Chapter 4: Signal Processing with Optics

OPTICAL INFORMATION PROCESSING

What is Optical Information Processing?

It is the use of light to process information.

Why use light anyways?

1) Information travels at the speed of light.
2) Parallel processing if arrays are used.
3) Immune to outside Electro-magnetic interference and crosstalk between waveguides and fiber optic cables.
4) No short circuiting problems.

Light by itself does not carry any information unless it is modulated by some means. Frequency, Polarization, Amplitude or the direct current of a semiconductor laser itself can be modulated, so that it may represent some form of information.
Once modulated, light can then be used for performing arithmetic operations for analog or digital information processing.

Analog Information Processing

Real Images are Analog in nature. By using Fourier Optics, we can perform arithmetic functions:

Multiplication- The output image is nothing but multiplication of all individual paths that make up the image.

Addition- If a hologram is used as the input transparency, then sum and product terms are present in the equation of a hologram. These are the signal and reference signals used to form a Hologram. Hence these addition and multiplication functions can be performed.

Division- An inverted or reciprocal of the input image is used in the second focal plane. This performs a division of the input image.

Subtraction- When an image is formed due to constructive (addition) and destructive (subtraction) interference produced due to different path lengths and related phase shifts.

What does Fourier Optics accomplish?

We can perform Convolution, Cross-correlation, Autocorrelation and Matched Filtering which are all properties of a Fourier Transform.

Fourier Transform

The one-dimensional Fourier transform pair is given by

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(k)e^{-ikx}\,dk \quad \ldots\ldots \quad (1)$$

$$g(k) = \int_{-\infty}^{+\infty} f(x)e^{ikx}\,dx \quad \ldots\ldots \quad (2)$$

Equation (1) states that an arbitrary, nonperiodic function *f(x)* can be synthesized by summing a continuous distribution of plane waves with amplitude distribution *g(k)* given by Eq. (2). The functions *f(x)* and *g(k)* are said to be a Fourier transform pair. Symbolically,

$$g(k) = \Im\{f(x)\} \quad \ldots\ldots \quad (3)$$

$$f(x) = \Im^{-1}\{g(k)\} \quad \ldots\ldots \quad (4)$$

Here $\Im$ and $\Im^{-1}$ represent, respectively, the Fourier-transform operation and its inverse. The inverse transform of the transform of a function *f(x)* returns the function *f(x)*. That is,

$$\Im^{-1}\{\Im(f(x))\} = \Im^{-1}\{g(k)\} = f(x) \quad \ldots\ldots \quad (5)$$

in accordance with Eqs. (3) and (4).

In two dimensions, the transform pair takes the form

$$f(x, y) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} g(k_x, k_y)e^{-i(xk_x+yk_y)}\,dk_x\,dk_y \quad \ldots\ldots \quad (6)$$

$$g(k_x, k_y) = \iint_{-\infty}^{+\infty} f(x, y) e^{i(xk_x + yk_y)}\, dx\, dy \qquad \ldots\ldots \quad (7)$$

Any nonperiodic function of two variables *f(x, y)* can thus be synthesized from a distribution of plane waves, each with amplitude and constant phase, such that

$$xk_x + yk_y = \text{constant} \qquad \ldots\ldots \quad (8)$$

The quantities and are the spatial frequency components needed in the expansion to represent the desired function *f(x, y)*.

FOURIER TRANSFORM PROCESSING

Optical data processing takes advantage of the fact that the simple lens constitutes a Fourier-transform computer, capable of transforming a complex two-dimensional pattern into a two-dimensional transform at very high resolution and at the speed of light.

