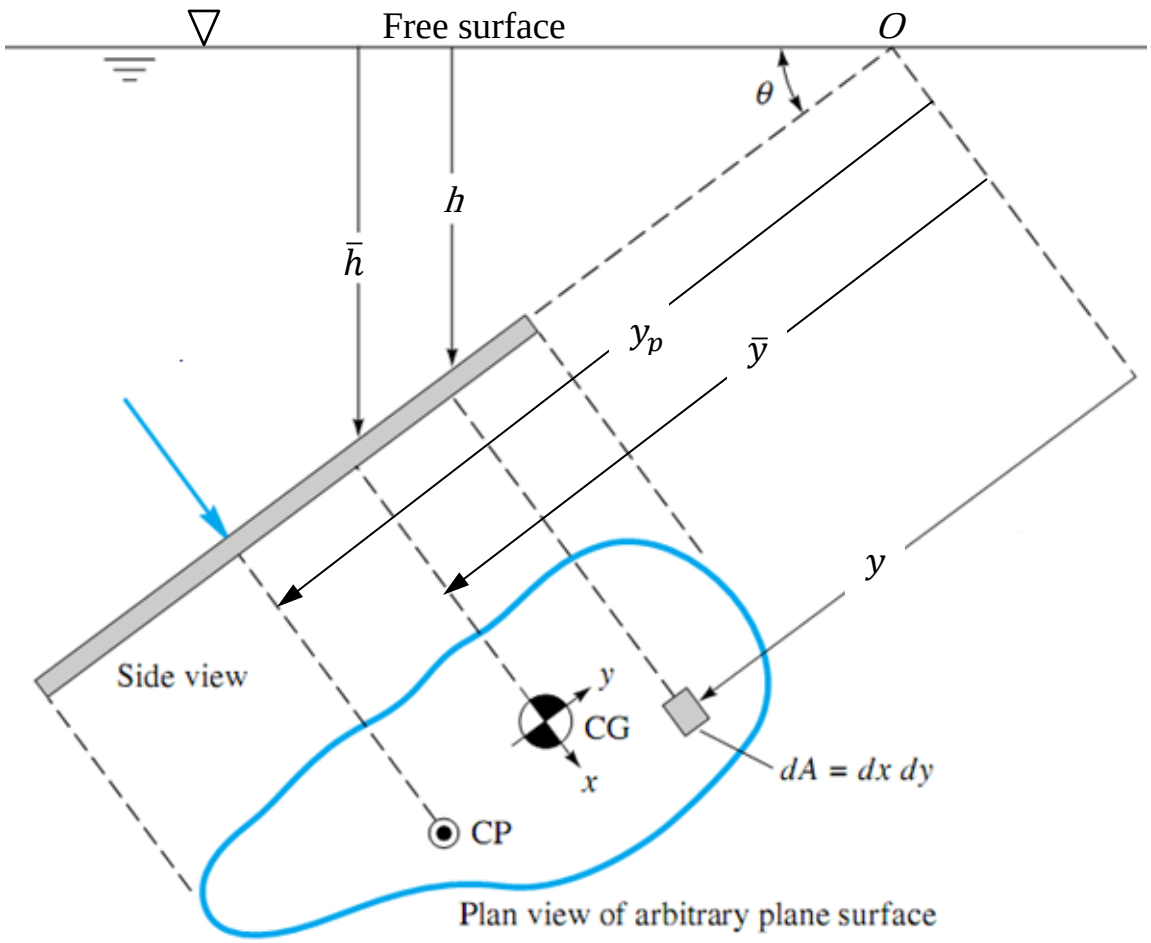
Chapter 3 Forces on Immersed Surfaces

In the design of submerged devices and objects, surfaces, dams, surfaces on ships, and holding tanks, it is necessary to calculate the magnitude and location of forces that act on both plane and curved surfaces. This subject will be divided into two titles; plane and curved surfaces.

