Subject: Electric Circuits Analysis	Theoretical: 3 hrs/wk
Code: EE202	Practical: ---
Class: 2nd Year	Tutorial: 1 hr/wk
Pre-requisite: EE101, EE102	Units: 3

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Syllabus

Resistive Circuits with Dependent Sources:
Dependent and independent sources, mesh analysis, super Mesh, nodal analysis, super node, Thevenin and Norton equivalent circuits, superposition analysis, maximum power transfer.

The Transient Circuits
RL, RC, RLC circuit in parallel and series and their complete response.

Sinusoidal Steady State Analysis
Sinusoidal analysis and phasor, mesh and nodal ac analysis, Thevenin and Norton ac analysis, superposition ac analysis, AC power calculation.

Poly-phase Circuits
Single-phase three wire system, 3-phase balance and unbalance systems with star and delta connections, power in 3-phase circuits.

Chapter One: Resistive Circuits with Dependent Sources

1.1 Voltage and Current Sources:

Independent Voltage Sources *is characterized by a terminal voltage which is completely independent of the current through it*; the circuit symbol is shown in Fig. 1.1.
The independent voltage source is an ideal source and does not represent exactly any real physical device, because the ideal source could theoretically deliver an infinite amount of energy from its terminals. This idealized voltage source does, however, furnish a reasonable approximation to several practical voltage sources. An automobile storage battery, for example, has a 12 V terminal voltage that remains essentially constant as long as the current through it does not exceed a few amperes.

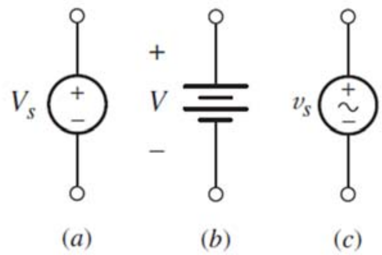


Fig. 1.1: (a) DC voltage source symbol; (b) battery symbol; (c) ac voltage source symbol.

Independent current source *is completely independent of the voltage across it*. The symbol is shown in Fig. 1.2.
Like the independent voltage source, the independent current source is at best a reasonable approximation for a physical element. In theory, it can deliver infinite power from its terminals because it produces the same finite current for any voltage across it, no matter how large that voltage may be. It is, however, a good approximation for many practical sources, particularly in electronic circuits.

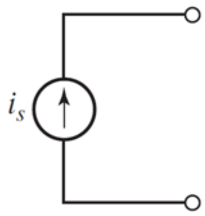


Fig. 1.2: Circuit symbol for the independent current source.

Dependent (or Controlled) Sources *in which the source quantity is determined by a voltage or current existing at some other location in the system being analysed*. The symbols are shown in Fig. 1.3.
Sources such as these appear in the equivalent electrical models for many electronic devices, such as transistors, operational amplifiers, and integrated circuits.

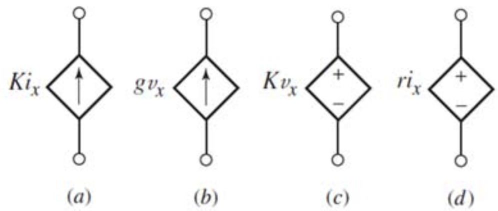


Fig. 1.3: The four different types of dependent sources: (a) current-controlled current source; (b) voltage-controlled current source; (c) voltage-controlled voltage source; (d) current controlled voltage source.

Example 1.1: Find the power absorbed by each element in the circuit in Fig. 1.4.

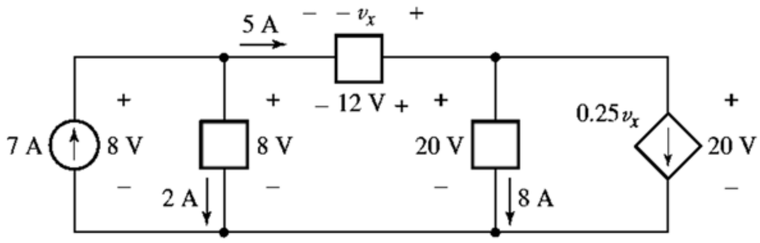


Fig. 1.4

Ans.: (left to right) −56 W; 16 W; −60 W; 160 W; −60 W.

1.2 Nodal Analysis:

We begin our study of general methods for methodical circuit analysis by considering a powerful method based on Kirchhoff's Current Law (KCL), namely ***nodal analysis.*** Specifically,

ΣCurrents entering the node from current sources = ΣCurrents leaving the node through resistors

An N-node circuit will need (N − 1) voltages and (N − 1) equations. Each equation is a simple KCL equation.

Example 1.2: To illustrate the basic technique, consider the three-node circuit shown in Fig. 1.5,

N = 3 → N-1 = 2 No. of equations.

At node 1

$$\frac{v_1}{2} + \frac{v_1 - v_2}{5} = 3.1 \qquad [1]$$

or

$$0.7v_1 - 0.2v_2 = 3.1 \qquad [2]$$

At node 2 we obtain

$$\frac{v_2}{1} + \frac{v_2 - v_1}{5} = -(-1.4) \qquad [3]$$

or

$$-0.2v_1 + 1.2v_2 = 1.4 \qquad [4]$$

Equations [2] and [4] are the desired two equations in two unknowns, and they may be solved easily. The results are v_1 = 5 V and v_2 = 2 V.

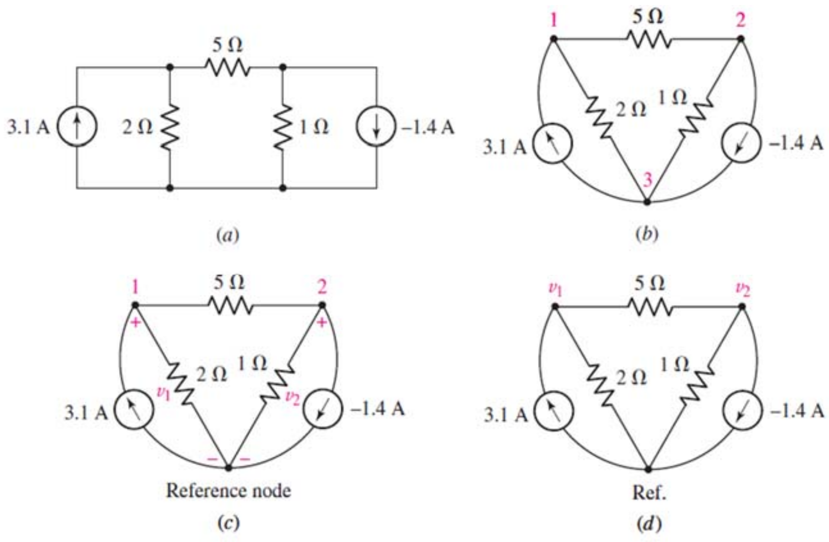


Fig. 1.5

Example 1.3: Determine the power supplied by the dependent source of Fig. 1.6.