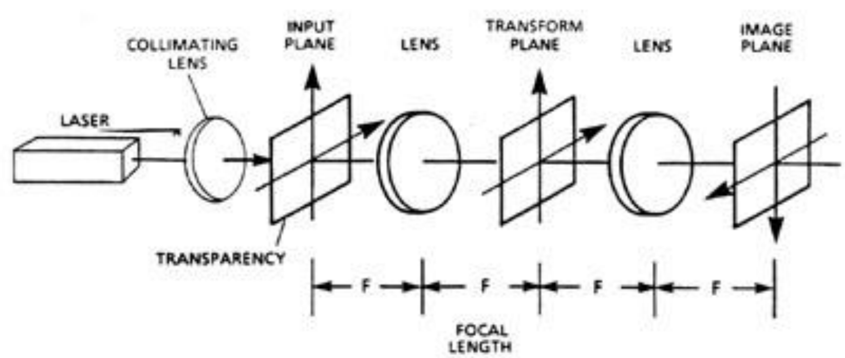

Fig. 1. Spatial and Fourier planes

In a nutshell, the FT provides information on the frequency content of the signal.

Coherent Laser Light illuminates the transparency at different path lengths. Each path diffracts through the lens at a different OPL causing phase shifts related to each path.

All these paths from each element of the transparency add up at the back focal plane, thereby forming a diffraction pattern which is the Fourier Transform (FT) of the input transparency. The back focal plane is hence known as the Fourier Plane of the lens.

This pattern may be manipulated in turn, using masks or filters to modify the final image produced by a second lens in a process called spatial filtering.

Since various details of the image can be modified by appropriate filtering, this technique is exploited in such areas as contrast enhancement and image restoration.

When the FT pattern goes through the second lens, it performs another FT and that is the restored image. Hence, FT of the FT is called the Convolution and the second lens is called the Convolver.

This is a completely reversible operation so if the FT of the signal is completely known, the signal is also completely determined.

A sinusoidal pattern appears; this indicates that points along the axis represent spatial frequency components in an image.

How is it parallel processing?

The input transparency is 2-Dimensional. Hence all illuminated elements of the transparency form an image on the focal plane simultaneously.

If the image is compared directly with a second object, the two may be optically correlated. Such correlation is applied, for example, in the problem of pattern recognition.

By such optical means, two-dimensional pictures or text are processed at once, without the necessity of sequential scanning of the object.

Optical data processing represents a fruitful convergence of the fields of optics, information science, and holography.

Optical Spectrum Analysis

The Fraunhofer diffraction pattern of a given aperture is most conveniently displayed using a positive lens, as in Figure 2. Light from a monochromatic (temporally coherent) point source (spatially coherent) is collimated by lens L1 and illuminates, in the input or aperture plane, a two-dimensional pattern whose transmittance varies across the aperture. Lens L2 forms the Fraunhofer pattern in the spectrum plane.

For simplicity we shall imagine the aperture function to vary like a square wave, such as would be produced by a Ronchi ruling, a grating of parallel straight lines with large grating space, whose opaque and transparent regions are of equal width.

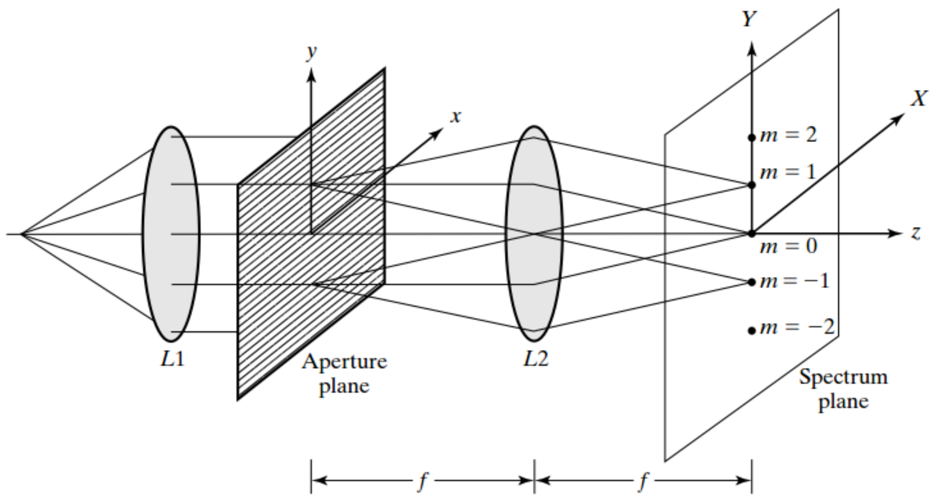

Fig. 2. Fraunhofer diffraction of a Ronchi ruling.

