Graph of Function رسـم الدوال

$Ex: \quad 9x^2 + 16y^2 = 144$

$1 - \ D_f \ \ and \ \ R_f$

$$\mathrm{D_f}: \ 16y^2 = 144 - 9x^2 \Longrightarrow y = \pm\sqrt{\frac{144 - 9x^2}{16}} \quad \Longrightarrow y = \pm\sqrt{\frac{9}{16}(16 - x^2)}$$

$$y = \pm\frac{3}{4}\sqrt{16 - x^2} \quad \Longrightarrow 16 - x^2 \geq 0$$

$$16 \geq x^2 \quad \Longrightarrow x^2 \leq 16 \quad \Longrightarrow |x| \leq 4$$

$$\therefore \quad D_{\mathrm{f}} \ : \ -4 \leq x \leq 4$$

$$R_f: 9x^2 = 144 - 16y^2 \quad \Longrightarrow x = \sqrt{\tfrac{16}{9}(9 - y^2)} \Longrightarrow x = \tfrac{4}{3}\sqrt{9 - y^2}$$

$$9 - y^2 \geq 0 \quad \Longrightarrow 9 \geq y^2 \quad \Longrightarrow y^2 \leq 9 \quad \Longrightarrow |y| \leq 3$$

$$\therefore \quad R_{\mathrm{f}} \ : \ -3 \leq y \leq 3$$

٢- نقاط تقاطع الدالة:

$$with\ \ x\text{-}axis \ \Longrightarrow y = 0 \ \ \Longrightarrow 9x^2 = 144 \ \ \Longrightarrow x^2 = \frac{144}{9} = 16$$

$x = \pm 4 \ \ \Longrightarrow (4\ ,0)\ and\ (-4\ ,0)$ x نقاط تقاطع الدالة مع محور

$$with\ y - axis \ \Longrightarrow x = 0 \ \ \Longrightarrow 16y^2 = 144 \ \ \Longrightarrow y^2 = \frac{144}{16} = 9$$

$y = \pm 3 \ \ \Longrightarrow (0,3)\ and\ (0,-3)$ y تقاطع الدالة مع محور

٣- تناظر الدالة:

$\text{with } x - \text{axis} \ \Longrightarrow \text{y} = -\text{y} \ \ \ \Longrightarrow 9x^2 + 16(-\text{y})^2 = 144$

$\Longrightarrow 9x^2 + 16y^2 = 144 = original$ x اذن يوجد تناظر مع محور

$\text{with y} - \text{axis} \ \Longrightarrow x = -x \ \ \Longrightarrow \ \ 9(-x)^2 + 16\text{y}^2 = 144$

$\Longrightarrow 9x^2 + 16y^2 = 144 = original$ y اذن يوجد تناظر مع محور

with origin(نقطة الأصل) $\Longrightarrow x = -x\ and\ y = -y \ \ \Longrightarrow 9(-x)^2 + 16(-\text{y})^2 = 144$

$\Longrightarrow 9x^2 + 16y^2 = 144 = original$ اذن يوجد تناظر مع نقطة الأصل

$$Find \ \frac{dy}{dx} \ for:$$

1. $y = (x^3+1)^2\sqrt{x^4+1}$

$$\frac{dy}{dx} = (x^3+1)^2.\frac{4x^3}{2\sqrt{x^4+1}} + \sqrt{x^4+1}\,.2(x^3+1).3x^2$$

2. $y = \dfrac{x^2+1}{x^2-1}$

$$\frac{dy}{dx} = \frac{(x^2-1)(2x)-(x^2+1)(2x)}{(x^2-1)^2} = \frac{4x}{(x^2-1)^2}$$

3. $x = y\sqrt{1-y^2}$

$$\frac{dx}{dy} = y.\frac{-2y}{2\sqrt{1-y^2}} + \sqrt{1-y^2}\,.1 = \frac{-y^2}{\sqrt{1-y^2}} + \sqrt{1-y^2}$$

$$\frac{dx}{dy} = \frac{-y^2+1-y^2}{\sqrt{1-y^2}} = \frac{1-2y^2}{\sqrt{1-y^2}}$$

$$\therefore \quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{1-2y^2}$$

مثال: جــد معادلة المستقيم المماس ومعادلة العمود المماس لمنحني الدالة $(y = x^2 + 2)$

في النقطة $(-1,3)$.