Solution:

At node 1

$$15 = (v_1 - v_2)/1 + v_1/2 \qquad [1]$$

At node 2

$$3i_1 = (v_2 - v_1)/1 + v_2/3 \qquad [2]$$

Unfortunately, we have only two equations but three unknowns; this is a direct result of the presence of the dependent current source, since it is not controlled by a nodal voltage. Thus, we need an additional equation that relates i_1 to one or more nodal voltages.

In this case, we find that

$$i_1 = v_1 / 2$$

which upon substitution into Eq. [2] yields (with a little rearranging)

$$3v_1 - 2v_2 = 30 \qquad [3]$$

and Eq. [1] simplifies to

$$-15v_1 + 8v_2 = 0 \qquad [4]$$

Solving, we find that $v_1 = -40\ V$, $v_2 = -75\ V$, and $i_1 = 0.5v_1 = -20\ A$. Thus, the power supplied by the dependent source is equal to $(3i_1)(v_2) = (-60)(-75) = 4.5\ kW$.

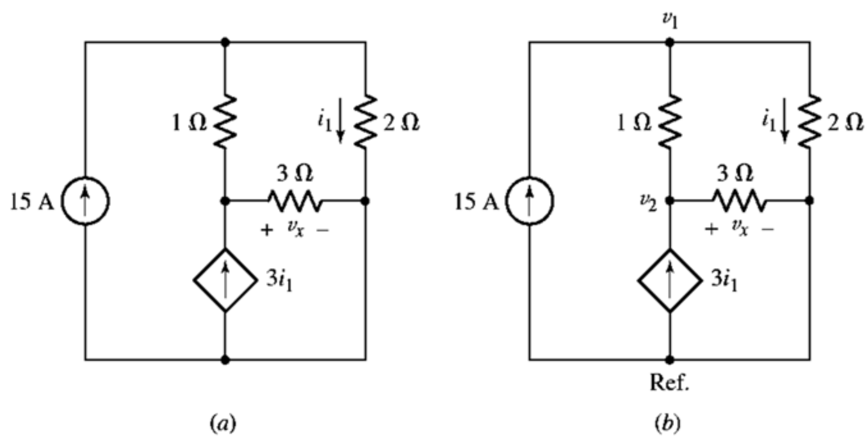


Fig. 1.6

Example 1.4: Determine the power supplied by the dependent source of Fig. 1.7.

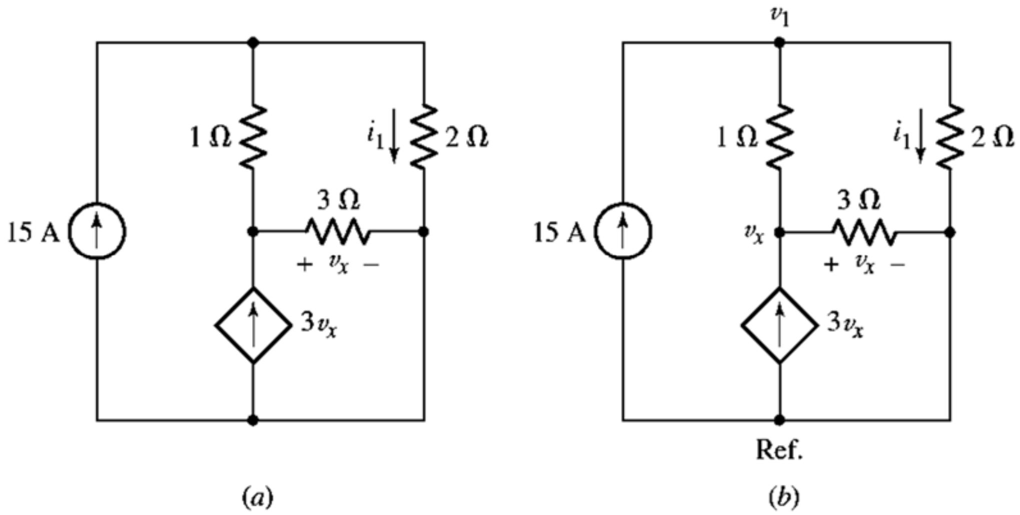


Fig. 1.7

Solution:

At node 1

$$15 = (v_1 - v_x)/1 + v_1/2 \qquad [1]$$

At node x

$$3v_x = (v_x - v_1)/1 + v_x/3 \qquad [2]$$

From Eqs [1] and [2]

$$v_1 = 50/7 \text{ and } v_x = -30/7$$

H.W.: For the circuit of Fig. 1.8, determine the nodal voltage v1 if A is (a) $2i_1$; (b) $2v_1$.

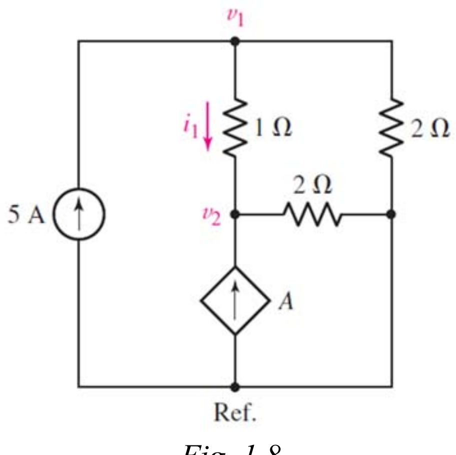


Fig. 1.8

1.3 Super Node:

For the circuit shown in Fig. 1.9, the first step to analysing circuit by nodal analyses is labelling the circuit nodes (N=4), and the next step is the application of KCL at each of the three non-reference nodes. If we try to do that, we see that we will run into some difficulty at both nodes 2 and 3, for we do not know what the current is in the branch with the voltage source.

The easier method is to treat node 2, node 3, and the voltage source together as a sort of ***super node*** and apply KCL to both nodes at the same time; the ***super node*** is indicated by the region enclosed by the broken line in Fig. 1.9.