Now, according to the grating equation,

$$m\lambda = d\sin\theta = d\frac{Y_m}{f} \quad \text{.....} \quad (9)$$

where d is the spatial period of the ruling. Spots appear at distances Y_m from the optical axis given by

$$Y_m = m\left(\frac{\lambda f}{d}\right) \quad \text{......} \quad (10)$$

Next, introducing the angular spatial frequencies, in the Y-direction these are given by

$$k_Y = \frac{kY}{f} \quad \text{......} \quad (11)$$

Let us introduce a wave number or "normalized" form of the spatial frequencies in the Fourier series by

$$\nu_Y \equiv \frac{1}{\lambda_Y} = \frac{k_Y}{2\pi} \quad \ldots\ldots \quad (12)$$

Then, substituting for from Eq. (11) and for Y from Eq. (10), we have, for the spectrum of spatial frequencies displayed in the diffraction pattern,

$$\nu_Y = \frac{m}{d} \quad \ldots\ldots \quad (13)$$

The central spot with $m = 0$ thus corresponds to a normalized spatial frequency $\nu_Y = 0$, the DC component, in analogy with electrical frequencies.

The first-order ($m = 1$) spots above and below the central spot represent the fundamental frequency $\nu_{Y1} = 1/d$.

Higher-order spots represent higher harmonics given by $m\nu_{Y1}$.

Example: Consider a Ronchi ruling with slits of width 0.1 mm illuminated by light of wavelength 488 nm. A lens of focal length 40 cm is used in a configuration like that shown in Figure 2.

a. Find the distances of the $m = 1$ and $m = 3$ spots from the central DC spot in the diffraction pattern on the screen in the spectrum plane.

b. Find the angular spatial frequencies associated with the $m = 1$ and $m = 3$ spots.

Optical Filtering

Optical filtering is the process of intentionally blocking certain portions—that is, certain spatial frequencies—present in the diffraction pattern, to manipulate the image.

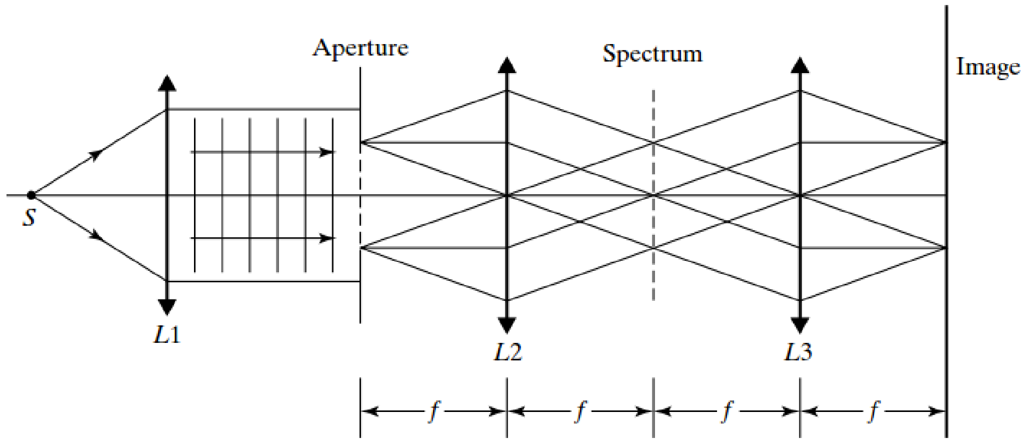

Fig. 3. Optical filter

Suppose, for example, that the aperture function is the superposition of two sine waves that are produced by back- to-back sinusoidal gratings with parallel rulings but different line spacings or spatial frequencies.

