Chapter Two: The Transient Circuits

The analysis of circuits containing inductors and/or capacitors is dependent upon the formulation and solution of the Integra differential equations that characterize the circuits. The solution of the differential equation represents a response of the circuit, and it is known by many names:

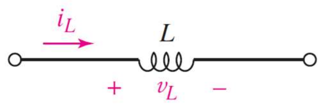
- *The source-free response may be called the **natural response**, the **transient response**, the **free response**, or the **complementary function**, but because of its more descriptive nature, we will most often call it the **natural response**.*
- *When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular source (or forcing function) used; this part of the response, called the particular solution, the **steady-state response**, or **the forced response**.*
- *In other words, the **complete response** is the sum of the natural response and the forced response.*

We will consider several different methods of solving these differential equations. The mathematical manipulation, however, is not circuit analysis.

2.1 RL Circuit:

Changing magnetic field could induce a voltage in a neighbouring circuit. This voltage is proportional to the time rate of change of the current producing the magnetic field. The constant of proportionality is what we now call the inductance, symbolized by L, and therefore

$$v = L\frac{di}{dt}$$



where we must realize that v and i are both functions of time.

Several examples of commercially available inductors are shown in Fig. 2.1.

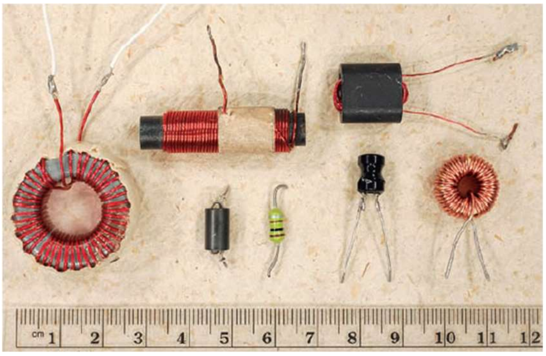


Fig. 2.1: Several different types of commercially available inductors.

We begin our study of transient analysis by considering the simple series RL circuit shown in Fig. 2.2. Let us designate the time-varying current as i (t); we will represent the value of i(t) at t = 0 as I_0; in other words, $i(0) = I_0$. We therefore have

$$Ri + v_L = Ri + L\,di/dt = 0$$

or

$$di/dt + (R/L)i = 0$$

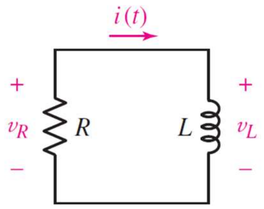


Fig. 2.2: RL circuit.

After separating the variables, the solution can be obtain by integrating the two sides of equation. Thus,

$$\int \frac{di}{i} = -\int \frac{R}{L} dt + K$$

$$\text{In}\, i(t) = -\frac{R}{L} t + K$$

At t = 0, $i(0) = I_0$

$\ln I_0 = K$

$$ln\, i(t) = -\frac{R}{L} t + lnI_0$$

$$\text{i(t)} = I_0 e^{-\frac{R}{L}t}$$

Let us now consider the nature of the response in the series RL circuit. We have found that the inductor current is represented by

$$\text{i(t)} = I_0 e^{-\frac{R}{L}t}$$

At t = 0, the current has value I_0, but as time increases, the current decreases and approaches zero. The shape of this decaying exponential is seen by the plot of $i(t)/I_0$ versus t shown in Fig. 2.3.

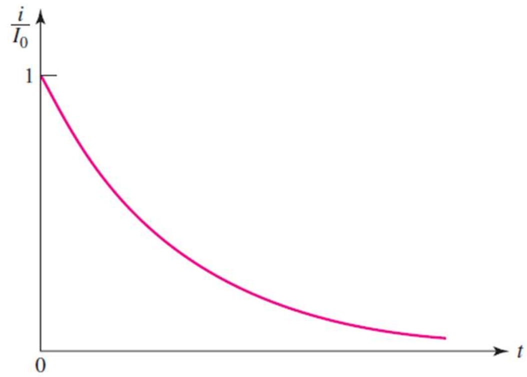


Fig. 2.3: The plot of $i(t)/I_0$ versus t.

We designate the value of time it takes for i/I_0 to drop from unity to zero, assuming a constant rate of decay, by the Greek letter τ (tau). Thus, τ = L/R

The ratio L/R has the units of seconds, since the exponent –(R/L)t must be dimensionless. This value of time τ is called the ***time constant*** and is shown pictorially in Fig. 2.4.

An equally important interpretation of the time constant τ is obtained by determining the value of $i(t)/I_0$ at t = τ . We have

$$i(\tau)/I_0 = e^{-1} = 0.3679$$

Thus, in one time constant the response has dropped to 36.8 percent of its initial value; the value of τ may also be determined graphically from this fact, as indicated by Fig. 2.4. It is convenient to measure the decay of the current at intervals of one time constant, and recourse to a hand calculator shows that $i(t)/I_0$ is 0.3679 at $t = \tau$, 0.1353 at $t = 2\tau$, 0.04979 at $t = 3\tau$, 0.01832 at $t = 4\tau$, and 0.006738 at $t = 5\tau$.