Example 1.5: Determine the value of the unknown node voltage v1 in the circuit of Fig. 1.9.

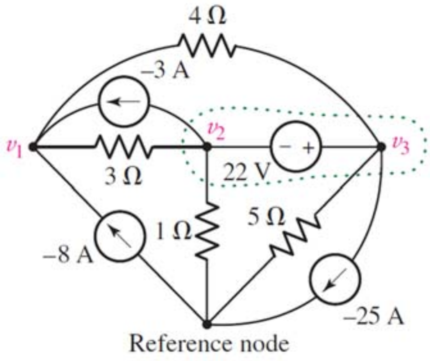


Fig. 1.9

Solution:

At node 1

$$-3-8 = (v_1 - v_2)/3 + (v_1 - v_3)/4$$

$$0.5833v_1 - 0.3333v_2 - 0.2500v_3 = -11 \qquad [1]$$

At super node 2-3

$$25+3 = (v_2 - v_1)/3 + (v_3 - v_1)/4 + v_2/1 + v_3/5$$

$$-0.5833v_1 + 1.3333v_2 + 0.45v_3 = 28 \qquad [2]$$

Finally, $v_3 - v_2 = 22$ [3]

By solving Eqs. [1] to [3], the solution for v_1 is 1.071 V.

Example 1.6: Determine the node-to-reference voltages in the circuit of Fig. 1.10.

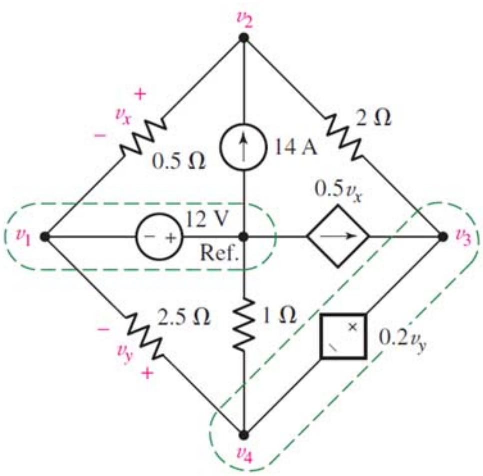


Fig. 1.10

Solution:
At node 1

$v_1 = -12$ V

At node 2

$14 = (v_2 - v_1)/0.5 + (v_2 - v_3)/2$ [1]

At super node 3-4

$0.5v_x = (v_3 - v_2)/2 + (v_4 - v_1)/2.5 + v_4/1$ [2]

Also,

$v_3 - v_4 = 0.2v_y$ [3]

and $v_y = v_4 - v_1$ [4]

$v_x = v_2 - v_1$ [5]

We can now eliminate v_x and v_y to obtain a set of four equations in the four node voltages:

$-2v_1 + 2.5v_2 - 0.5v_3 = 14$

$0.1v_1 - v_2 + 0.5v_3 + 1.4v_4 = 0$

$v_1 = -12$

$0.2v_1 + v_3 - 1.2v_4 = 0$

Solving, $v_1 = -12$ V, $v_2 = -4$ V, $v_3 = 0$ V, and $v_4 = -2$ V.

H.W.: Determine the nodal voltages in the circuit of Fig. 1.11.

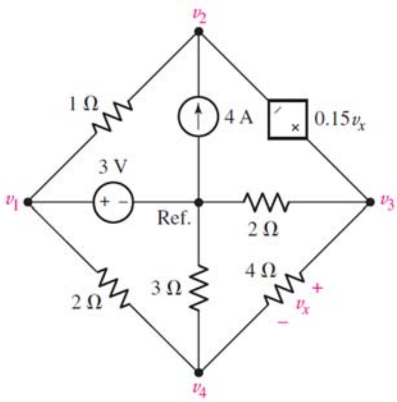


Fig. 1.11

1.4 Mesh Analysis:

In mesh analysis, the circuit will be analysed based on Kirchhoff's Voltage Law (KVL) which if it is possible to draw the diagram of a circuit on a plane surface in such a way that no branch passes over or under any other branch, then that circuit is said to be a **planar circuit**. Thus, Fig. 1.12a shows a planar network, Fig. 1.12b shows a nonplanar network, and Fig. 1.12c also shows a planar network, although it is drawn in such a way as to make it appear nonplanar at first glance.

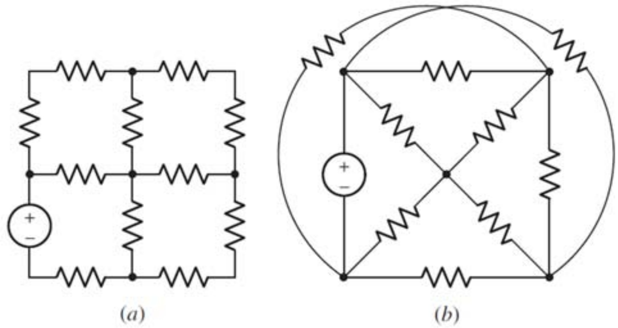


Fig. 1.12: Examples of planar and nonplanar networks; crossed wires without a solid dot are not in physical contact with each other.

The ***mesh*** is a property of a planar circuit and is undefined for a nonplanar circuit. We define a mesh as *a loop that does not contain any other loops within it*, (see Fig. 1.13).

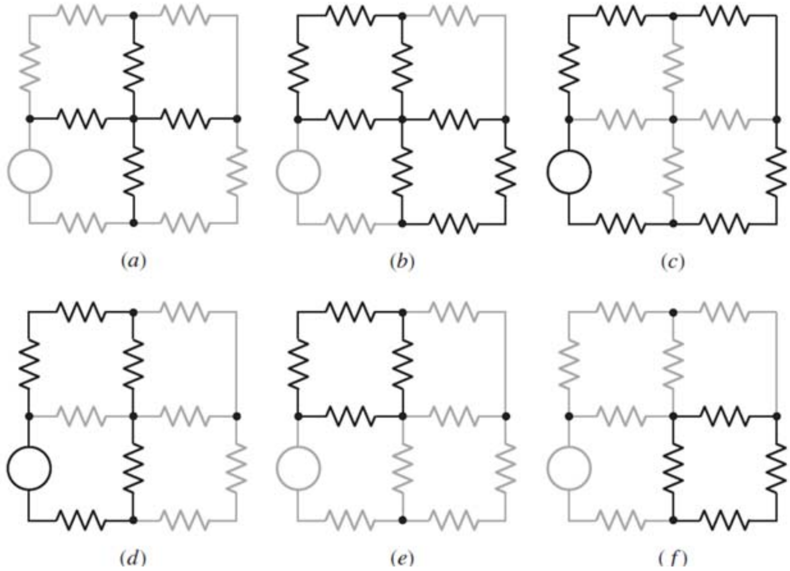


Fig. 1.13: (a) The set of branches identified by the heavy lines is neither a path nor a loop. (b) The set of branches here is not a path, since it can be traversed only by passing through the central node twice. (c) This path is a loop but not a mesh, since it encloses other loops. (d) This path is also a loop but not a mesh. (e, f) Each of these paths is both a loop and a mesh.

