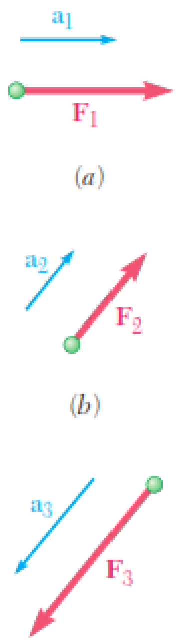
Kinetics of Particles: Newton's Second Law

12.2 NEWTON'S SECOND LAW OF MOTION

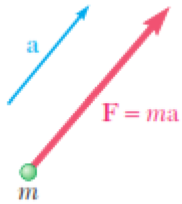


Newton's second law can be stated as follows:

If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \frac{F_3}{a_3} = \cdots = \text{constant}$$

$$\mathbf{F} = m\mathbf{a}$$



$$\Sigma\mathbf{F} = m\mathbf{a}$$

where $\Sigma\mathbf{F}$ represents the sum, or resultant, of all the forces acting on the particle.

12.4 SYSTEMS OF UNITS

International System of Units (SI Units).

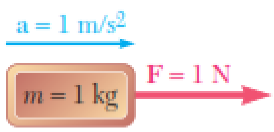
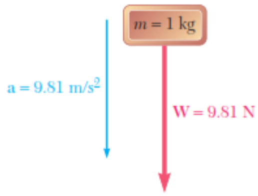


Fig. 12.4

$$1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{kg} \cdot \text{m/s}^2$$

$$W = mg$$

Recalling that $g = 9.81 \text{ m/s}^2$, we find that the weight of a body of mass 1 kg (Fig. 12.5) is



$$W = (1 \text{ kg})(9.81 \text{ m/s}^2) = 9.81 \text{ N}$$

Multiples and submultiples of the units of length, mass, and force are frequently used in engineering practice. They are, respectively, the *kilometer* (km) and the *millimeter* (mm); the *megagram*† (Mg) and the *gram* (g); and the *kilonewton* (kN). By definition,

$$1 \text{ km} = 1000 \text{ m} \qquad 1 \text{ mm} = 0.001 \text{ m}$$
$$1 \text{ Mg} = 1000 \text{ kg} \qquad 1 \text{ g} = 0.001 \text{ kg}$$
$$1 \text{ kN} = 1000 \text{ N}$$

U.S. Customary Units.

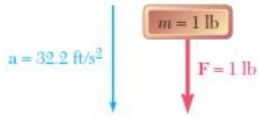
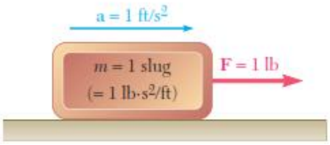


Fig. 12.6



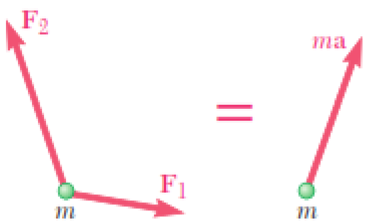
$$F = ma \qquad 1 \text{ lb} = (1 \text{ slug})(1 \text{ ft/s}^2)$$

$$1 \text{ slug} = \frac{1 \text{ lb}}{1 \text{ ft/s}^2} = 1 \text{ lb} \cdot \text{s}^2/\text{ft}$$

Length:	$1 \text{ ft} = 0.3048 \text{ m}$
Force:	$1 \text{ lb} = 4.448 \text{ N}$
Mass:	$1 \text{ slug} = 1 \text{ lb} \cdot \text{s}^2/\text{ft} = 14.59 \text{ kg}$

12.5 EQUATIONS OF MOTION

$$\Sigma \mathbf{F} = m\mathbf{a}$$



Rectangular Components. Resolving each force **F** and the acceleration **a** into rectangular components, we write