1- Plane Surfaces



Cp: Center of pressure force

CG: center of geometry

The total force of the liquid on the plane surface is found by integrating the pressure over area.

$$F = \int_A P\, dA$$

Using gauge pressure, the local pressure is

$$P = \gamma h = \gamma\, y\, sin\theta$$

$$\therefore F = \int_A \gamma\, y\, sin\theta\, dA$$

$$\therefore F = \gamma\, sin\theta \int_A y\, dA$$

h is measured vertically down from the free surface and *y* is measured from point *O* on the free surface.

We know that the distance to a centroid is defined as:

$$\bar{y} = \frac{1}{A}\int_A y\, dA$$

$$\therefore F = \gamma\, sin\theta\, \bar{y}A = \gamma\, \bar{h}\, A = P_c A$$

Where Pc is the pressure at the centroid.

How to find the location of the resultant force *F*?

Generally, we termed to the location of the resultant force by y_p. Firstly, we should defined the well known rule that says: *the sum of the moments of all the infinitesimal forces acting on the area A must equal the moment of the resultant force.*

$$y_p F = \int_A y \; PdA = \int_A y \; \gamma \, sin\theta \, y \, dA = \gamma \, sin\theta \int_A y^{\,2} \, dA$$

But, $I_x = \int_A \; y^{\,2} \, dA$ is the second moment of area about x-axis

$$y_p F = \gamma \, sin \, \theta \, I_x$$

$$\therefore y_p = \frac{\gamma \, sin\theta \, I_x}{\gamma \, \bar{y} \, A \, sin\theta}$$

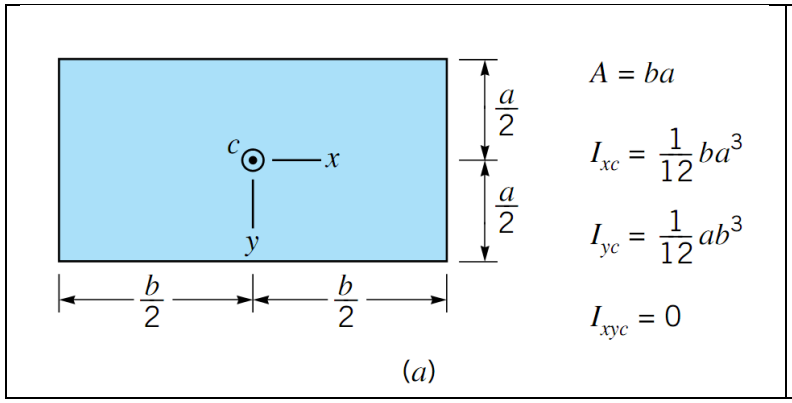
$I_x = I_{xc} + A\bar{y}^2$ (Parallel axis theorem),

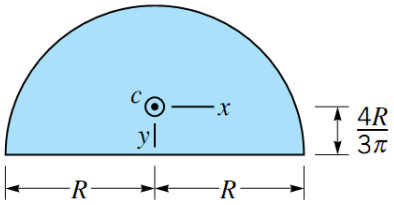
Where I_{xc} is the second moment of area about the centroid axis.

$$\therefore y_p = \frac{I_{xc} + A\bar{y}^2}{\bar{y} \, A}$$

$$\therefore y_p = \bar{y} + \frac{I_{xc}}{\bar{y} \, A}$$

This equation clearly shows that the resultant force F doesn't pass through the centroid but it always below it.





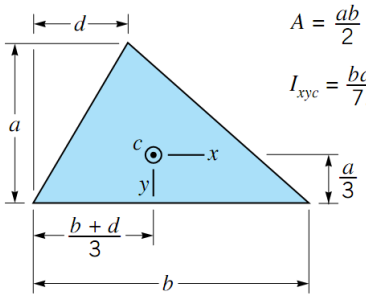
$A = \frac{\pi R^2}{2}$

$I_{xc} = 0.1098R^4$

$I_{yc} = 0.3927R^4$

$I_{xyc} = 0$

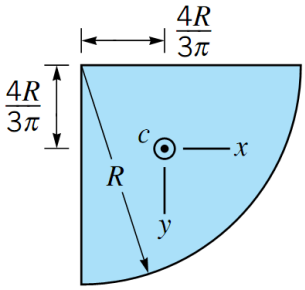
(c)