***If a network is planar, mesh analysis can be used to accomplish the analysis. This technique involves the concept of a* mesh current.**

Example 1.7: we will introduce mesh current analysis by considering the analysis of the two-mesh circuit of Fig. 1.14.

Solution:

We apply KVL to the left-hand mesh,

$$-42 + 6i_1 + 3(i_1 - i_2) = 0$$

or

$$9i_1 - 3i_2 = 42 \qquad [1]$$

Applying KVL to the right-hand mesh,

$$-3(i_1 - i_2) + 4i_2 - 10 = 0$$

or

$$-3i_1 + 7i_2 = 10 \qquad [2]$$

With two equations [1] and [2] and two unknowns, the solution is easily obtained:

$i_1 = 6$ A $\qquad i_2 = 4$ A $\qquad$ and $(i_1 - i_2) = 2$ A

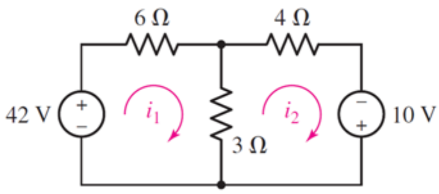


Fig. 1.14

Example 1.8: Determine the current i_1 in the circuit of Fig. 1.15.

Solution:

For the left mesh, KVL yields

$$-5 - 4i_1 + 4(i_2 - i_1) + 4i_2 = 0 \qquad [1]$$

and for the right mesh we find

$$4(i_1 - i_2) + 2i_1 + 3 = 0 \qquad [2]$$

Grouping terms, these equations may be written more compactly as

$$-8i_1 + 8i_2 = 5$$

and

$$6i_1 - 4i_2 = -3$$

Solving, $i_2 = 375$ mA, so $i_1 = -250$ mA.

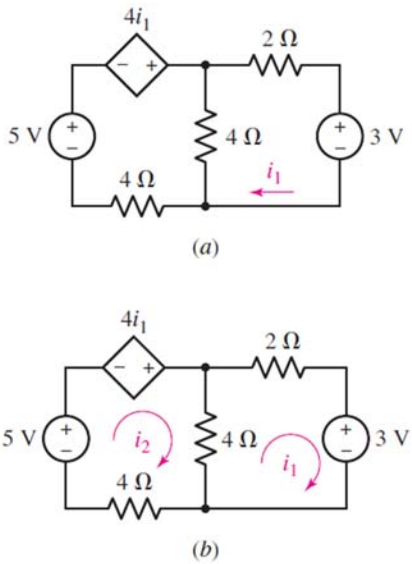


Fig. 1.15

Example 1.9: Determine the current i1 in the circuit of Fig. 1.15.

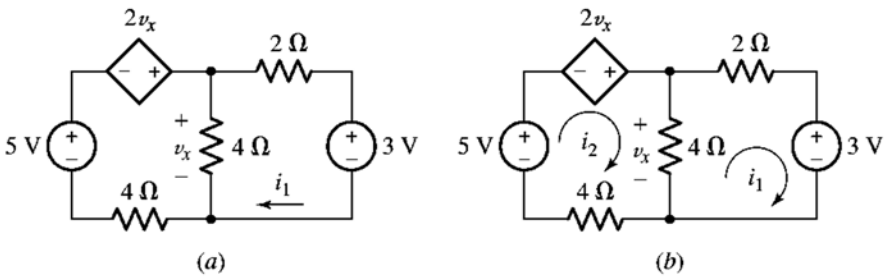


Fig. 1.15

Solution:

For the left mesh

$$-5 - 2v_x + 4(i_2 - i_1) + 4i_2 = 0 \qquad [1]$$

and for the right mesh

$$4(i_1 - i_2) + 2i_1 + 3 = 0 \qquad [2]$$

Also v_x is given by

$$v_x = 4(i_2 - i_1) \qquad [3]$$

We simplify our system of equations by substituting Eq. [3] into Eq. [1], resulting in $4i_1 = 5$

Solving, we find that $i_1 = 1.25$ A.

H.W.: Determine i_1 in the circuit of Fig. 1.16 if the controlling quantity A is equal to (a) $2i_2$; (b) $2v_x$.

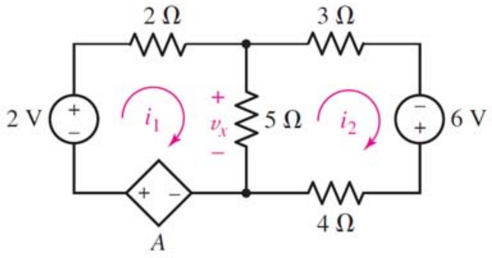


Fig. 1.16

1.5 Super Mesh:

Now we create a kind of "**supermesh**" from two meshes that have a current source as a common element; the current source is in the interior of the supermesh.

We thus reduce the number of meshes by 1 for each current source present. If the current source lies on the perimeter of the circuit, then the single mesh in which it is found is ignored. Kirchhoff's voltage law is thus applied only to those meshes or supermeshes in the reinterpreted network.

Example 1.10: Evaluate the three unknown currents in the circuit of Fig. 1.17.