$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}) = m(a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

from which it follows that

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \qquad (12.8)$$

Recalling from Sec. 11.11 that the components of the acceleration are equal to the second derivatives of the coordinates of the particle, we have

$$\Sigma F_x = m\ddot{x} \qquad \Sigma F_y = m\ddot{y} \qquad \Sigma F_z = m\ddot{z} \qquad (12.8')$$

Consider, as an example, the motion of a projectile. If the resistance of the air is neglected, the only force acting on the projectile after it has been fired is its weight $\mathbf{W} = -W\mathbf{j}$. The equations defining the motion of the projectile are therefore

$$m\ddot{x} = 0 \qquad m\ddot{y} = -W \qquad m\ddot{z} = 0$$

and the components of the acceleration of the projectile are

$$\ddot{x} = 0 \qquad \ddot{y} = -\frac{W}{m} = -g \qquad \ddot{z} = 0$$

where g is 9.81 m/s^2 or 32.2 ft/s^2

Tangential and Normal Components. Resolving the forces and the acceleration of the particle into components along the tangent to the path (in the direction of motion) and the normal (toward the inside of

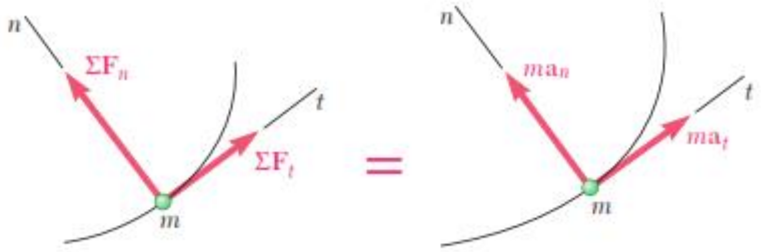


Fig. 12.9

the path) (Fig. 12.9), and substituting into Eq. (12.2), we obtain the two scalar equations

$$\Sigma F_t = ma_t \qquad \Sigma F_n = ma_n \qquad (12.9)$$

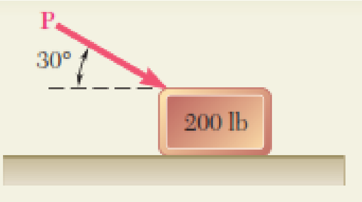
Substituting for a_t and a_n from Eqs. (11.40), we have

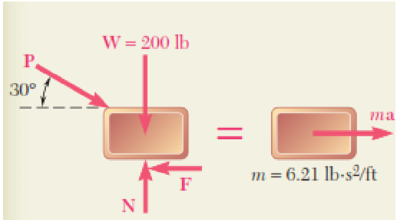
$$\Sigma F_t = m\frac{dv}{dt} \qquad \Sigma F_n = m\frac{v^2}{\rho} \qquad (12.9')$$

The equations obtained may be solved for two unknowns.

SAMPLE PROBLEM 12.1

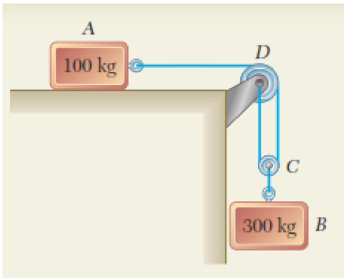
A 200-lb block rests on a horizontal plane. Find the magnitude of the force **P** required to give the block an acceleration of 10 ft/s^2 to the right. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.25$.

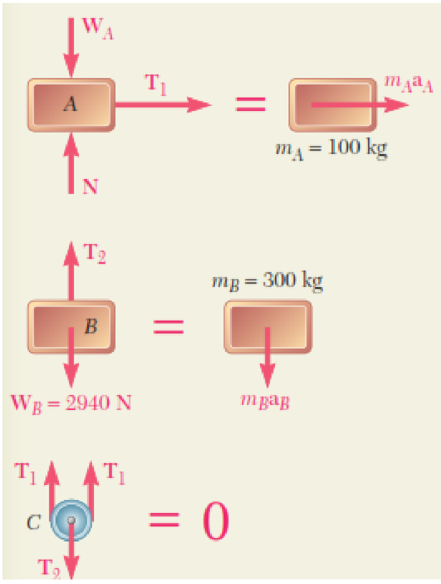




SAMPLE PROBLEM 12.3

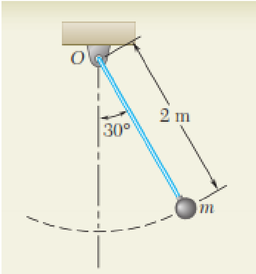
The two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in each cord.