The diffraction pattern consists, in addition to the direct beam, of two pairs of light spots, each pair due to one of the spatial frequencies present. If one of these pairs is blocked, that frequency is eliminated, or filtered from the illumination. The image is a sinusoidal pattern of the other frequency.

This example shows how optical filtering is applied to the extraction of desired periodic signals from background noise or, on the other hand, to the elimination of periodic noise from a desirable signal.

A diaphragm, which blocks all but those frequencies near the direct beam, functions as a low-pass optical filter. A diaphragm, which blocks only those frequencies near the direct beam, functions as a high-pass optical filter; and a clear annular ring, which blocks the lowest and the highest frequencies, functions as a band-pass filter.

A case in point is the suppression of low spatial frequencies, or high-pass optical filtering, to enhance the contrast in a photograph.

Optical Correlation

Optical correlator formed by the combination of an optical filter and a spectrum analyzer as shown in figure 4.

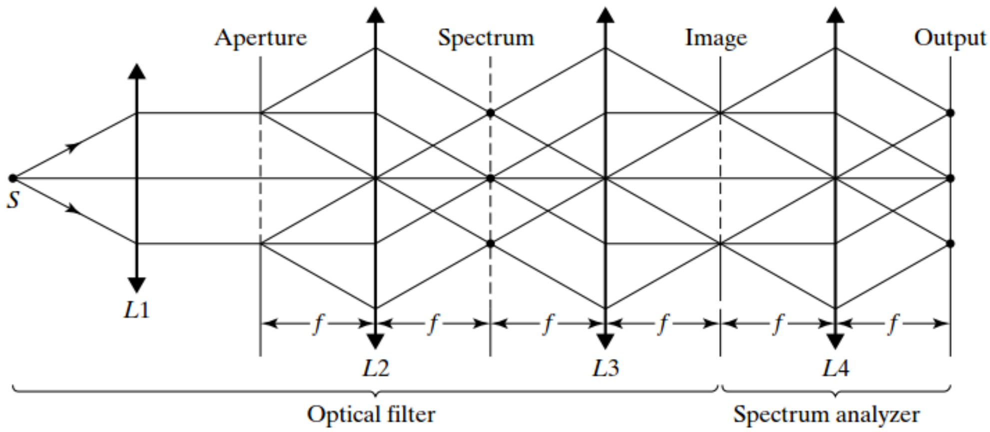

Fig. 4. Optical correlation

In the position of the image plane, we insert a mask containing a pattern which transmits light at certain positions and blocks light at other positions.

The image of the object in the aperture plane is thus superimposed over the pattern on the partially transparent mask.

Let the light so transmitted be intercepted by an additional lens L4, as shown in figure 4. The transmitted light is then monitored by a light detector placed in the second focal plane of L4.

In the output plane where the detector is placed, we expect to measure the spectrum or Fourier transform of the transmission function represented by the light transmitted through the mask located in the image plane.

This system provides an experimental means of comparing, or correlating, the light pattern in the image of the object in the aperture plane and the pattern contained on the mask.

If the two patterns are identical, for example, and so situated that the image of the object in the aperture plane coincides with the transmission pattern on the mask, then maximum light throughput occurs, a case of maximum correlation.

If one pattern is translated relative to the other, however, the bright points of the image no longer all coincide with the transparent regions of the mask, and light throughput and correlation are reduced.

If the object in the aperture plane is a photographic image of the block letter A and the mask in the image plane contains a pattern of similar shape, a high degree of correlation should be obtained when the objects are properly positioned.

On the other hand, if the mask contains a pattern of the letter B, the maximum light throughput and correlation should be significantly reduced.

This technique of pattern recognition is applied, for example, to the recognition and counting of small particles with different shapes, as in the case of blood cells, or to the search for characteristic patterns in aerial photographs, medical X-rays, and fingerprint files.