Solution:

At mesh 1

$$i_1 = 15 \text{ A}.$$

We find that because we now know one of the two mesh currents relevant to the dependent current source, there is no need to write a supermesh equation about meshes 1 and 3. Instead, we simply relate i_1 and i_3 to the current from the dependent source using KCL:

$$v_x/9 = i_3 - i_1$$

also

$$v_x = 3(i_3 - i_2)$$

which can be written more compactly as

$$-i_1 + (1/3)i_2 + (2/3)i_3 = 0 \text{ or}$$

$$(1/3)i_2 + (2/3)i_3 = 15 \qquad [1]$$

With one equation in two unknowns, all that remains is to write a KVL equation about mesh 2:

$$1(i_2 - i_1) + 2i_2 + 3(i_2 - i_3) = 0$$

or

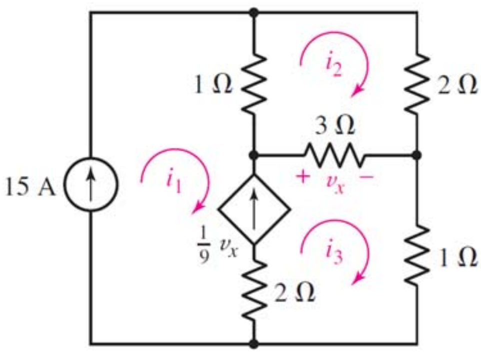


Fig. 1.17

$$6i_2 - 3i_3 = 15 \qquad [2]$$

Solving Eqs. [1] and [2], we find that $i_2 = 11$ A and $i_3 = 17$ A; we already determined that $i_1 = 15$ A by inspection.

H.W.: Determine v_3 in the circuit of Fig. 1.18.

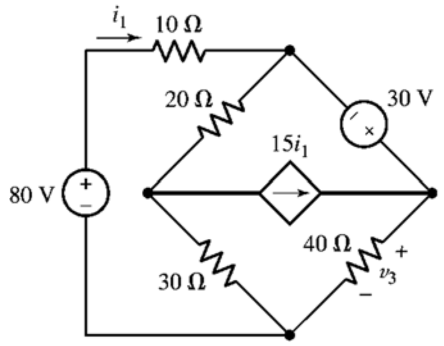


Fig. 1.18

1.6 Superposition Analysis:

The **superposition analysis** can be used only for linear circuits, which

- **The linear circuit** is defined as a circuit composed entirely of independent sources, linear dependent sources, and linear elements.
- **The linear element** is defined as a passive element that has a linear voltage-current relationship.
- **The linear dependent source** is defined as a dependent current or voltage source whose output current or voltage is proportional only to the first power of a specified current or voltage variable in the circuit (or to the sum of such quantities).

*In any **linear resistive network**, the voltage across or the current through any resistor or source may be calculated by adding algebraically all the individual voltages or currents caused by the separate independent sources acting alone, with all other independent voltage sources replaced by short circuits and all other independent current sources replaced by open circuits as shown in fig.1.19.*

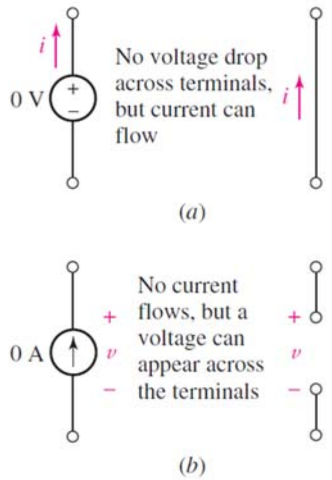


Fig. 1.19: (a) A voltage source set to zero acts like a short circuit. (b) A current source set to zero acts like an open circuit.

Example 1.11: In the circuit of Fig. 1.20, use the superposition principle to determine the value of i_x.

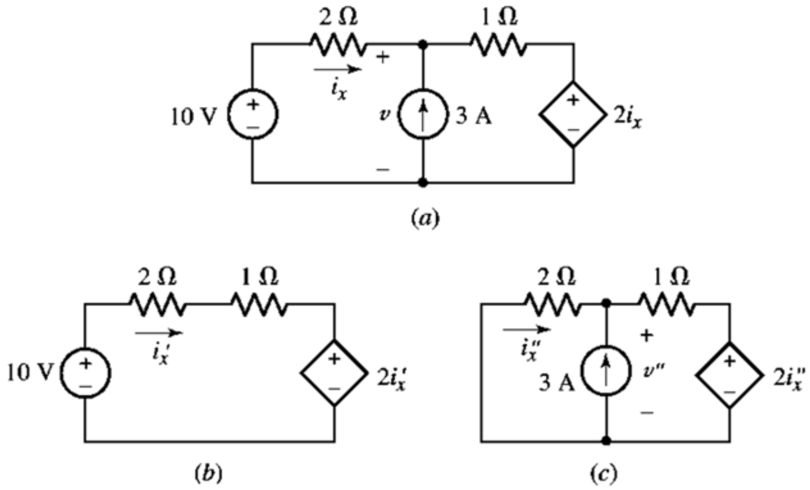


Fig. 1.20

Solution:

First open-circuit the 3 A source (Fig. 1.20b). The single mesh equation is

$$-10 + 2i_x' + i_x' + 2i_x' = 0$$

so that, $i_x' = 2$ A

Next, short-circuit the 10 V source (Fig. 1.20c) and write the single node equation

$$v''/2 + (v'' - 2i_x'')/1 = 3$$

and relate the dependent-source-controlling quantity to v'':

$$v'' = 2(-i_x'')$$

Solving, we find, $i_x'' = -0.6$ A

and, thus, $i_x = i_x' + i_x'' = 2 + (-0.6) = 1.4$ A

H.W.: For the circuit of Fig. 1.21, use superposition to obtain the voltage across each current source.

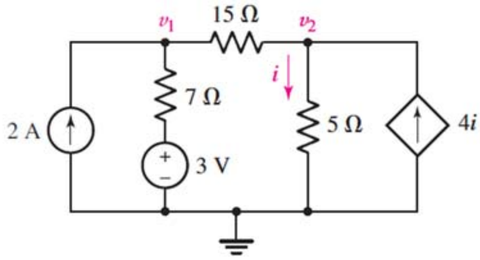


Fig. 1.21

1.7 Thevenin and Norton Equivalent Circuits:

Thevenin's theorem tells us that it is possible to replace everything except the load resistor with an independent voltage source in series with a resistor (Fig. 1.22b); the response measured at the load resistor will be unchanged. Using ***Norton's theorem***, we obtain an equivalent composed of an independent current source in parallel with a resistor (Fig. 1.22c).