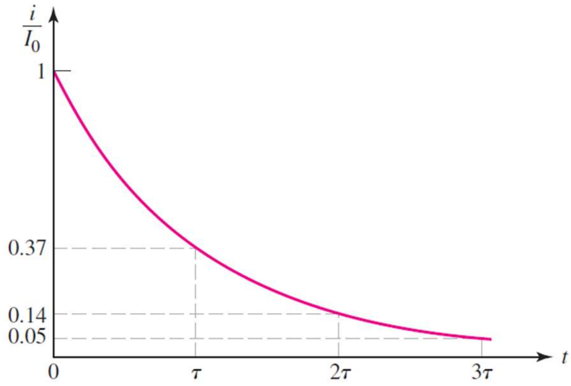


Fig. 2.4: The current in a series RL circuit is reduced to 37 percent of its initial value at t = τ, 14 percent at t = 2τ, and 5 percent at t = 3τ.

H.W.: In a source-free series RL circuit, find the numerical value of the ratio: (a) $i(2\tau)/i(\tau)$; (b) $i(0.5\tau)/i(0)$; (c) t/τ if $i(t)/i(0) = 0.2$; (d) t/τ if $i(0) - i(t) = i(0) \ln 2$.

General RL Circuits

As an example, consider the circuit shown in Fig. 2.5. The equivalent resistance the inductor faces is

$$R_{eq} = R_3 + R_4 + (R_1R_2/(R_1 + R_2))$$

and the time constant is therefore

$$\tau = L/R_{eq}$$

If several inductors are present in a circuit and can be combined using series and/or parallel combination, then time constant can be further generalized to

$$\tau = L_{eq}/R_{eq}$$

where L_{eq} represents the equivalent inductance.

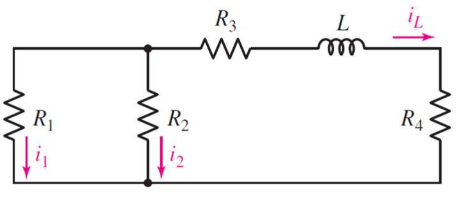
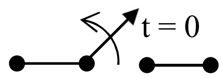


Fig. 2.5

The Distinction between 0^+ and 0^-



Switch

$T = 0^-$ represents the instant before the event. In this case, the switch has been closed for some considerable time.

T = 0 is the Initial condition. The switch is closed i.e., switched on. (Short circuit)

$T = 0^+$ is the instant after the event that means the switch has just opened. (Open circuit)

Example 2.1: Determine both i_1 and i_L in the circuit shown in Fig. 2.6a for $t > 0$.

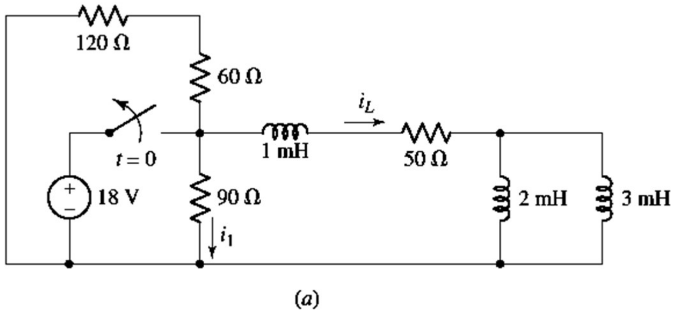


Fig. 2.6

Solution:

After t = 0, when the voltage source is disconnected as shown in Fig. 2.6b, we easily calculate an equivalent inductance,

$L_{eq} = 2 \times 3/(2 + 3) + 1 = 2.2$ mH

an equivalent resistance, in series with the equivalent inductance,

$R_{eq} = 90(60 + 120)/(90 + 180) + 50 = 110\ \Omega$

and the time constant,

$\tau = Leq/Req = 2.2 \times 10^{-3}/110 = 20\ \mu s$

Thus, the form of the natural response is $Ke^{-50{,}000t}$, where K is an unknown constant. Considering the circuit just prior to the switch opening ($t = 0^-$), $i_L = 18/50$ A. Since $i_L(0^+) = i_L(0^-)$, we know that $i_L = 18/50$ A or 360 mA at $t = 0^+$ and so

$$i_L = \begin{cases} 360\ mA & t < 0 \\ 360e^{-50{,}000t}\ mA & t \geq 0 \end{cases}$$

There is no restriction on i_1 changing instantaneously at t = 0, so its value at $t = 0^-$ (18/90 A or 200 mA) is not relevant to finding i_1 for t > 0. Instead, we must find $i_1(0^+)$ through our knowledge of $i_L(0^+)$.

Using current division,

$i_1(0^+) = -i_L(0^+)(120 + 60)/(120 + 60 + 90) = -240$ mA

Hence,

$$i_1 = \begin{cases} 200\ mA & t < 0 \\ -240e^{-50{,}000t}\ mA & t \geq 0 \end{cases}$$

H.W.: At t = 0.15 s in the circuit of Fig. 2.7, find the value of (a) i_L; (b) i_1; (c) i_2.

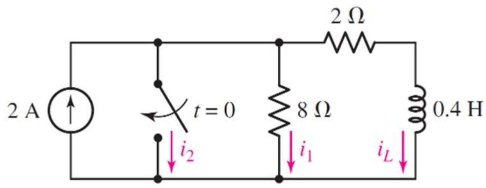
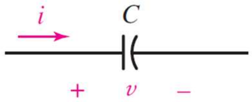


Fig. 2.7.

2.2 RC Circuit:

We now introduce a new passive circuit element, the capacitor. We define capacitance C by the voltage-current relationship

$$i = C\frac{dv}{dt} \qquad [1]$$



where v and i satisfy the conventions for a passive element. From Eq. [1], we may determine the unit of capacitance as an ampere-second per volt, or coulomb per volt. We will now define the farad (F) as one coulomb per volt, and use this as our unit of capacitance.

Fig. 2.8: Several examples of commercially available capacitors.

