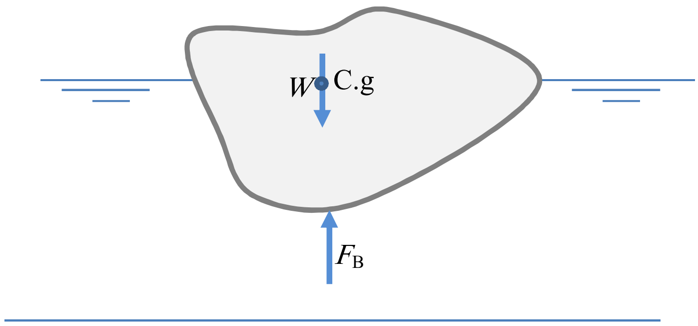
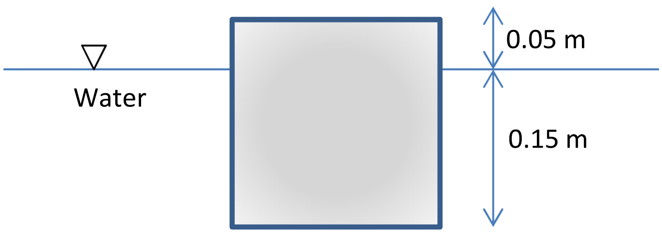
Chapter 4- Buoyancy and Stability

Buoyancy: resultant force exerted on a body by static fluid which is submerged or floating. It always acts vertically upward.



- The buoyancy force acts through the centroid of the displaced liquid volume.
- It can be proven that the Buoyancy force equals the weight of the displaced liquid.
- For equilibrium, $F_B = W$, $F_B = \gamma\ V_{\text{displaced liquid}}$

Example: A 0.2 m cube is floating as shown, find the density of the cube material.



Hydrometer: an instrument used to measure the specific gravity of liquids. It consists of bulb and constant area stem. When placed in pure water the specific gravity is marked to read 1.0. The force balance is

$$W = \gamma_{water} V_{displaced}$$

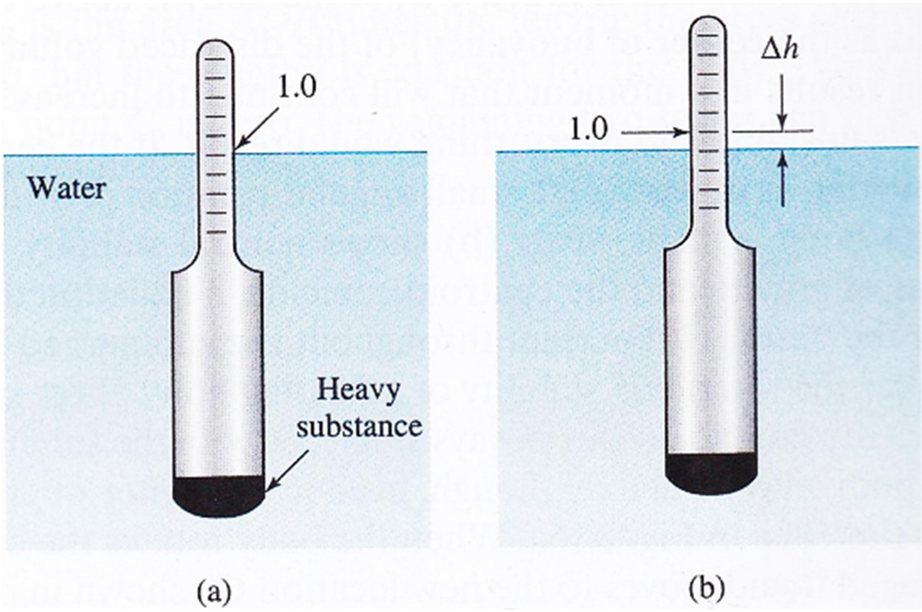
Where W is the weight of the hydrometer and V is the submerged volume below the S=1.0 line. In an unknown liquid of specific gravity, γ_x, a force balance would be:

$$W = \gamma_x (V - A\,\Delta h)$$

Where A is the cross-sectional area of the stem. Equating the two equations above gives