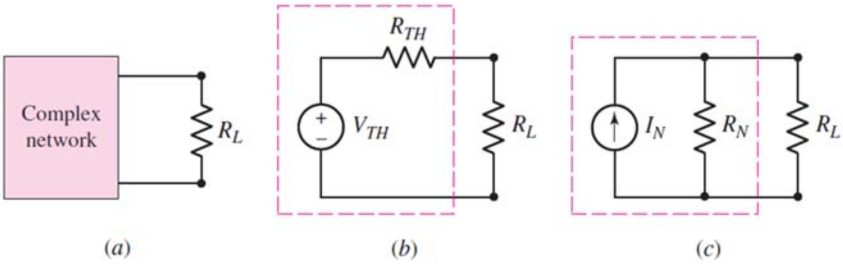


Fig. 1.22: (a) A complex network including a load resistor RL. (b) A Thevenin equivalent network connected to the load resistor RL. (c) A Norton equivalent network connected to the load resistor RL.

Example 1.12: Consider the circuit shown in Fig. 1.23a. Determine the Thevenin equivalent of network A, and compute the power delivered to the load resistor R_L.

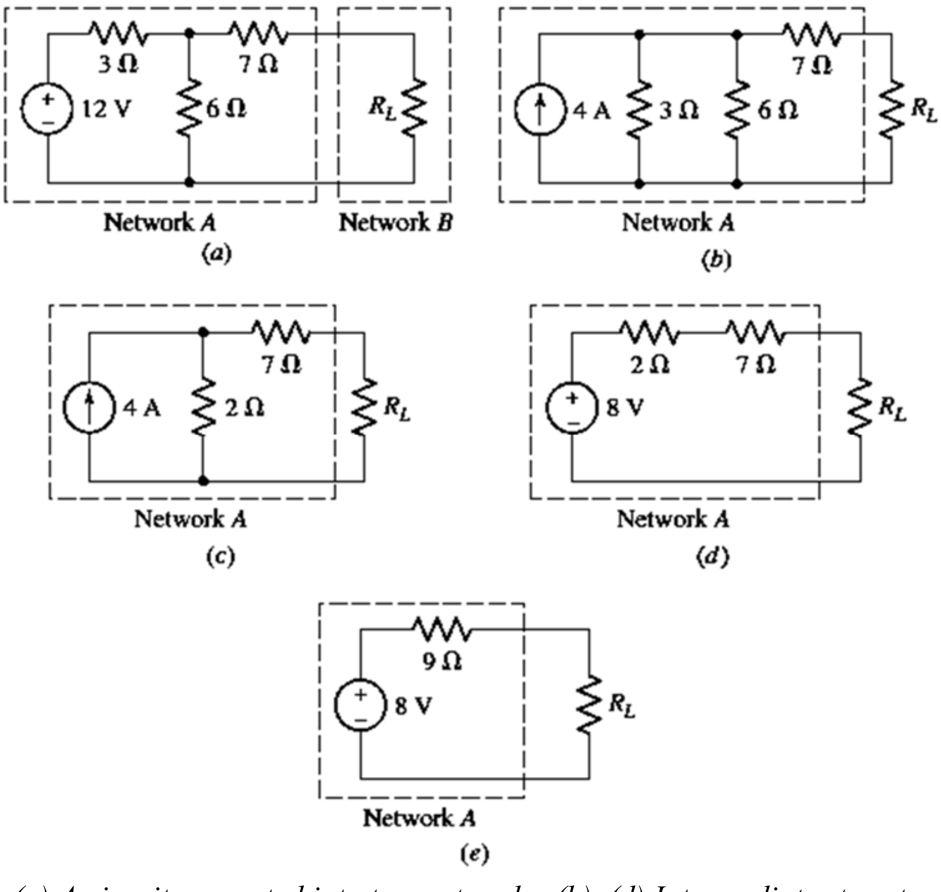


Fig. 1.23: (a) A circuit separated into two networks. (b)–(d) Intermediate steps to simplifying network A. (e) The Thevenin equivalent circuit.

We first treat the 12 V source and the 3Ω resistor as a practical voltage source and replace it with a practical current source consisting of a 12/3=4 A source in parallel with 3Ω (Fig. 1.23b). The parallel resistances are then combined into 3//6=2Ω (Fig. 1.23c), and the practical current source that results is transformed back into a practical voltage source 4*2=8V (Fig. 1.23d). The final result is shown in Fig. 1.23e.

From the viewpoint of the load resistor R_L, this network A (the Thevenin equivalent) is equivalent to the original network A; from our viewpoint, the circuit is much simpler, and we can now easily compute the power delivered to the load:

$$P_L = (8/(9 + R_L))^2 * R_L$$

H.W.: Using repeated source transformations, determine the Norton equivalent of the highlighted network in the circuit of Fig. 1.24.

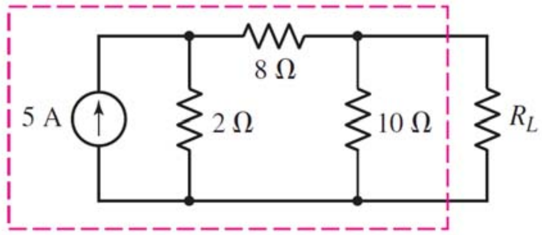


Fig. 1.24

1.7.1 Thevenin's Theorem

A Statement of Thevenin's Theorem (see Fig. 1.23):

1. **Given any linear circuit, rearrange it in the form of two networks, A and B, connected by two wires**. Network A is the network to be simplified; B will be left untouched.
2. **Disconnect network B.** Define a voltage v_{oc} as the voltage now appearing across the terminals of network A.
3. **Turn off or "zero out" every independent source in network A to form an inactive network**. Leave dependent sources unchanged.
4. **Connect an independent voltage source with value v_{oc} in series with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network B to the terminals of the new network A**. All currents and voltages in B will remain unchanged.

Example 1.13: Use Thevenin's theorem to determine the Thevenin equivalent for that part of the circuit in Fig. 1.25 to the left of RL.