Note: *Circuits based on resistor-capacitor combinations are more common than their resistor-inductor analogues. The principal reasons for this are the smaller losses present in a physical capacitor, lower cost, better agreement between the simple mathematical model and the actual device behaviour, and also smaller size and lighter weight, both of which are particularly important for integrated-circuit applications.*

Let us see how closely the analysis of the parallel (or is it series?) RC circuit shown in Fig. 2.9 corresponds to that of the RL circuit. We will assume an initial stored energy in the capacitor by selecting $v(0) = V_0$

The total current leaving the node at the top of the circuit diagram must be zero, so we may write

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

Division by C gives us

$$\frac{dv}{dt} + \frac{v}{RC} = 0$$

The response of the RC circuit is

$$v = V_0 e^{-\frac{t}{RC}}$$

The time constant of the RC circuit given by

$$\tau = RC$$

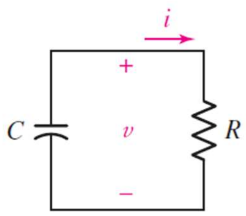


Fig. 2.9: RC circuit.

Our familiarity with the negative exponential and the significance of the time constant τ enables us to sketch the response curve readily (Fig. 2.10). Larger values of R or C provide larger time constants and slower dissipation of the stored energy.

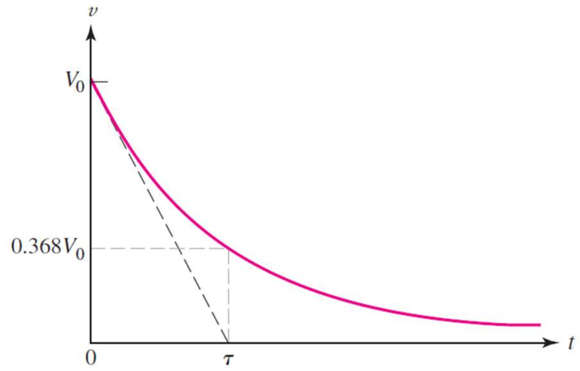


Fig. 2.10: RC circuit response.

General RC Circuits

If the circuit has more than one resister and more than one capacitor, but they may be replaced somehow using series and/or parallel combinations with an equivalent resistance R_{eq} and equivalent capacitance C_{eq}, then the circuit has an effective time constant given by

$$\tau = R_{eq}C_{eq}$$

Example 2.2: Find $v(0^+)$ and $i_1(0^+)$ for the circuit shown in Fig. 2.11a if $v(0^-) = V_0$.

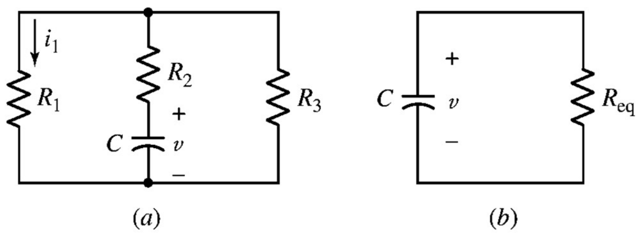


Fig. 2.11

Solution:

We first simplify the circuit of Fig. 2.11a to that of Fig. 2.11b, enabling us to write

$$v = V_0e^{-t/(R_{eq}C)}$$

where

$$v(0^+) = v(0^-) = V_0 \text{ and } R_{eq} = R_2 + R_1R_3/(R_1 + R_3)$$

Every current and voltage in the resistive portion of the network must have the form $Ae^{-t/(R_{eq}C)}$, where A is the initial value of that current or voltage. Thus, the current in R_1, for example, may be expressed as

$$i_1 = i_1(0^+)e^{-t/\tau}$$

where

$$\tau = (R_2 + R_1//R_3)C$$

and $i_1(0^+)$ remains to be determined from the initial condition. Any current flowing in the circuit at t = 0^+ must come from the capacitor. Therefore, since v cannot change instantaneously, $v(0^+) = v(0^-) = V_0$ and

$$i_1(0^+) = \left(\frac{V_o}{\left(R_2 + \frac{R_1R_3}{R_1 + R_3}\right)}\right)\left(\frac{R_3}{R_1 + R_3}\right)$$

H.W.: Find values of v_C and v_o in the circuit of Fig. 2.12 at t equal to (a) 0^-; (b) 0^+; (c) 1.3 ms.

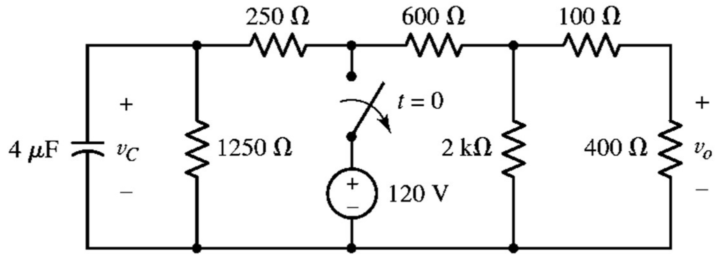


Fig. 2.12

Example 2.3: For the circuit of Fig. 2.13a, find the voltage labelled v_C for $t > 0$ if $v_C(0^-) = 2$ V.