$$\Delta h = \frac{V}{A}\left(1 - \frac{1}{S_x}\right)$$

Where $S_x = \gamma_x / \gamma_{water}$

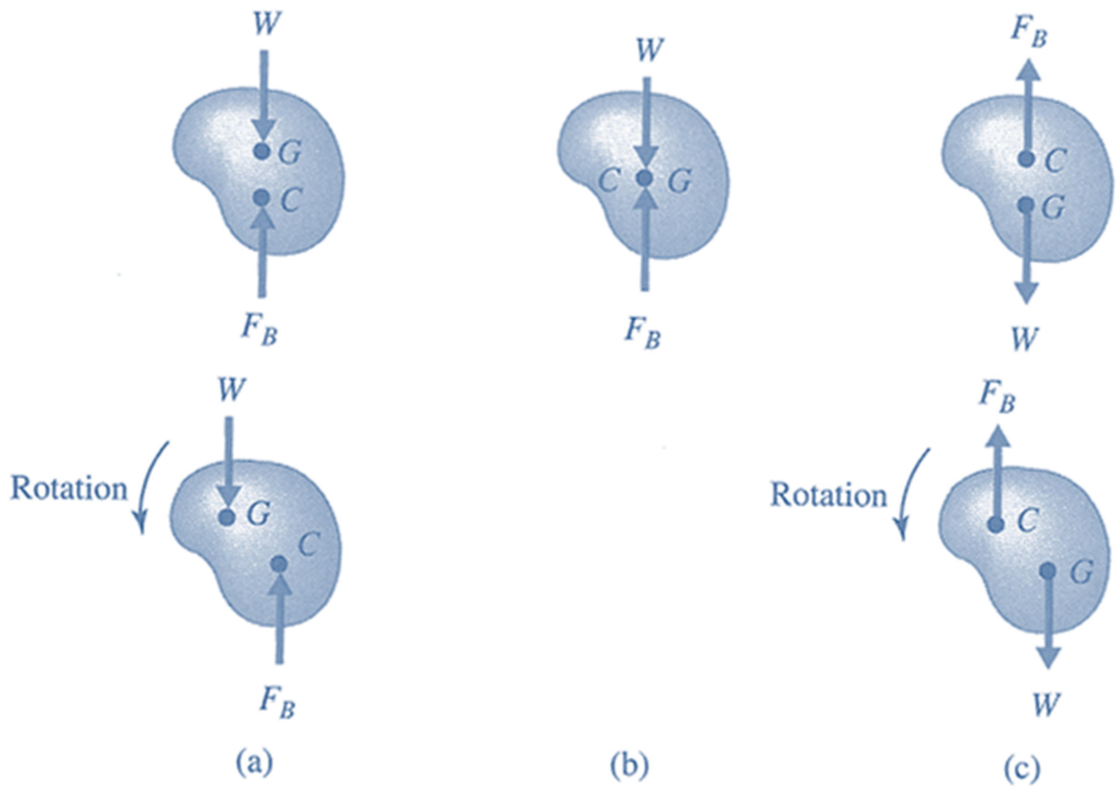


Hydrometer: (a) in water, (b) in unknown liquid

Stability

Stability becomes an important consideration when floating bodies such as a boat or ferry is designed. It is an obvious requirement that a floating body such as a boat does not topple when slightly disturbed. We say that a body is in stable equilibrium if it is able to return to its position when slightly disturbed. Failure to do so denotes unstable equilibrium

