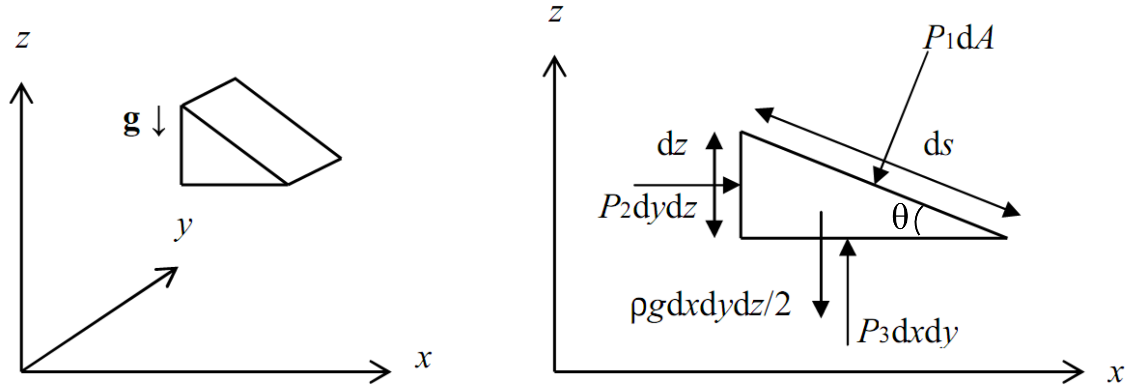
Chapter 2- Fluid Statics

Pressure acting on a point

It can be proven that the pressures acting on a point at rest, has the same value in all directions. Let us assume a particle of a fluid at rest, with free body diagram shown in figure.



$dA = ds \cdot dy = dy \cdot dz/\sin\theta$

$$\sum \mathbf{F} = 0$$

$$F_x = P_2 \mathrm{d}y\mathrm{d}z - P_1 \mathrm{d}A \sin\theta = 0$$

$$P_2 \mathrm{d}y\mathrm{d}z = P_1 \mathrm{d}y \frac{\mathrm{d}z}{\sin\theta} \sin\theta$$

$$P_2 = P_1$$

$$F_z = P_3 \mathrm{d}y\mathrm{d}x = \frac{1}{2}\rho g \mathrm{d}x\mathrm{d}y\mathrm{d}z + P_1 \mathrm{d}y \frac{\mathrm{d}x}{\cos\theta} \cos\theta$$

$$P_3 = P_1 + \frac{1}{2}\rho g \mathrm{d}z$$

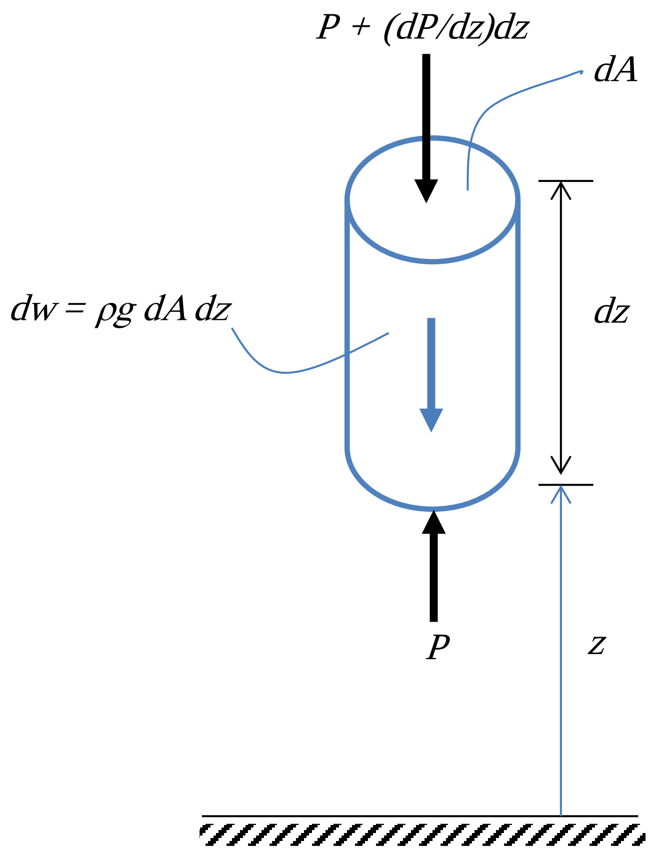
$$\mathrm{d}z \to 0,\ P_3 = P_1$$

$$\therefore P_1 = P_2 = P_3$$

Pressure variation with depth

Assuming a small element with a cross sectional area dA and length dz. The upward acting pressure is P and the downward acting pressure is $P + \frac{dP}{dz}dz$.