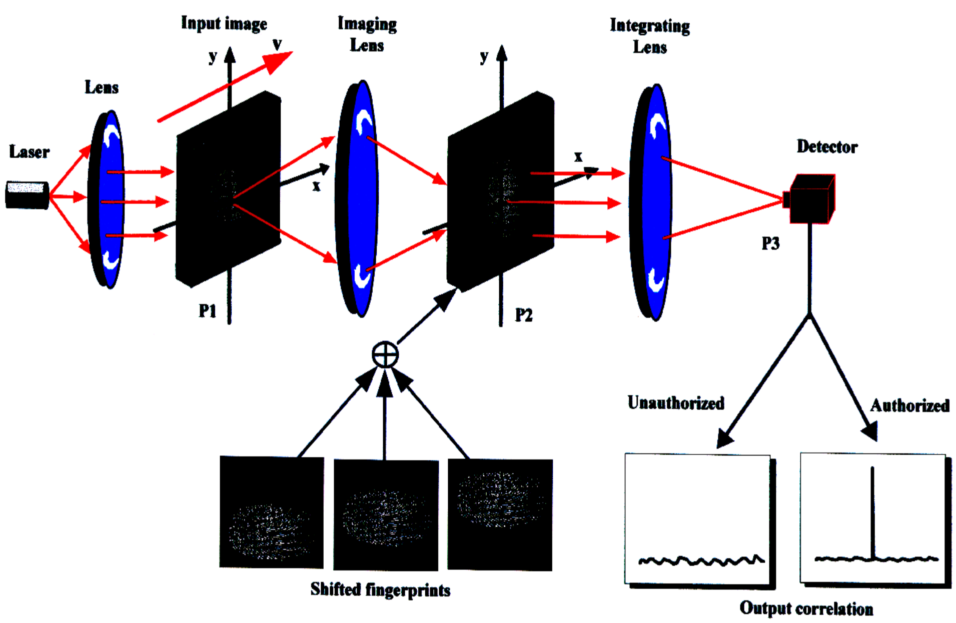

Fig. 5. Fourier Transform and Correlation Property in Pattern Recognition

Convolution

Consider a two-dimensional aperture (xy-plane) and its image (XY- plane) formed by some intervening optical system.

Let the irradiance of such a perfect image be given $I_o(X,Y) \equiv I_o(x,y)$.

In a linear system, these elementary irradiance patterns are simply additive. Let the actual irradiance over the image plane be given by $I_i(X,Y)$.

The transformation from $I_i(X,Y)$ to $I_o(X,Y)$ clearly characterizes the optical system and is accomplished by a third function, called the point spread function, $G(x, y, X, Y)$.

If we assume the point spread function to be space-invariant (independent of object point coordinates)

$$G(x, y, X, Y) = G(X - x, Y - y) \quad \ldots\ldots \ (14)$$

Further, if the light from the object plane is incoherent, irradiances add, and we can write for the irradiance at the image point (X, Y) due to all object points (x, y):

$$\underbrace{I_i(X, Y)}_{\substack{\text{image} \\ \text{irradiance}}} = \iint \underbrace{I_0(x, y)}_{\substack{\text{object} \\ \text{irradiance}}} \underbrace{G(X - x, Y - y)}_{\substack{\text{point spread} \\ \text{function}}} \, dx \, dy \quad \ldots\ldots \ (15)$$

The integral in Eq. (15) is called the convolution of the functions and G, usually abbreviated by

$$I_i = I_0 \otimes G \quad \ldots\ldots \ (16)$$

Suppose that we calculate the Fourier transform of each of these functions, represented by $\Im(I_o)$, $\Im(I_i)$ and $\Im(G)$. The *convolution theorem* states that the Fourier transform of the convolution of two functions is equal to the product of their individual transforms. Symbolically,

$$\Im(I_i) = \Im(I_0 \otimes G) = \Im(I_0) \times \Im(G) \quad \ldots\ldots \ (17)$$

The content of Eqs. (16) and (17) can be succinctly summarized by stating that convolution in real space corresponds to multiplication in Fourier space.