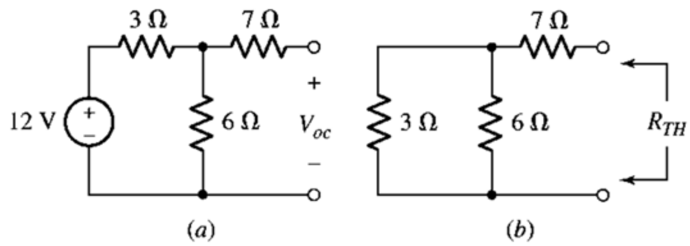


Fig. 1.25

Solution:

$$v_{oc} = 12*6/(3+6) = 8 \text{ V}$$

$$R_{TH} = 3//6 + 7 = 9\ \Omega$$

1.7.2 Norton's Theorem:

A Statement of Norton's Theorem (see Fig. 1.23):

1. **Given any linear circuit, rearrange it in the form of two networks, A and B, connected by two wires.** Network A is the network to be simplified; B will be left untouched. As before, if either network contains a dependent source, its controlling variable must be in the same network.
2. **Disconnect network B, and short the terminals of A.** Define a current i_{sc} as the current now flowing through the shorted terminals of network A.
3. **Turn off or "zero out" every independent source in network A to form an inactive network**. Leave dependent sources unchanged.
4. **Connect an independent current source with value i_{sc} in parallel with the inactive network.** Do not complete the circuit; leave the two terminals disconnected.
5. **Connect network B to the terminals of the new network A.** All currents and voltages in B will remain unchanged.

Example 1.14: Find the Norton equivalent circuits for the network faced by the 1 kΩ resistor in Fig. 1.26.

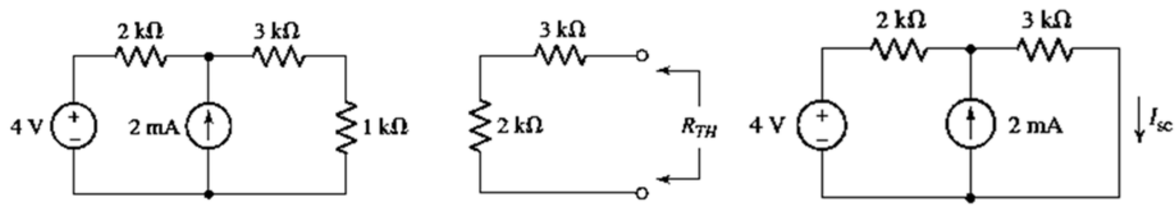


Fig. 1.26

Solution:

$$i_{sc} = i_{sc|4V} + i_{sc|4mA} = 4/(2k+3k) + 2m*(2k/(2k+3k)) = 0.8m + 0.8\ m = 1.6\ mA$$

$$R_{TH} = 2k + 3k = 5\ k\Omega$$

1.7.3 When Dependent Sources Are Present:

If network A contains a dependent source, then again we must ensure that the controlling variable and its associated element(s) cannot be in network B. In the following examples, we consider various means of reducing networks with dependent sources and resistors into a single resistance.

Example 1.15: Determine the Thevenin equivalent of the circuit in Fig. 1.27a.

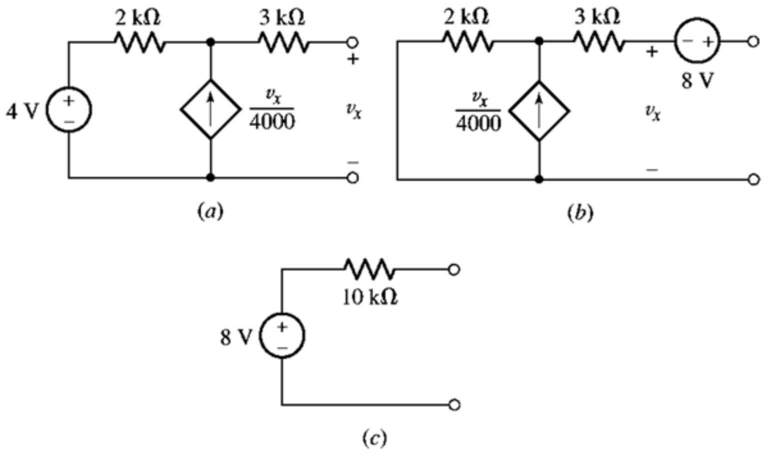


Fig. 1.27

Solution:

To find V_{oc} we note that $v_x = V_{oc}$. Using KVL around the outer loop:

$$-4 + 2 \times 10^3 (-v_x/4000) + 3 \times 10^3(0) + v_x = 0$$

And $v_x = 8$ V $= V_{oc}$

By Thevenin's theorem, then, the equivalent circuit could be formed with the inactive A network in series with an 8 V source, as shown in Fig. 1.27b. This is correct, but not very simple and not very helpful; in the case of linear resistive networks, we really want a simpler equivalent for the inactive A network, namely, R_{TH}.

The dependent source prevents us from determining R_{TH} directly for the inactive network through resistance combination; we therefore seek I_{sc}. Upon short-circuiting the output terminals in Fig. 1.27a, it is apparent that $v_x = 0$ and the dependent current source is not active.

$$I_{sc} = 4/(5 \times 10_3) = 0.8 \text{ mA. Thus,}$$

$$R_{TH} = V_{oc} / I_{sc} = 8 / 0.8 \times 10^{-3} = 10 \text{ k}\Omega$$

Example 1.16: Find the Thévenin equivalent of the circuit shown in Fig. 1.28a.

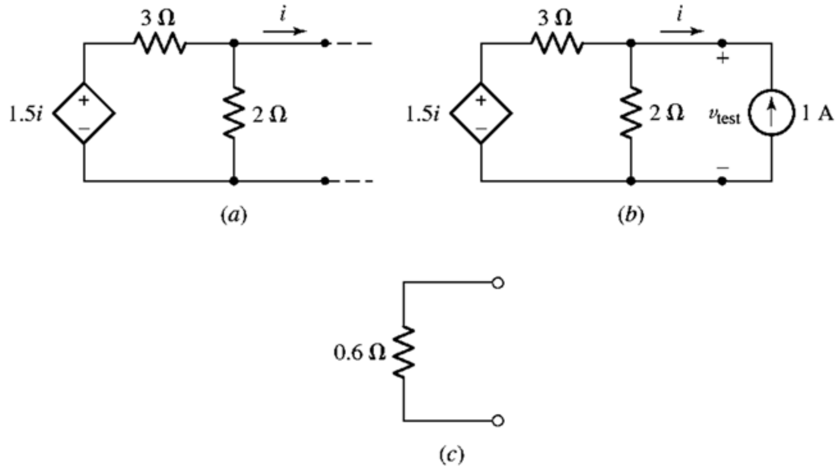


Fig. 1.28

Solution:
The rightmost terminals are already open-circuited, hence i = 0. Consequently, the dependent source is inactive, so $v_{oc} = 0$. We next seek the value of R_{TH} represented by this two-terminal network. However, we cannot find v_{oc} and i_{sc} and take their quotient, for there is no independent source in the network and both v_{oc} and i_{sc} are zero. Let us, therefore, be a little tricky.
We apply a 1 A source externally, measure the voltage v_{test} that results, and then set $R_{TH} = v_{test}/1$. Referring to Fig. 1.28b, we see that i = −1 A. Applying nodal analysis,

$$(v_{test} - 1.5(-1))/3 + v_{test}/2 = 1$$

so that

$$v_{test} = 0.6\ \text{V}$$

and thus

$$R_{TH} = 0.6\ \Omega$$

The Thevenin equivalent is shown in Fig. 1.28c.