Stability of submerged bodies



Unstable Neutral Stable

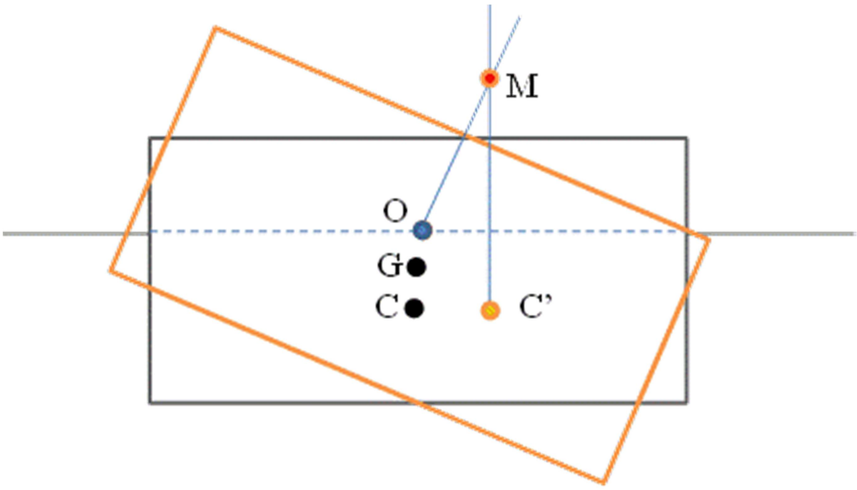
Stability of floating bodies: in this case, the stability is more complicated to deal with.

When the body is slightly rotated about O,

1- The center of gravity remains unchanged.
2- The center of buoyancy is changed to C'

The center of the buoyancy (C, the centroid of the displaced volume of fluid) of a floating body depends on the shape of the body and on the position in which it is floating.

Extending a line from C' vertically. It will intercept with a line extended from the point O (axis of rotation) at a point M. This point M is called the Metacenter.

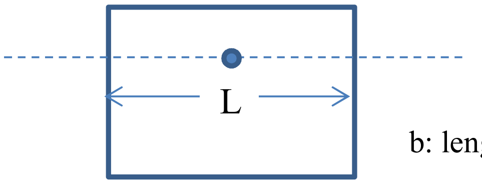


Now:

If M is above G, the body is stable, otherwise, it is unstable and according to the following relation:

$$\overline{GM} = \frac{I_o}{V_{displac}} - \overline{CG}$$

Where $\overline{GM}$: distance between G and M



$$I_o = \frac{bL^3}{12}$$

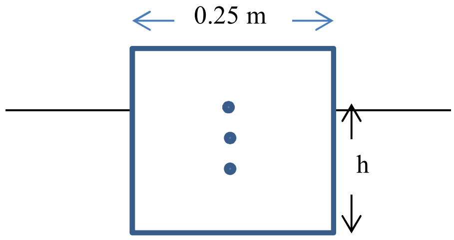
b: length into the paper

$\overline{CG}$ distance between C and G

I_o: second moment area of the waterline area about an axis passing through the Origin O.

$V_{displac}$: Volume of displaced liquid or (submerged body)

<u>Example:</u> A 0.25 long cylinder with 0.25 m diameter composed of material with density of ρ = 815 kg/m^3. Will it float on water on its base?