Imaging

Sampling theory is involved with the collection, analysis, and interpretation of data. The data in which we are interested are image data collected from photonic systems, and this includes images formed by scanning and staring devices. A scanning device has one sensor or a small array of sensors that it moves in order to collect an array of data. A staring sensor has as many sensor elements as data in the array that it records and so does not move.

Pixel

A pixel is a conjunction of the words picture element, in which the term picture is synonymous with image. Imagine a set of white marbles set into holes on a wooden board. The holes have been drilled to form a square array, and so a view of the marbles would look something like Figure 6.

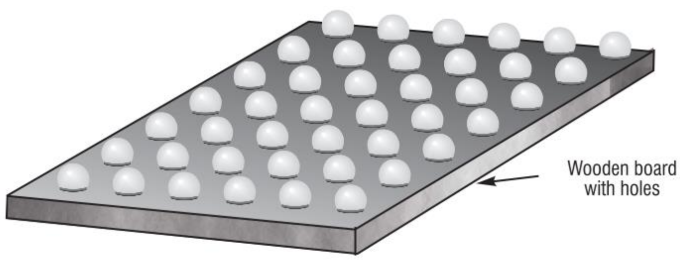

Fig. 6. "Marble" array

Now imagine that we have 256 marbles and they are arranged as 16 rows by 16 columns. If we replace some of the white marbles with black ones, we can produce an image using the marbles, like that shown in Figure 7.

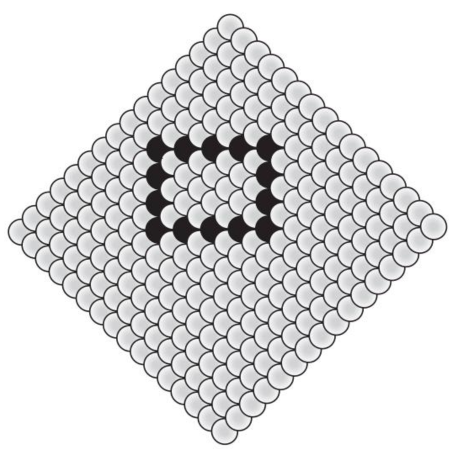

Fig. 7. "Marble" array image

The pixels in the image are exemplified by the marbles. The image produced by the marbles is called a binary image, since each of the pixels (marbles) can be one of two values (black or white).

The range of grays from black to white is called the grayscale, and a black and white image of this sort is sometimes called a grayscale image.

If the pixels can take on color values, we have a color image.

Pixels are not restricted to visible light, but can be variations of ink on a printed page. Pixels can also represent signals that cannot be viewed directly by the human eye, such as the pixels in an infrared or laser radar image.

Quantization

The values that a pixel can represent have to do with the quantization of the pixel, expressed in terms of bits. The images that we have been discussing are digital images, or images that take on discrete values. Each pixel is a discrete component of the image, and a fundamental assumption is that the image will be stored, manipulated, and displayed by a computer.

The pixels may be only one of two (discrete) values.

In a grayscale image, the pixel takes on a set of values that are typically defined by a power of 2, such as 4, 8, 16, 32, 64, 128, and 256. This is because each pixel is represented by a binary number in the computer. The more bits that represent the pixel, the more grayscale values it can take on.

The two most common image quantizations are 8-bit grayscale images and what are called 24-bit truecolor images. To compute the quantization of a pixel, we simply raise 2 by the number of bits. So, the 8-bit image will have $2^8 = 256$ gray levels. The 24-bit truecolor image is a bit different. Here each pixel is actually three pixels, one each of red, green, and blue.

Color images are more complex than black and white images because they require a combination of the three primary colors, red, green, and blue.