$$\frac{dy}{dx} = 2x \quad ,at \quad x = -1 \quad \rightarrow \quad \frac{dy}{dx} = -2 = m$$

$$y - y_1 = m(x - x_1) \ \rightarrow \ y - 3 = -2(x + 1)$$

$$y - 3 = -2x - 2 \quad \rightarrow \quad y + 2x - 1 = 0$$ معادلة المستقيم المماس

$$y - y_1 = -\frac{1}{m}(x - x_1) \ \rightarrow \ y - 3 = -\frac{1}{-2}(x + 1)$$

$$2y - 6 = x + 1 \quad \rightarrow \quad 2y - x - 7 = 0$$ معادلة المستقيم العمودي على المماس

$$Ex:\ if\ \ y = t - t^3 \quad and \quad x = t - t^2 \quad Find\ \ \frac{d^2y}{dx^2}$$

$$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 - 3t^2}{1 - 2t}$$

$$\rightarrow\ \frac{d\acute{y}}{dt} = \frac{(1 - 2\mathrm{t})(-6\mathrm{t}) - (1 - 3\mathrm{t}^2)(-2)}{(1 - 2\mathrm{t})^2}$$

$$\rightarrow\ \frac{d\acute{y}}{dt} = \frac{6\mathrm{t}^2 + 6\mathrm{t} + 2}{(1 - 2\mathrm{t})^2}$$

$$\therefore\ \frac{d^2y}{dx^2} = \frac{\frac{d\acute{y}}{dt}}{\frac{dx}{dt}} = \frac{\frac{6\mathrm{t}^2 + 6\mathrm{t} + 2}{(1 - 2\mathrm{t})^2}}{1 - 2t} = \frac{6\mathrm{t}^2 + 6\mathrm{t} + 2}{(1 - 2\mathrm{t})^3}$$

$$Ex: \int \frac{x^2}{\sqrt{x^3+1}}\,dx$$

$$\int x^2(x^3+1)^{-\frac{1}{2}}\,dx \quad , m = -\frac{1}{2}$$

$$\frac{1}{3}\int 3x^2(x^3+1)^{-\frac{1}{2}}\,dx \qquad , u = x^3+1\ , \qquad \mathrm{du} = 3x^2$$

$$\Rightarrow\ \frac{1}{3}\,\frac{(x^3+1)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c \ \Rightarrow\ \frac{1}{3}\,\frac{(x^3+1)^{\frac{1}{2}}}{\frac{1}{2}}+c \ \Rightarrow\ \frac{2}{3}\sqrt{x^3+1}+c$$

المعادلة التفاضلية $\boldsymbol{Differential\ Equation\ (D.E)}$

هي معادلة تحتوي على مشتقة من الرتبة الأولى $(\frac{dy}{dx})$(1st D.E) أو الرتبة الثانية $(\frac{d^2y}{dx^2})$ (2nd D.E) ويمكن حلها بالتكامل.

مثال: حل المعادلة التفاضلية $\frac{\mathrm{dy}}{\mathrm{d}x} = x\sqrt{y}$

لحل المعادلة التفاضلية نجعل كل حدود (x) مع dx على جهة وحدود (y) مع (dy) على جهة ثانية قبل إجراء عملية التكامل.

$$\frac{\mathrm{dy}}{\mathrm{d}x} = xy^{\frac{1}{2}} \Rightarrow \int y^{-\frac{1}{2}}\, dy = \int x\, dx$$

$$\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + c_1 = \frac{x^2}{2} + c_2 \Rightarrow 2\sqrt{y} - \frac{1}{2}x^2 = c$$ الحل العام (G.S)، حيث $c = c_2 - c_1$

الدوال اللوغاريتمية والأسية والمثلثية

$$Find\ \frac{dy}{dx}\ for:$$

$$\boldsymbol{Ex}:\ \boldsymbol{ln}(\boldsymbol{x}+\boldsymbol{y}) = \boldsymbol{e}^{\boldsymbol{xy}}$$

$$\frac{(1+\acute{y})}{x+\mathrm{y}} = e^{xy}\,(x\acute{y}+y.1)$$

$$(1+\acute{y}) = x\acute{y}\,(x+\mathrm{y})\ e^{xy} + y(x+\mathrm{y})e^{xy}$$

$$\acute{\mathrm{y}} - x\acute{y}\,(x+\mathrm{y})\ e^{xy} = y(x+\mathrm{y})e^{xy} - 1$$

$$\acute{y}\ [1 - x(x+\mathrm{y})\ e^{xy}] = y(x+\mathrm{y})e^{xy} - 1$$

$$\frac{dy}{dx} = \frac{y(x+\mathrm{y})e^{xy} - 1}{1 - x(x+\mathrm{y})\ e^{xy}}$$

$$\boldsymbol{Ex}: \quad \boldsymbol{y^{\frac{2}{3}} = \frac{(x^2+1)(3x+4)^{\frac{1}{2}}}{\sqrt[5]{(2x-3)(x^2-4)}}} \Rightarrow ln y^{\frac{2}{3}} = ln\frac{(x^2+1)(3x+4)^{\frac{1}{2}}}{\sqrt[5]{(2x-3)(x^2-4)}}$$