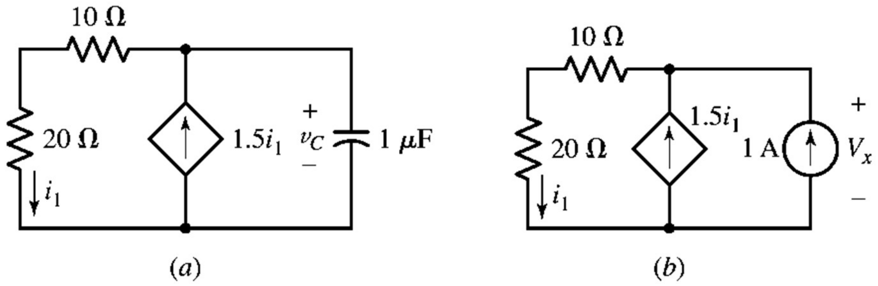


Fig. 2.13

Solution:

The dependent source is not controlled by a capacitor voltage or current, so we can start by finding the Thévenin equivalent of the network to the left of the capacitor. Connecting a 1Atest source as in Fig. 2.13b,

$$V_x = (1 + 1.5i_1)(30)$$

where

$$i_1 = V_x\, 20/(20(10+20)) = V_x/30$$

Performing a little algebra, we find that $V_x = -60$ V, so the network has a Thévenin equivalent resistance of −60 Ω (unusual, but not impossible when dealing with a dependent source). Our circuit therefore has a negative time constant

$$\tau = -60(1 \times 10^{-6}) = -60\ \mu s$$

The capacitor voltage is therefore

$$v_C(t) = Ae^{t/60\times10^{-6}} \quad V$$

where $A = v_C(0^+) = v_C(0^-) = 2$ V. Thus,

$$v_c(t) = 2e^{t/60\times10^{-6}} \quad V$$

which, interestingly enough is unstable: it grows exponentially with time. This cannot continue indefinitely; one or more elements in the circuit will eventually fail.

H.W.: (a) Regarding the circuit of Fig. 2.14, determine the voltage $v_C(t)$ for $t > 0$ if $v_C(0^-) = 11$ V. (b) Is the circuit "stable"?

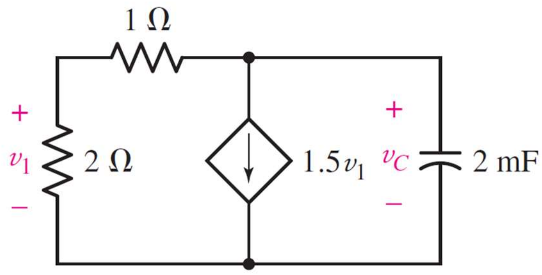


Fig. 2.14

<u>*2.3 The Unit-Step Function:*</u>

We define the unit-step forcing function (u(t)) as a function of time which is zero for all values of its argument less than zero and which is unity for all positive values of its argument.
If we let $(t - t_0)$ be the argument and represent the unit-step function by u, then $u(t - t_0)$ must be zero for all values of t less than t_0, and it must be unity for all values of t greater than t_0.
At $t = t_0$, u(t − t0) changes abruptly from 0 to 1. Its value at $t = t_0$ is not defined, but its value is known for all instants of time that are arbitrarily close to $t = t_0$. We often indicate this by writing $u(t_0^-) = 0$ and $u(t_0^+) = 1$. The concise mathematical definition of the unit-step forcing function is

$$u(t - t_0) = \begin{cases} 0 \; at \; t < t_0 \\ 1 \; at \; t > \; t_0 \end{cases}$$

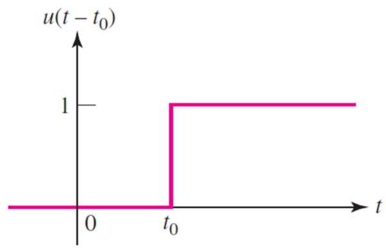


Fig. 2.15: The unit-step forcing function, u(t − t0).

To obtain an exact equivalent for the voltage-step forcing function, we may provide a single-pole double-throw switch.

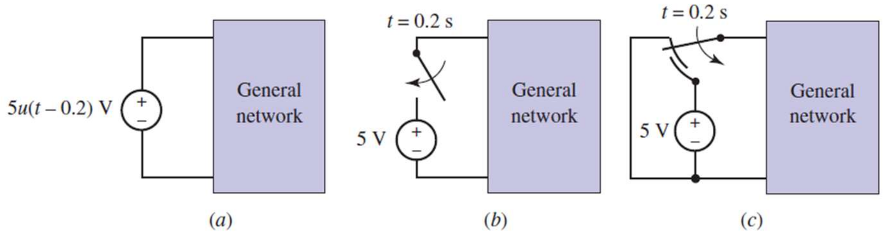


Fig 2.16 (a) A voltage-step forcing function is shown as the source driving a general network. (b) A simple circuit which, although not the exact equivalent of part (a), may be used as its equivalent in many cases. (c) An exact equivalent of part (a).

Figure 2.17 shows a current-step forcing function driving a general network.

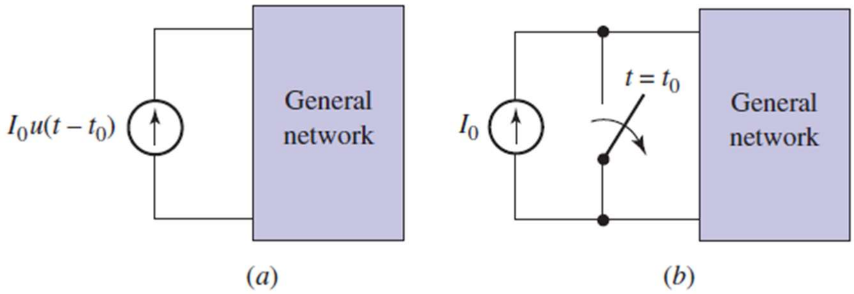


Fig. 2.17 (a) A current-step forcing function is applied to a general network. (b) A simple circuit which, although not the exact equivalent of part (a), may be used as its equivalent in many cases.