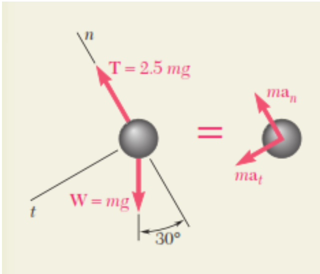




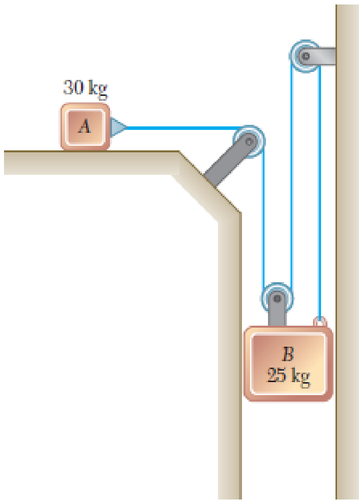
SAMPLE PROBLEM 12.5

The bob of a 2-m pendulum describes an arc of circle in a vertical plane. If the tension in the cord is 2.5 times the weight of the bob for the position shown, find the velocity and the acceleration of the bob in that position.

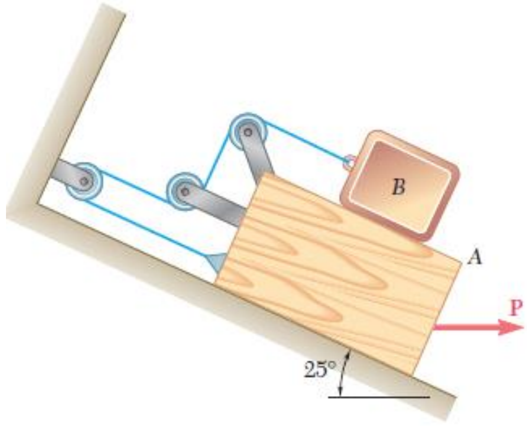




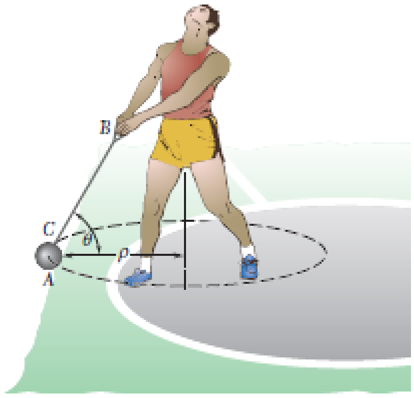
The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficients of friction between block A and the horizontal surface are $\mu_s = 0.25$ and $\mu_k = 0.20$, determine (*a*) the acceleration of each block, (*b*) the tension in the cable.



Block A has a mass of 40 kg, and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are $\mu_s = 0.20$ and $\mu_k = 0.15$. If $P = 40$ N $\rightarrow$, determine (*a*) the acceleration of block B, (*b*) the tension in the cord.



During a hammer thrower's practice swings, the 7.1-kg head A of the hammer revolves at a constant speed v in a horizontal circle as shown. If $\rho = 0.93$ m and $\theta = 60°$, determine (*a*) the tension in wire BC, (*b*) the speed of the hammer's head.



A single wire ACB passes through a ring at C that is attached to a 12-lb sphere which revolves at a constant speed v in the horizontal circle shown. Knowing that $\theta_1 = 50°$ and $d = 30$ in. and that the tension in both portions of the wire is 7.6 lb, determine (*a*) the angle θ_2 (*b*) the speed v.