$$\frac{2}{3}\ln y = ln(x^2+1) + \frac{1}{2}ln\,(3x+4) - \frac{1}{5}[ln(2x-3) + ln(x^2-4)]$$

$$\frac{2}{3}.\frac{1}{y}\frac{dy}{dx} = \frac{2\text{x}}{x^2+1} + \frac{3}{2(3x+4)} - \frac{2}{5(2x-3)} - \frac{2x}{5(x^2-4)}$$

$$\frac{\text{dy}}{\text{d}x} = \frac{3y}{2}\left[\frac{2x}{x^2+1} + \frac{3}{2(3x+4)} - \frac{2}{5(2x-3)} - \frac{2x}{5(x^2-4)}\right]$$

$\boldsymbol{Ex}$: $\int \frac{\boldsymbol{x}}{\boldsymbol{x+1}}\,\boldsymbol{dx}$

$$\int \frac{x+1-1}{x+1}dx \quad \Rightarrow \quad \int \frac{x+1}{x+1}dx - \int \frac{1}{x+1}dx$$

$$\Rightarrow \quad \int dx - \int \frac{1}{x+1}dx \quad \Rightarrow x - ln|x+1| + c$$

Ex: $\int \frac{\ln x}{x}dx$

$$\int (\ln x)^1 . \frac{1}{x} dx \quad \Rightarrow \quad \frac{(\ln x)^2}{2} + c$$

مشتقة وتكامل الدوال المثلثية:

	Derivative	Integral
1	$\frac{d(\sin u)}{dx} = \cos u . \frac{du}{dx}$	$\int \cos u \, du = \sin u + c$
2	$\frac{d(\cos u)}{dx} = -\sin u . \frac{du}{dx}$	$\int \sin u \, du = -\cos u + c$
3	$\frac{d(\tan u)}{dx} = \sec^2 u . \frac{du}{dx}$	$\int \sec^2 u \, du = \tan u + c$
4	$\frac{d(\cot u)}{dx} = -\csc^2 u . \frac{du}{dx}$	$\int \csc^2 u \, du = -\cot u + c$
5	$\frac{d(\sec u)}{dx} = \sec u . \tan u . \frac{du}{dx}$	$\int \sec u . \tan u \, du = \sec u + c$
6	$\frac{d(\csc u)}{dx} = -\csc u . \cot u . \frac{du}{dx}$	$\int \csc u . \cot u \, du = -\csc u + c$

$Find\ \frac{dy}{dx}\ for$

$\boldsymbol{Ex}: \mathbf{1}.\ \ y = sin\sqrt{3x} \ \ \Longrightarrow \frac{\mathrm{dy}}{\mathrm{d}x} = \cos\sqrt{3x}.\frac{3}{2\sqrt{3x}}$

$\boldsymbol{Ex}: \mathbf{2}.\ \ y = 3^{\sec\,x}$

$\ln y = \ln 3^{\sec\,x} \quad \Rightarrow \quad \ln y = \sec x \ln 3$

$$\frac{1}{y}\frac{dy}{dx} = \ln 3 \sec x . \tan x \quad \Longrightarrow \quad \frac{dy}{dx} = y(\ln 3 \sec x . \tan x\ )$$

$\boldsymbol{Ex}: \mathbf{3}.\ \ y = e^{1+\tan\,2x}$

$\frac{dy}{dx} = e^{1+\tan\,2x}[sec^2(2x).2]$

1. $\int e^{2x} \cos e^{2x}\, dx$

$$\frac{1}{2}\int 2e^{2x} \cos e^{2x}\, dx \implies \frac{1}{2}\sin e^{2x} + c$$

2. $\int x\, sec^2(x^2)\ tan^3(x^2)\ dx$

$$\frac{1}{2}\int 2x\, sec^2(x^2)\ [tan(x^2)]^3\ dx \implies \frac{1}{2}.\frac{[tan(x^2)]^4}{4} + c$$

3. $\int \frac{cos\sqrt{x}}{\sqrt{x}}\ dx$

$$2\int \frac{cos\sqrt{x}}{2\sqrt{x}}\ dx \implies 2\ sin\sqrt{x} + c$$

4. $\int \frac{sec^2 x}{\sqrt{1+2\tan x}}\, dx$

$$\frac{1}{2}\int 2sec^2 x(1+2\tan x)^{-\frac{1}{2}}\, dx \quad \Rightarrow \frac{1}{2}.\frac{(1+2\tan x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+c$$

$$\Rightarrow \ \frac{1}{2}.\frac{(1+2\tan x)^{\frac{1}{2}}}{\frac{1}{2}}+\mathrm{c} \Rightarrow \ \sqrt{1+2\tan x}+\mathrm{c}$$