Some very useful forcing functions may be obtained by manipulating the unit-step forcing function. Let us define ***a rectangular voltage pulse*** by the following conditions:

$$v(t) = V_0u(t-t_0) - V_0u(t-t_1) = \begin{cases} 0 \; at \; t < t_0 \\ V_0 \; at \; t_0 < t < t_1 \\ 0 \; at \; t > t_1 \end{cases}$$

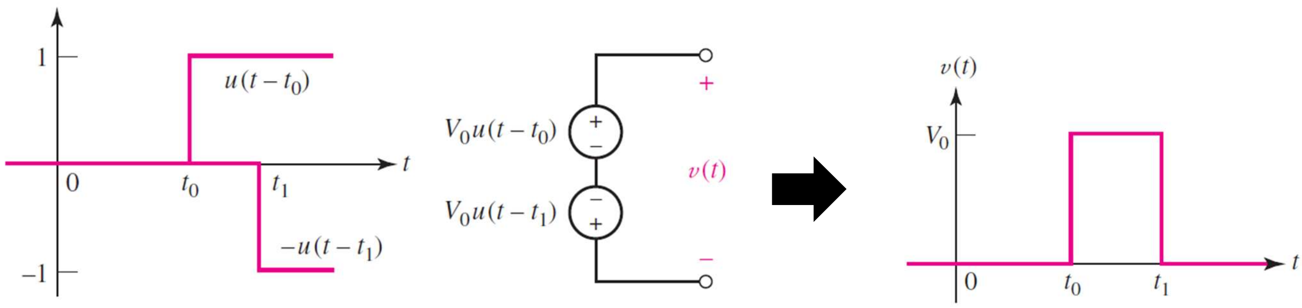
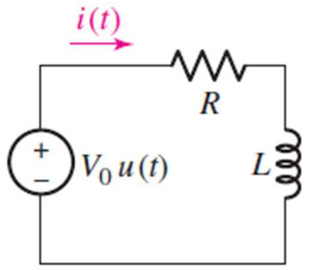


Fig. 2.18: A useful forcing function, the rectangular voltage pulse.

H.W.: Evaluate each of the following at t = 0.8: (a) 3u(t) − 2u(−t) + 0.8u(1 − t); (b) [4u(t)]u(−t); (c) 2u(t) sin πt.

- ***The complete response*** *is composed of two parts, the natural response and the forced response.*
- ***The natural response*** *is a characteristic of the circuit and not of the sources. Its form may be found by considering the source-free circuit, and it has an amplitude that depends on both the initial amplitude of the source and the initial energy storage.*
- ***The forced response*** *has the characteristics of the forcing function; it is found by pretending that all switches were thrown a long time ago. Since we are presently concerned only with switches and dc sources, the forced response is merely the solution of a simple dc circuit problem.*

<u>Driven RL circuits consist of a battery whose voltage is V_0 in series with a switch, a resistor R, and an inductor L.</u>



Applying Kirchhoff's voltage law to the circuit

$$Ri + L di/dt = V_0\, u(t)$$

Since the unit-step forcing function is discontinuous at t = 0, we will first consider the solution for t < 0 and then for t > 0. The application of zero voltage since t = −∞ forces a zero response, so that

$$i(t) = 0 \qquad t < 0$$

For positive time, however, u(t) is unity and we must solve the equation

$$Ri + L di/dt = V_0 \qquad t > 0$$

The variables may be separated in several simple algebraic steps, yielding

$$L\, di/(V_0 - Ri) = dt$$

and each side may be integrated directly:

$$-(L/R)\ln(V_0 - Ri) = t + k$$

In order to evaluate k, an initial condition must be invoked. Prior to t = 0, i (t) is zero, and thus $i(0^-) = 0$. Since the current in an inductor cannot change by a finite amount in zero time without being associated with an infinite voltage, we thus have $i(0^+) = 0$. Setting i = 0 at t = 0, we obtain

$$-(L/R)\ln V_0 = k$$

and, hence,

$$-(L/R)[\ln(V_0 - Ri) - \ln V_0] = t$$

Rearranging,

$$(V_0 - Ri)/V_0 = e^{-Rt/L}$$

or

$$i = (V0/R) - (V0/R)e^{-Rt/L} \qquad t > 0$$

Thus, an expression for the response valid for all t would be

$$i = [(V_0/R) - (V_0/R)e^{-Rt/L}]\, u(t)$$

Example 2.4: For the circuit of Fig. 2.19, find i(t) for t=∞, 3^-, 3^+, and 100 µs after the source changes value.

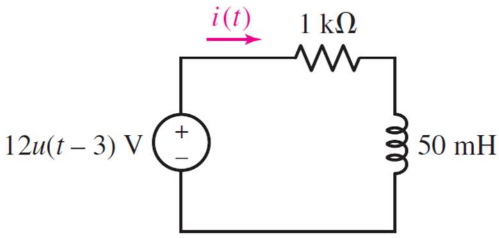


Fig. 2.19

Solution:

Long after any transients have died out (t →∞), the circuit is a simple dc circuit driven by a 12 V voltage source. The inductor appears as a short circuit, so

i (∞) = 12/1000 = 12 mA

What is meant by i (3^-)? This is simply a notational convenience to indicate the instant before the voltage source changes value. For t < 3, u(t − 3) = 0. Thus, i (3^-) = 0 as well.

At t = 3^+, the forcing function 12u(t − 3) = 12 V. However, since the inductor current cannot change in zero time, i (3^+) = i (3^-) = 0.