H.W.: Find the Thevenin equivalent for the network of Fig. 1.29. (Hint: a quick source transformation on the dependent source might help.)

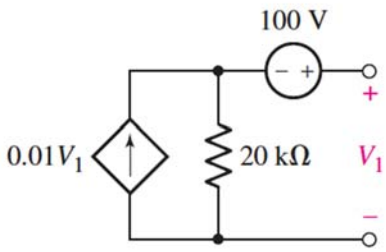


Fig. 1.29

1.8 Maximum Power Transfer:

For the practical voltage source (Fig. 1.30), the power delivered to the load RL is

$$P_L = i_L^2 R_L = (v_s^2/(R_s+R_L)^2)R_L \qquad [1]$$

$$\frac{dp_L}{dR_L} = \frac{(R_s+R_L)^2 v_s^2 - v_s^2 R_L(2)(R_s+R_L)}{(R_s+R_L)^4}$$

and equate the derivative to zero, obtaining

$$2R_L (R_s + R_L) = (R_s + R_L)^2$$

or

$$R_s = R_L$$

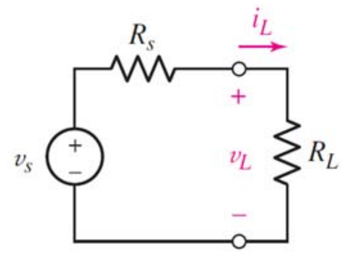


Fig. 1.30 A practical voltage source connected to a load resistor R_L.

Since the values $R_L = 0$ and $R_L = \infty$ both give a minimum ($p_L = 0$), and since we have already developed the equivalence between practical voltage and current sources, we have therefore proved the following **maximum power transfer theorem**:
An independent voltage source in series with a resistance R_s , or an independent current source in parallel with a resistance R_s, delivers maximum power to a load resistance R_L such that $R_L = R_s$.

A minor amount of algebra applied to Eq. [1] coupled with the maximum power transfer requirement that $R_L = R_s = R_{TH}$ will provide

$$p_{max|\text{delivered to load}} = v_s^2 / 4R_s = v_{TH}^2 / 4R_{TH}$$

Example 1.17: The circuit shown in Fig. 1.31 is a model for the common-emitter bipolar junction transistor amplifier. Choose a load resistance so that maximum power is transferred to it from the amplifier, and calculate the actual power absorbed.

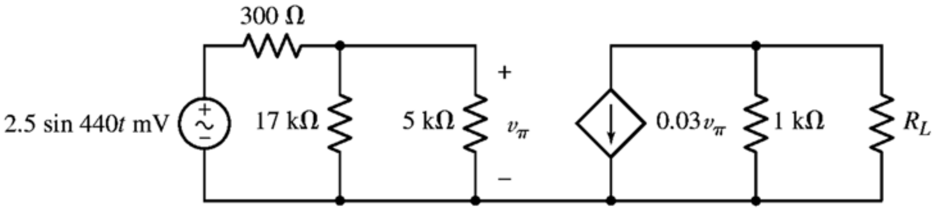


Fig. 1.31

Solution:

Since it is the load resistance we are asked to determine, the maximum power theorem applies. The first step is to find the Thevenin equivalent of the rest of the circuit.

We first determine the Thevenin equivalent resistance, which requires that we remove RL and short-circuit the independent source as in Fig. 1.32a.

Since $v_\pi = 0$, the dependent current source is an open circuit, and R_{TH} = 1 kΩ. This can be verified by connecting an independent 1A current source across the 1 kΩ resistor; v_π will still be zero, so the dependent source remains inactive and hence contributes nothing to R_{TH}.

In order to obtain maximum power delivered into the load, R_L should be set to R_{TH} = 1 kΩ.

To find v_{TH} we consider the circuit shown in Fig. 1.32b. We may write

$$v_{oc} = -0.03v_\pi (1000) = -30v_\pi$$

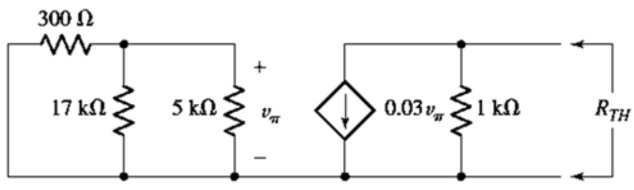
where the voltage v_π may be found from simple voltage division:

$$v_\pi = (2.5 \times 10^{-3} \sin 440t)(3864/(300 + 3864))$$

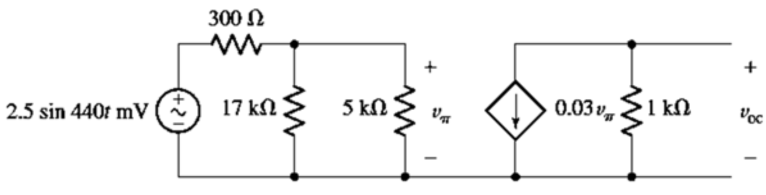
so that our Thevenin equivalent is a voltage −69.6 sin 440t mV in series with 1 kΩ.

The maximum power is given by

$$p_{max} = v_{TH}^2/4R_{TH} = 1.211 \sin^2 440t \ \mu W$$



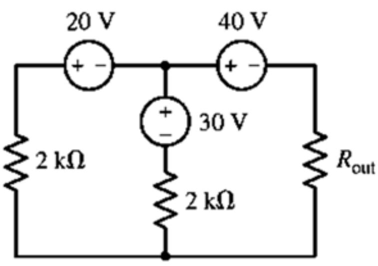
(*a*)



(*b*)

Fig. 1.32

H.W.: Consider the circuit of Fig. 1.33.



(a) If R_{out} = 3 kΩ, find the power delivered to it.

(b) What is the maximum power that can be delivered to any R_{out}?

(c) What two different values of Rout will have exactly 20 mW delivered to them?