The force balance gives:



$$PdA - \left(P + \frac{dP}{dz}dz\right)dA - \rho g dAdz = 0$$

$$\frac{dP}{dz}dzdA = -\rho g dAdz$$

$$\therefore dP = -\rho g dz$$

$$P = -\rho g \int dz$$

$$P = -\rho g z + c$$

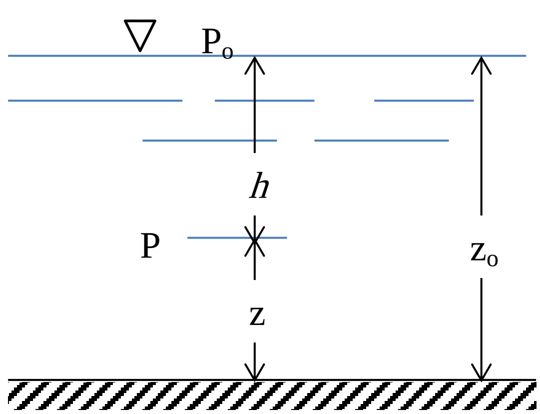
To find the constant c, we need a pressure value at a known elevation.

At $z = z_o$, $P = P_o$

$$\therefore c = P_o + \rho g z_o$$

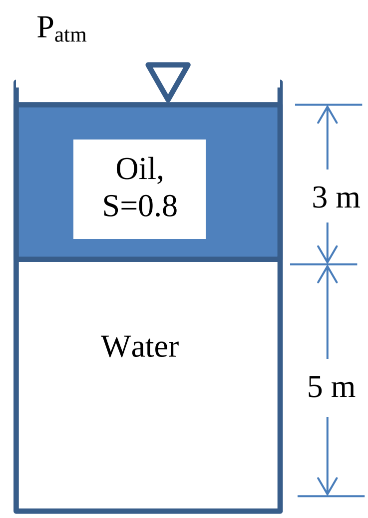
$$\therefore P = P_o + \rho g (z_o - z)$$