$A = \frac{ab}{2}$ $I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

(d)



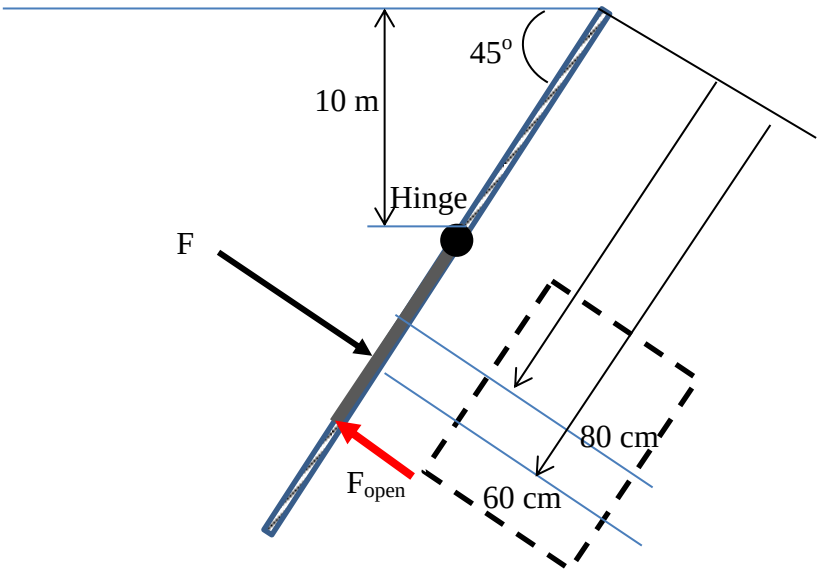
$A = \frac{\pi R^2}{4}$

$I_{xc} = I_{yc} = 0.05488R^4$

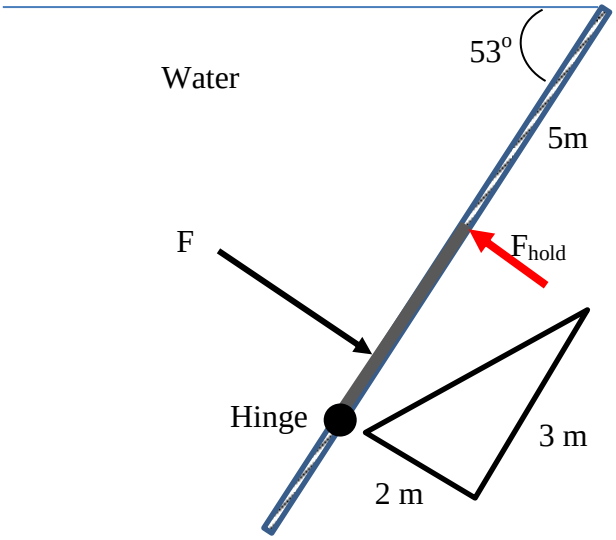
$I_{xyc} = -0.01647R^4$

(e)

Example: A 60 x 80 cm window on a submersible lake. If it is on a 45° angle with horizontal, what force applied normal to the window at the bottom edge in needed to just open the window, if is hinged at the top edge when the top edge is 10 m below the surface?



Example: find the force necessary to hold the gate in the position shown in figure.