The most straightforward approach to analysing the circuit for t > 3 s as

$$i(t') = \left(\frac{V_0}{R} - \frac{V_0}{R} e^{-Rt'/L}\right) u(t')$$

and note that this equation applies to our circuit as well if we shift the time axis such that

t' = t − 3

Therefore, with V_0/R = 12 mA and R/L = 20,000 s^{-1},

$$i(t-3) = \left(12 - 12e^{-20000(t-3)}\right)u(t-3) \qquad \text{mA} \qquad [1]$$

which can be written more simply as

$$i(t) = \left(12 - 12e^{-20000(t-3)}\right)u(t-3) \qquad \text{mA} \qquad [2]$$

since the unit-step function forces a zero value for t < 3, as required. Substituting t = 3.0001 s into Eq. [1] or [2], we find that i = 10.38 mA at a time 100 µs after the source changes value.

H.W.: The voltage source 60 − 40u(t) V is in series with a 10 Ω resistor and a 50 mH inductor. Find the magnitudes of the inductor current and voltage at t equal to (a) 0^-; (b) 0^+; (c) ∞; (d) 3 ms.

Example 2.5: Determine i(t) for all values of time in the circuit of Fig. 2.20.

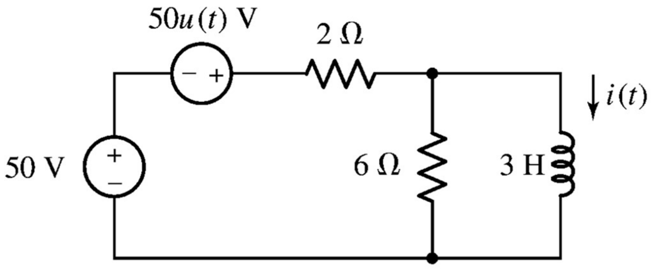


Fig. 2.20

Solution:

The circuit contains a dc voltage source as well as a step-voltage source. We might choose to replace everything to the left of the inductor by the Thevenin equivalent, but instead let us merely recognize the form of that equivalent as a resistor in series with some voltage source. The circuit contains only one energy storage element, the inductor. We first note that

$$\tau = L/R_{eq} = 3/1.5 = 2 \text{ s}$$

and recall that $i = i_f + i_n$

The natural response is therefore a negative exponential as before:

$$i_n = Ke^{-t/2} \quad \text{A} \quad t > 0$$

Since the forcing function is a dc source, the forced response will be a constant current. The inductor acts like a short circuit to dc, so that $i_f = 100/2 = 50$ A

Thus,

$$i = 50 + Ke^{-0.5t} \quad \text{A} \quad t > 0$$

In order to evaluate K, we must establish the initial value of the inductor current. Prior to t = 0, this current is 25 A, and it cannot change instantaneously.

Thus,

$$25 = 50 + K \text{ or } K = -25$$

Hence,

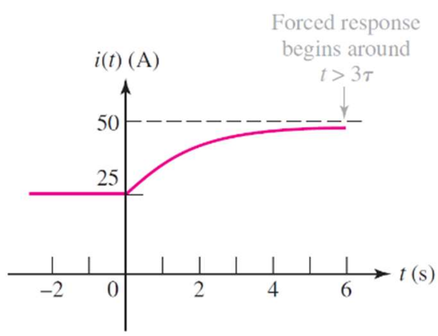
$$i = 50 - 25e^{-0.5t} \quad \text{A} \quad t > 0$$

We complete the solution by also stating

$$i = 25 \quad \text{A} \quad t < 0$$

or by writing a single expression valid for all t,

$$i = 25 + 25(1 - e^{-0.5t})u(t) \quad \text{A}$$



The complete response is sketched in Fig. 2.21. Note how the natural response serves to connect the response for t < 0 with the constant forced response.

H.W.: A voltage source, vs = 20u(t) V, is in series with a 200 Ω resistor and a 4 H inductor. Find the magnitude of the inductor current at t equal to (a) 0^-; (b) 0^+; (c) 8 ms; (d) 15 ms.

Example 2.6: Find the current response in a simple series RL circuit when the forcing function is a rectangular voltage pulse of amplitude V_0 and duration t_0.

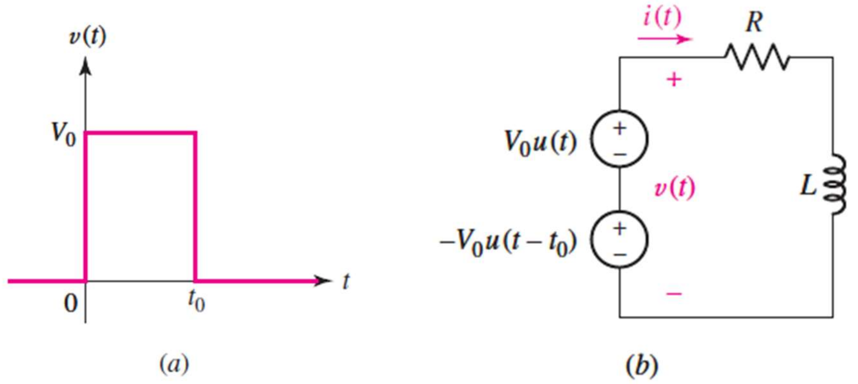


Fig. 2.22

Solution:
We represent the forcing function as the sum of two step-voltage sources $V_0u(t)$ and $-V_0u(t - t_0)$, as indicated in Fig. 2.22a and b, and we plan to obtain the response by using superposition. Let $i_1(t)$ designate that part of i (t) which is due to the upper source $V_0u(t)$ acting alone, and let $i_2(t)$ represent that part due to $-V_0u(t - t_0)$ acting alone. Then,

$$i(t) = i_1(t) + i_2(t)$$

Our object is now to write each of the partial responses i_1 and i_2 as the sum of a natural and a forced response. The response $i_1(t)$ is familiar:

$$i_1(t) = (V_0/R)(1 - e^{-Rt/L}) \qquad t > 0$$

Note that this solution is only valid for $t > 0$ as indicated; $i_1 = 0$ for $t < 0$.