$$P = P_o + \rho g\, h$$

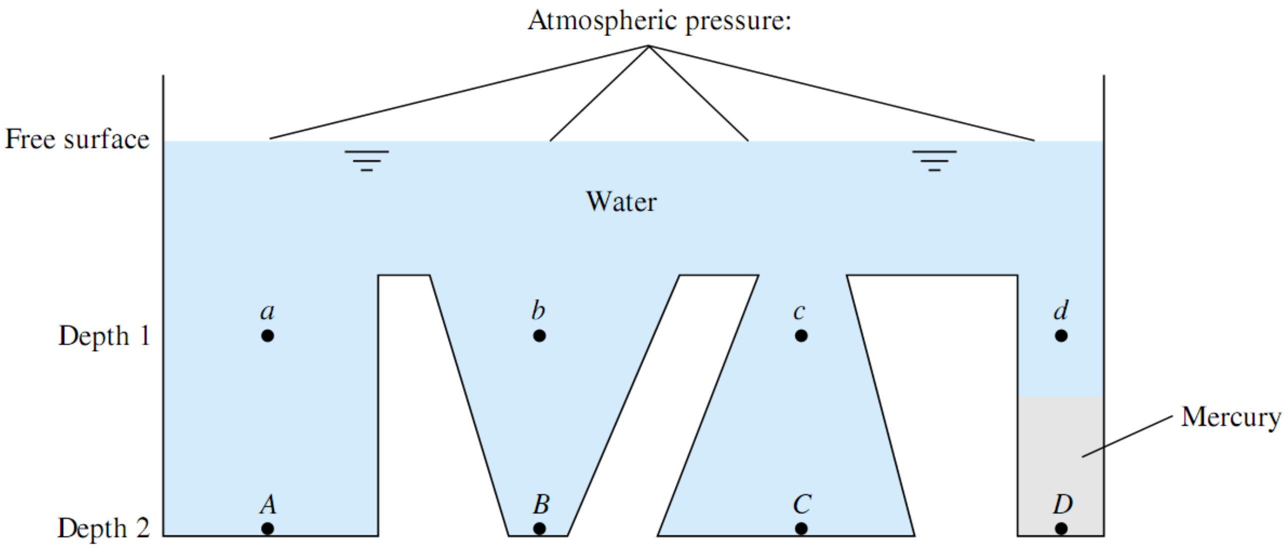


Example 2.1 Determine the pressure of sea water at 10 m under sea level. Given the sea water density as 1020 kg/m^3.

Example 2.2 Determine the pressure at the base of the tank shown in figure below.

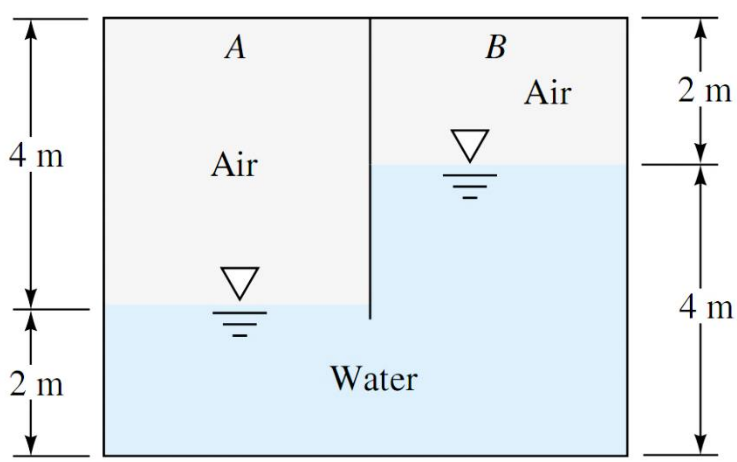


Note: Pressure doesn't vary horizontally, provided that the fluid is connected. To illustrate this statement, we may refer to the figure below.



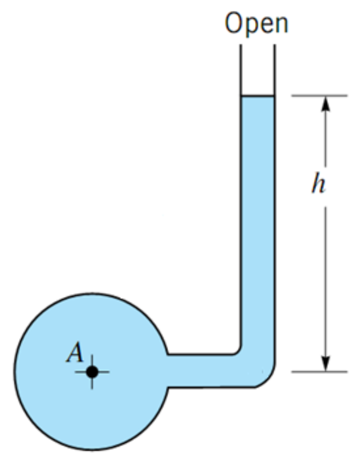
Points *a*, *b*, *c*, and *d* are at equal depths in water and therefore have identical pressures. Points *A*, *B*, and *C* are also at equal depths in water and have identical pressures higher than *a*, *b*, *c*, and *d*. Point *D* has a different pressure from *A*, *B*, and *C* because it is not connected to them by a water path.

Example 2.3: For the closed tank shown in figure, the pressure at point *A* is 95 kPa absolute, what is the absolute pressure at point *B*?

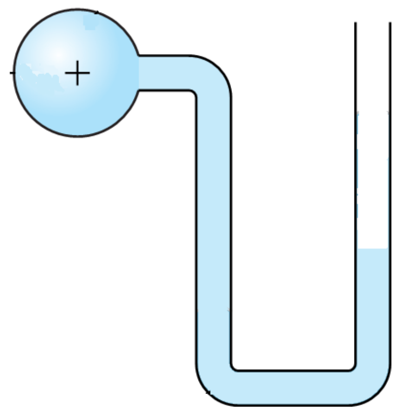


Manometers: devices that employ liquid columns for determining differences in pressure:

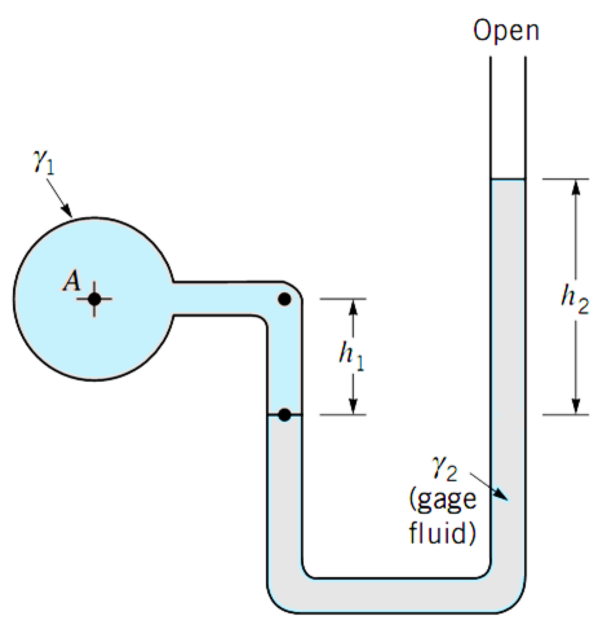
1- Piezometer Manometer: The simplest type of manometer consists of a vertical tube ,open at the top, and attached to the container in which the pressure is required ,it is used for small positive pressures.



2- U-Tube Manometer: This type of manometer consists of a tube formed into the shape of a U filled with the same fluid to be measured. It is used for small positive and negative pressures.



3- U-Tube Manometer with Multi-Liquids: It is U tube with using another liquid(s) of greater gravity. It is used for greater positive and negative pressure.



General Procedure in Working with Manometers Problems.

1- Start at one end and write the pressure there.

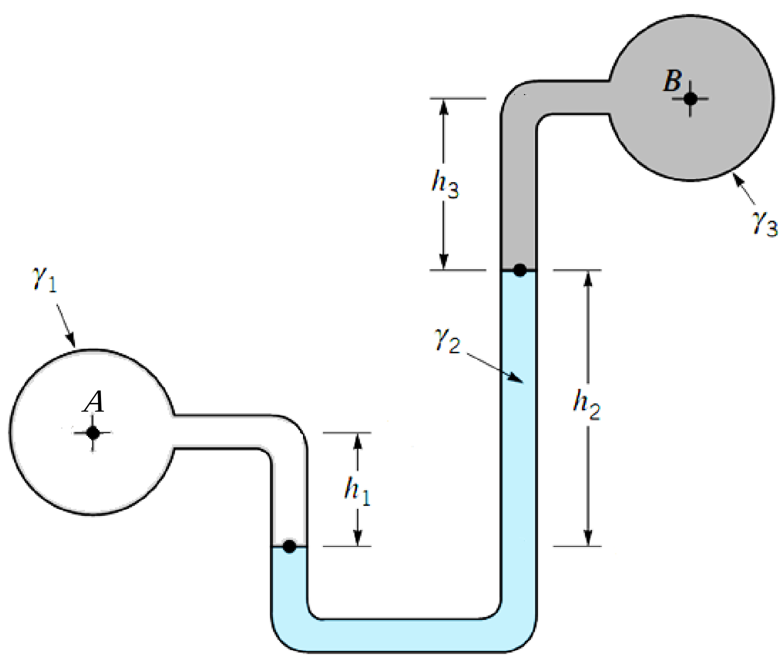
2- Add to the started pressure the change in pressure in the same unit from one meniscus (liquid surface) to the next (plus for lower meniscus and minus for higher)
3- Continue until the other end of the gage, and equate the expression to the pressure at that point.

$$P_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = P_B$$

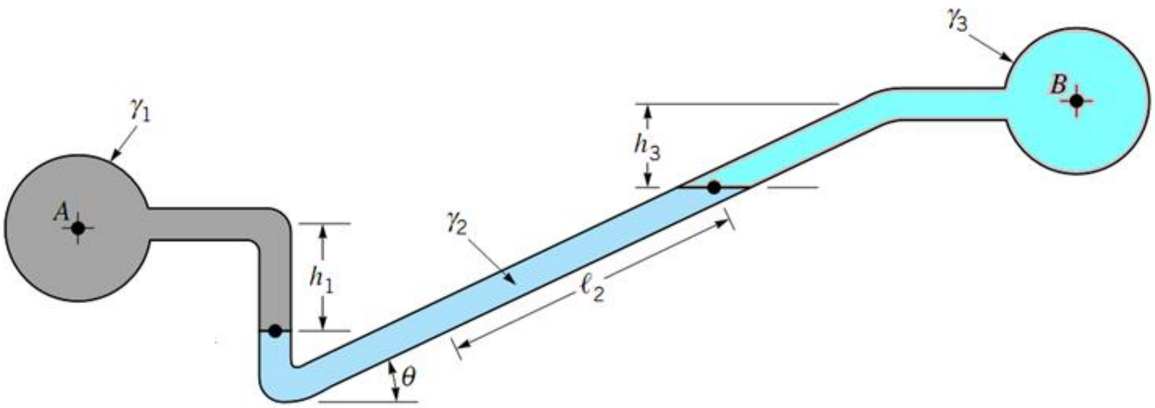
Or, $P_A - P_B = -\gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3$