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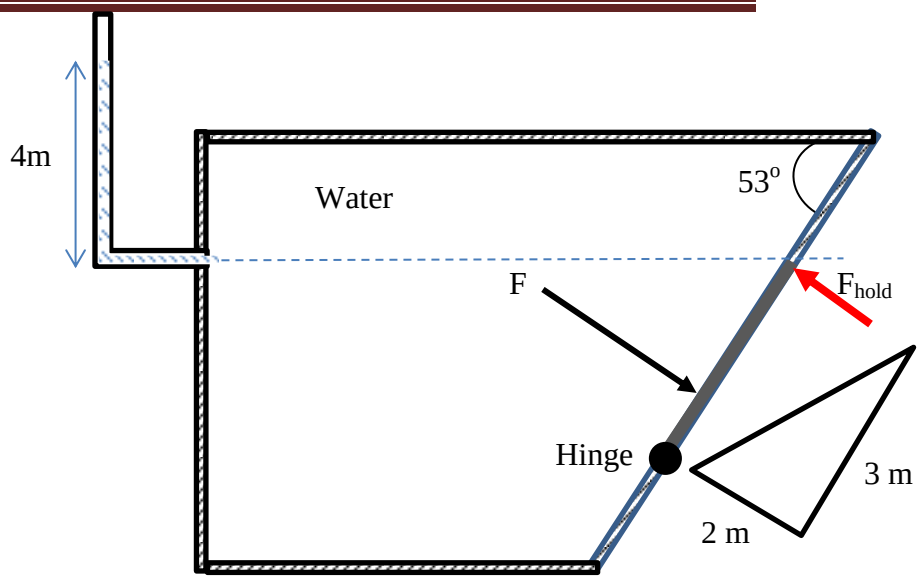


2- Curved Surfaces

We know that the pressure force is normal on each element of the surface. For curved surfaces, we calculate the *components* (horizontal and vertical) rather than the resultant, this for simplicity.

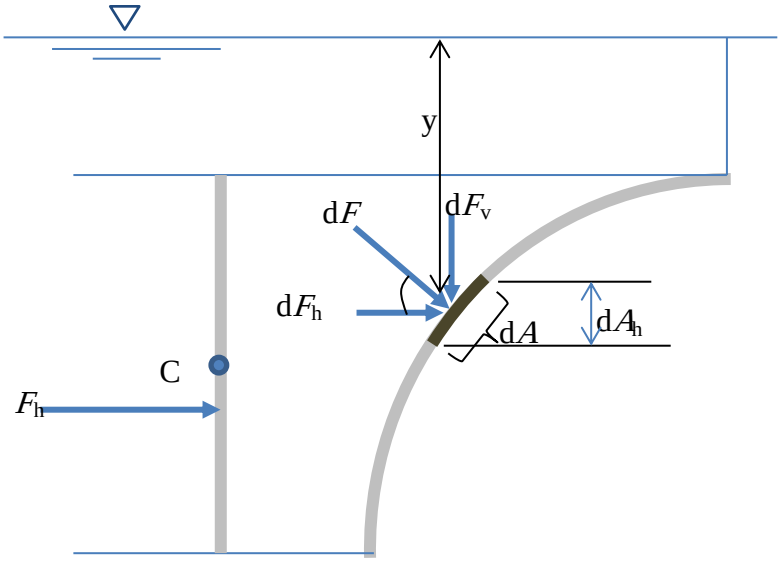
2-1 The horizontal component

$$F_h = \int_A dF_h = \int_A P\, dA\, cos\theta = \int_A \gamma\, y\, dA\, cos\theta$$

$$F_h = \gamma \int_A y\, dA_h$$

$$\therefore F_h = \gamma\, \bar{y}\, A_h$$

Where A_h is the vertical projection of the curved area and $\bar{y}$ is the centroid of the projected area.



2-2 The vertical component

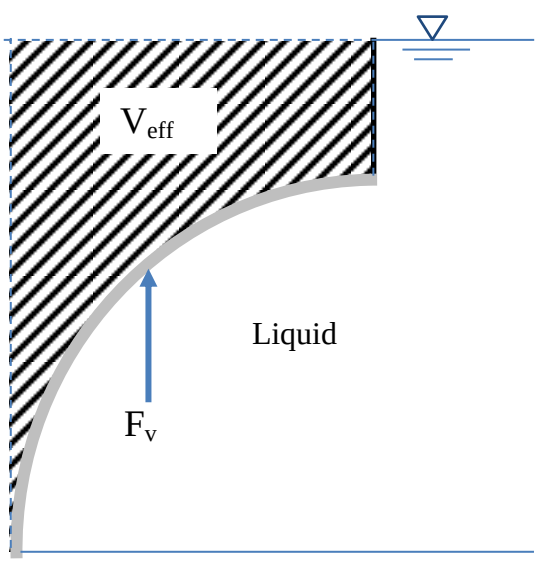
$$F_V = \int dF_V = \int P\, dA\, sin\theta = \int \gamma\, y\, dA\, sin\theta$$