We now turn our attention to the other source and its response $i_2(t)$. Only the polarity of the source and the time of its application are different. There is no need therefore to determine the form of the natural response and the forced response; the solution for $i_1(t)$ enables us to write

$$i_2(t) = -(V_0/R)[1 - e^{-R(t-t_0)/L}] \qquad t > t_0$$

where the applicable range of t, t > t0, must again be indicated; and $i_2 = 0$ for $t < t_0$.

We now add the two solutions, but do so carefully, since each is valid over a different interval of time. Thus,

$$i(t) = 0 \qquad t < 0 \qquad [1]$$

$$i(t) = (V_0/R)(1 - e^{-Rt/L}) \qquad 0 < t < t_0 \qquad [2]$$

and

$$i(t) = (V_0/R)(1 - e^{-Rt/L}) - (V_0/R)(1 - e^{-R(t-t_0)/L}) \qquad t > t_0$$

or more compactly,

$$i(t) = (V_0/R)e^{-Rt/L}(e^{Rt_0/L} - 1) \quad t > t0 \qquad [3]$$

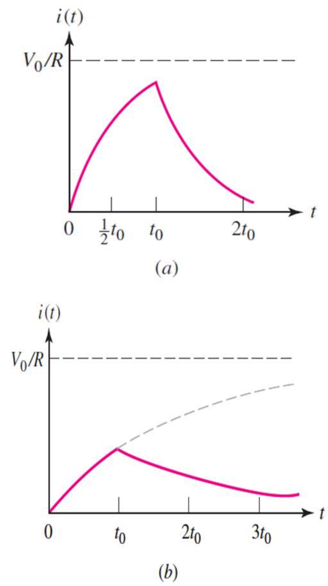


Fig. 2.23. Two possible response curves are shown for the circuit of Fig. 8.39b. (a) τ is selected as $t_0/2$. (b) τ is selected as $2t_0$.

H.W.: The circuit shown in Fig. 2.24 has been in the form shown for a very long time. The switch opens at t = 0. Find i_R at t equal to (a) 0^-; (b) 0^+; (c) ∞; (d) 1.5 ms.

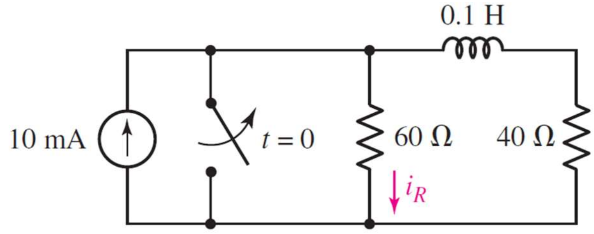


Fig. 2.24

Example 2.7: Find the capacitor voltage $v_C(t)$ and the current i(t) in the 200 Ω resistor of Fig. 2.25 for all time.

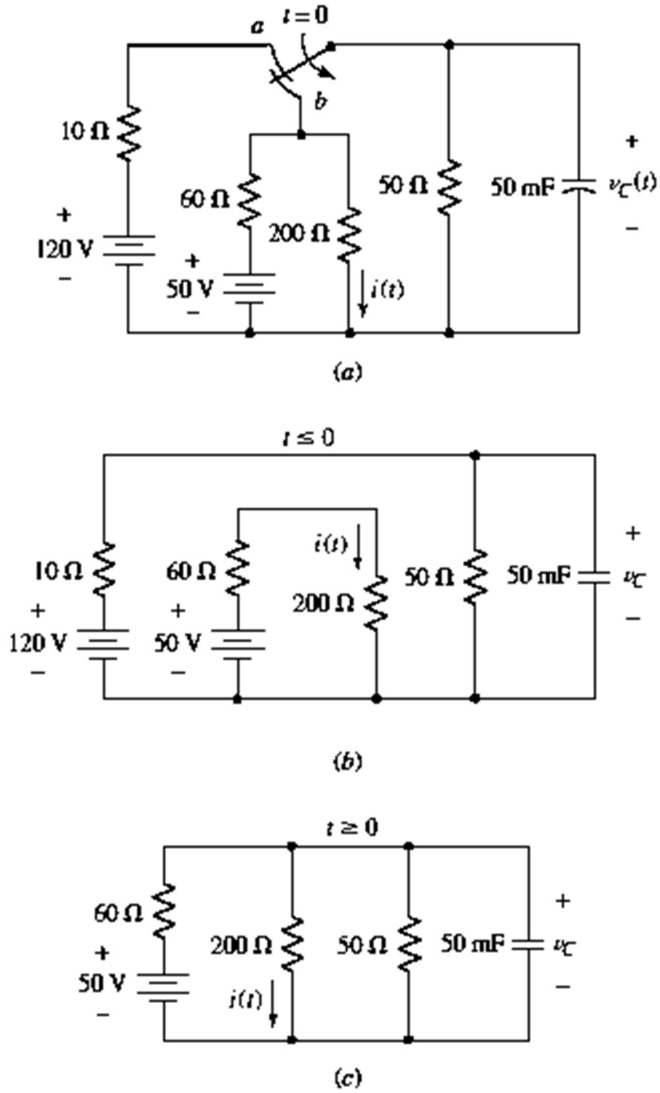


Fig. 2.25

Solution:

We begin by considering the state of the circuit at t < 0, corresponding to the switch at position a as represented in Fig. 2.25b. As usual, we assume no transients are present, so that only a forced response due to the 120 V source is relevant to finding $v_C(0^-)$. Simple voltage division then gives us the initial voltage,

$$v_C(0) = (50/(50 + 10))(120) = 100 \text{ V}$$

Since the capacitor voltage cannot change instantaneously, this voltage is equally valid at $t = 0^-$ and $t = 0^+$.