Note: If any tube section is filled with gas, then the elevation in this section can be ignored because the specific weight (γ) of gases is much less than liquids. For example, in the figure shown, if fluid 1 is a gas, then the manometer relation will be:

$$P_A - P_B = \gamma_2 h_3 + \gamma_3 h_3$$

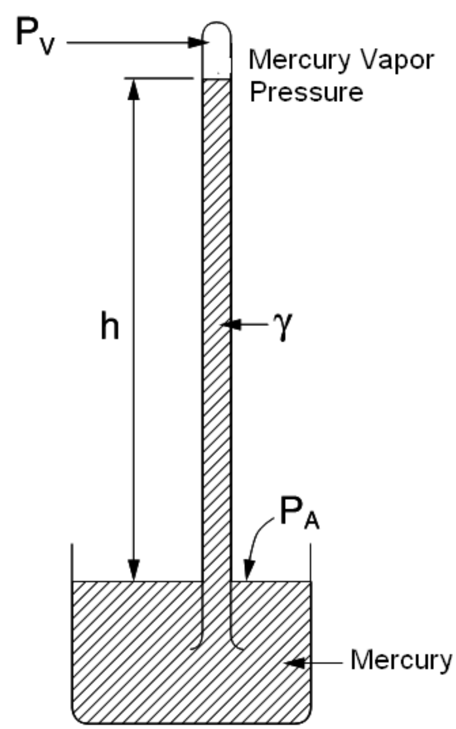


Inclined Tube Manometer: this type of manometer is designed to increase the accuracy of pressure measurements.



$$P_A + \gamma_1 h_1 - \gamma_2 l \sin\theta - \gamma_3 h_3 = P_B$$

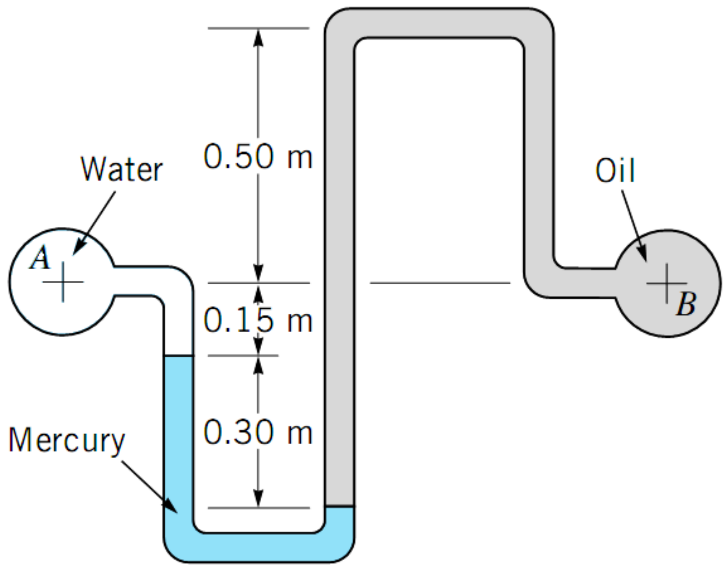
Mercury Barometer: it consists of a glass tube closed at one end and filled with mercury, and inverted so that the open end is submerged in mercury. It is used to measure the atmospheric pressure, P_A



$$P_A = \gamma_{Hg} h + P_V$$

P_V: is the pressure of mercury vapor

<u>Example 2.4</u>: The mercury manometer shown indicates a differential reading of 0.30 m. Determine the differential pressure between pipe *A* and pipe *B*. What is the pressure value in pipe *B* when the pressure in pipe *A* is 30-mm Hg vacuum.



Example 2.5: For the inverted manometer shown in figure, if P_B-P_A= 90 kPa, what must the height *H* be?