$$F_V = \gamma \int y\, dA_V$$

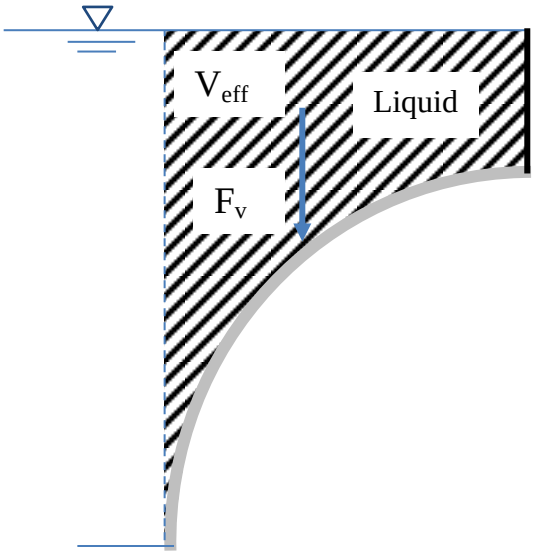
The last integral represents the fluid volume over the curved surface until the free surface (at which the pressure is atmospheric), hence we can say that the vertical component is the fluid weight over the curved surface.

$$F_V = \gamma\, V$$

- The fluid volume V is found by extending the curved surface to the free surface level ($P = P_{atm} = 0$).
- When the liquid is below the curved surface, an imaginary or equivalent free surface can be constructed. The weight of the imaginary volume of liquid vertically above the curved surface is then the vertical component of pressure force on the curved surface.
- The imaginary liquid must be of the same specific weight as the liquid in contact with curved surface.



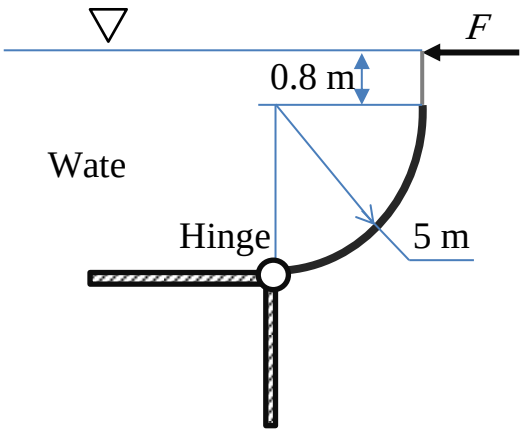
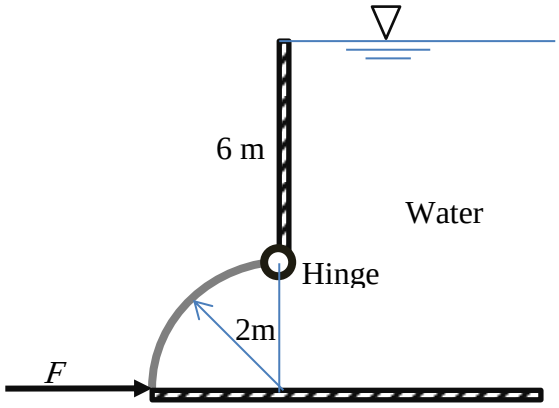
Liquid below the curved surface



Liquid above the curved surface

Note: the location of the vertical component action must pass through the centroid of the effective volume.

Example: Find the force *F* required to hold the gate in the position shown in figure. The gate is 5 m wide.



Example: Find the force F needed to just open the gate shown. The gate is 4 m wide.