Resolution and Spatial Frequency

Resolution has to do with the fineness of detail the image can represent, or the fineness of detail the camera can record or the display system display. The more pixels per unit area an image has, the higher resolution.

There are a number of ways to define resolution in terms of imaging. One way is by using the following equation:

Resolution = number of pixels/area.

For an image that is 3 × 3 inches square and contains 900 by 900 pixels, the resolution is (900 × 900) / (3 × 3) = 90000 pixels per square inch.

We can consider resolution to be a measure of sampling capability, where each pixel is a sample. The maximum number of lines that can be represented, i.e. the maximum spatial frequency, is measured in lines per unit distance and is just one-half the resolution. This can be expressed analytically as:

Maximum spatial frequency (lines/distance) = 1/2 resolution (lines/distance).

Bandwidth

Bandwidth is defined as the amount of information that can be transmitted across a channel in a fixed amount of time.

Image bandwidth (lines/distance) = 1/2 resolution (lines/distance).

CCD Cameras

Charge-coupled device (CCD) camera uses an array of light-sensitive cells formed in silicon. The cells can be thought of as miniature capacitors, where each capacitor is a pixel in the image created by the array.

When the array is exposed to light, the capacitors charge up proportional to the intensity of the light falling on the array. This charge, which is discriminated by values of voltage, is then read off of the array, converted to a digital signal, and transferred into a computer.

CCD cameras have electronic shutters that control the integration time at each pixel. This means that the charging time of the capacitor is controlled. If the capacitor is allowed to charge a long time, the pixel will have more time to charge.

Figure 8 shows the basics of how a CCD array works.

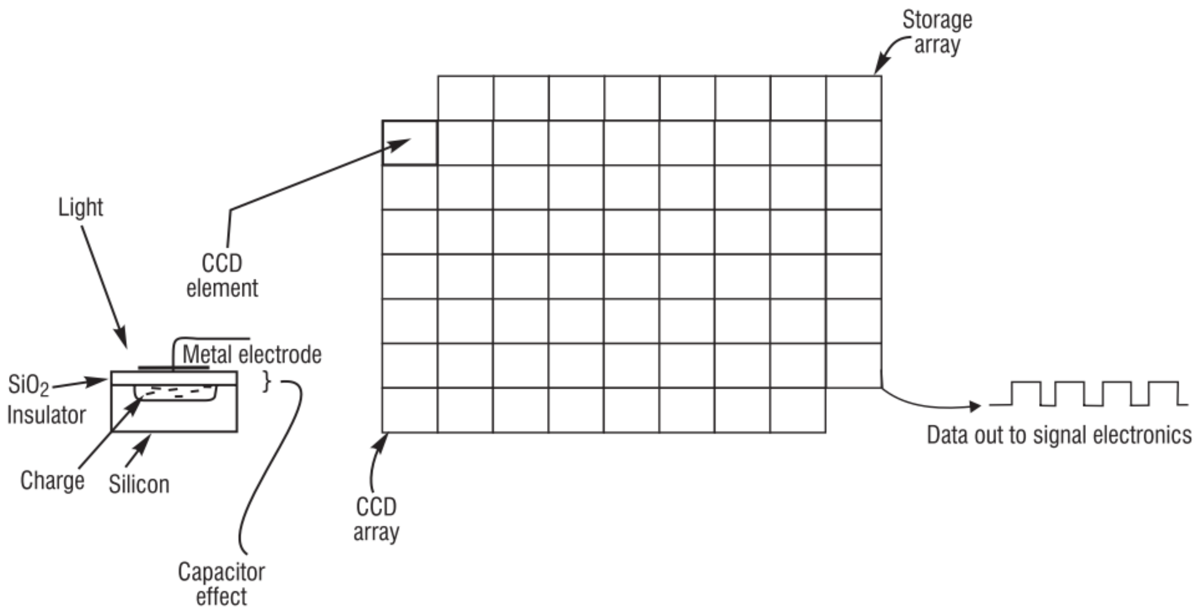

Fig. 8. CCD array