The switch is now thrown to b, and the complete response is

$$v_C = v_{Cf} + v_{Cn}$$

The corresponding circuit has been redrawn in Fig. 2.25c for convenience. The form of the natural response is obtained by replacing the 50 V source by a short circuit and evaluating the equivalent resistance to find the time constant (in other words, we are finding the Thevenin equivalent resistance "seen'' by the capacitor):

$$R_{eq} = 50//200//60 = 24\ \Omega$$

Thus,

$$v_{Cn} = Ae^{-t/R_{eq}C} = Ae^{-t/1.2}$$

In order to evaluate the forced response with the switch at b, we wait until all the voltages and currents have stopped changing, thus treating the capacitor as an open circuit, and use voltage division once more:

$$v_{Cf} = 50((200//50)/(60 + 200//50)) = 20 \text{ V}$$

Consequently,

$$v_C = 20 + Ae^{-t/1.2} \text{ V}$$

and from the initial condition already obtained, 100 = 20 + A or

$$v_C = 20 + 80e^{-t/1.2} \text{ V} \quad t \geq 0$$

and

$$v_C = 100 \text{ V} \quad t < 0$$

This response is sketched in Fig. 2.26a; again the natural response is seen to form a transition from the initial to the final response. Next we attack i (t). This response need not remain constant during the instant of switching. With the contact at a, it is evident that i = 50/260 = 192.3 milliamperes. When the switch moves to position b, the forced response for this current becomes

$$i_f = (50/(60 + 50*200/(50 + 200)))(50/(50 + 200) = 0.1$$ ampere

The form of the natural response is the same as that which we already determined for the capacitor voltage:

$$i_n = Ae^{-t/1.2}$$

Combining the forced and natural responses, we obtain

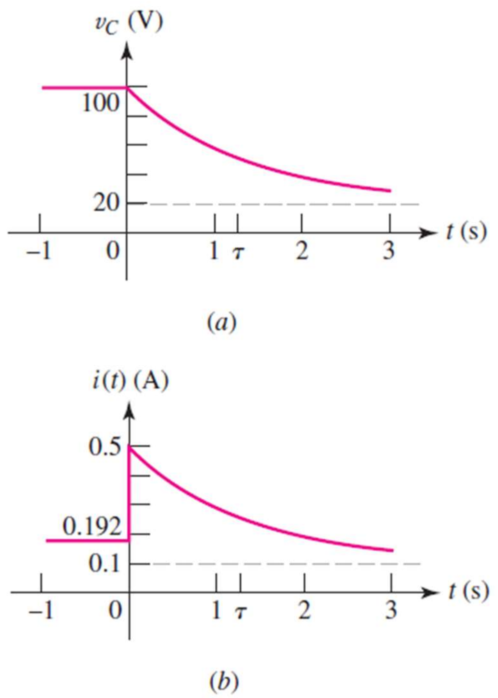


Fig. 2.26

$i = 0.1 + Ae^{-t/1.2}$ amperes

To evaluate A, we need to know i (0^+). This is found by fixing our attention on the energy-storage element (the capacitor). The fact that v_C must remain 100 V during the switching interval is the governing condition which establishes the other currents and voltages at $t = 0^+$.

Since $v_C(0^+) = 100$ V, and since the capacitor is in parallel with the 200 Ω resistor, we find i (0^+) = 0.5 ampere, A = 0.4 ampere, and thus

$i(t) = 0.1923$ ampere $t < 0$

$i(t) = 0.1 + 0.4e^{-t/1.2}$ ampere $t > 0$

or

$i(t) = 0.1923 + (-0.0923 + 0.4e^{-t/1.2})u(t)$ amperes

where the last expression is correct for all t.

The complete response for all t may also be written concisely by using u(−t), which is unity for t < 0 and 0 for t > 0. Thus,

$i(t) = 0.1923u(-t) + (0.1 + 0.4e^{-t/1.2})u(t)$ amperes

This response is sketched in Fig. 2.26b. Note that only four numbers are needed to write the functional form of the response for this singleenergy-storage-element circuit, or to prepare the sketch: the constant value prior to switching (0.1923 ampere), the instantaneous value just after switching (0.5 ampere), the constant forced response (0.1 ampere), and the time constant (1.2 s). The appropriate negative exponential function is then easily written or drawn.

H.W.: For the circuit of Fig. 2.27, find $v_C(t)$ at t equal to (a) 0^-; (b) 0^+; (c) ∞; (d) 0.08 s.

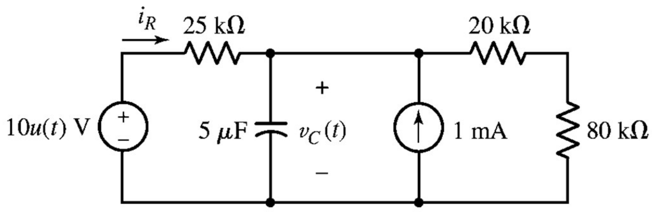


Fig. 2.27