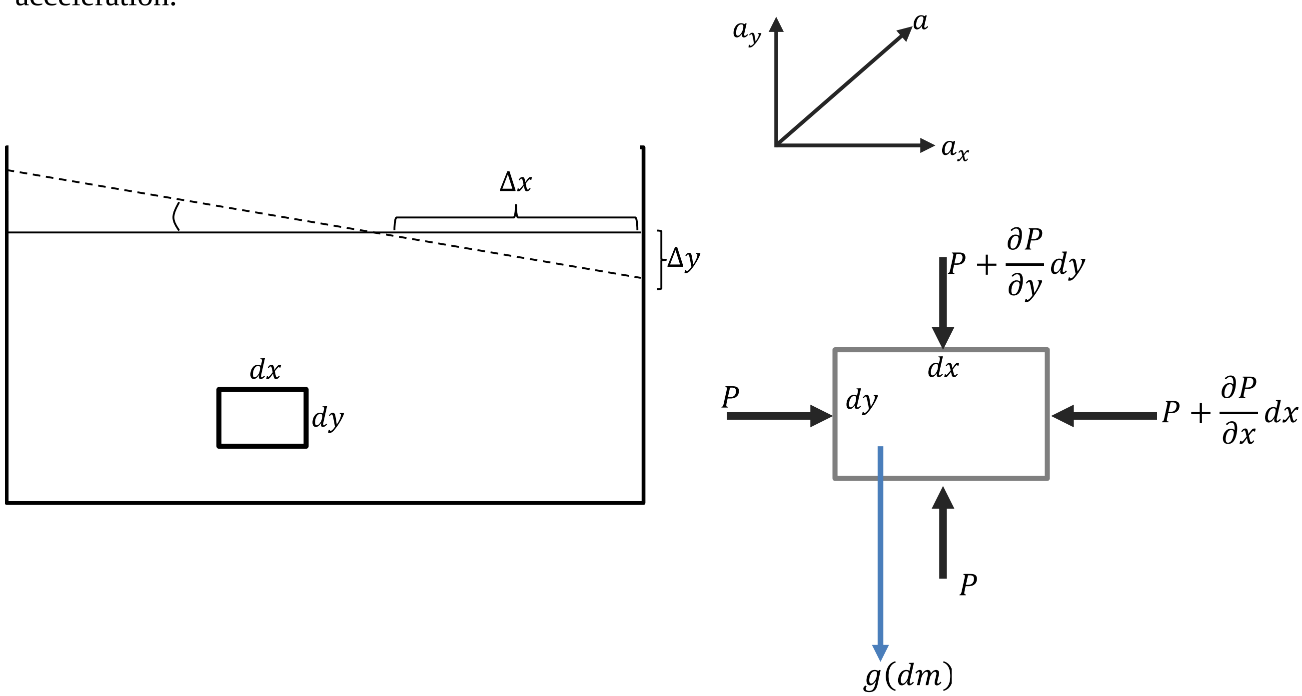


Chapter-5 Accelerated Fluid

When a fluid mass is moving with constant acceleration, we assume no relative motion between the fluid layers, i.e. no shear stress.

1- <u>Linear motion with constant acceleration.</u>

Assume a fluid in a vessel (of unit width), the vessel is moving with constant acceleration.



<u>Equation of Newton 2nd law in x-direction</u>

$$ma_x = \sum F_x$$

$$d_m a_x = P d_y - \left(P + \frac{\partial P}{\partial x} dx\right) dy$$

$$d_m a_x = -\frac{\partial P}{\partial x} dx dy$$

$$\therefore a_x = -\frac{1}{\rho} \frac{\partial P}{\partial x} \qquad (1)$$

Equation of Newton 2nd law in y-direction

$ma_y = \sum F_y$

$d_m a_y = Pd_x - \left(P + \frac{\partial P}{\partial y}dy\right)dx - gdm$

$$d_m a_y = -\frac{\partial P}{\partial y}dxdy - gdm$$

$dm = \rho(dx * dy * 1)$

$$\therefore a_y = -g - \frac{1}{\rho}\frac{\partial P}{\partial y} \qquad (2)$$

Note: if $a_y = 0$, the pressure along y direction will vary hydrostatically i.e. P = γh.

But, $dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$

Hence, from equations (1) and (2),

$$\boldsymbol{dP = -\rho a_x d_x - (\rho g + \rho a_y)dy} \qquad (3)$$

The line of constant pressure, can be found from the above equation, by setting $dP = 0$

⇨ $\rho a_x d_x = -\rho(g + a_y)dy$

$$\therefore \frac{dy}{dx} = -\frac{a_x}{g + a_y} \quad \text{(negative slope).}$$

The line of constant pressure is free surface itself.

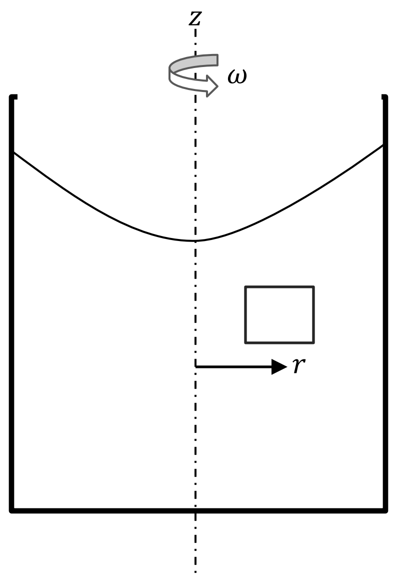
2- Rotation with constant acceleration

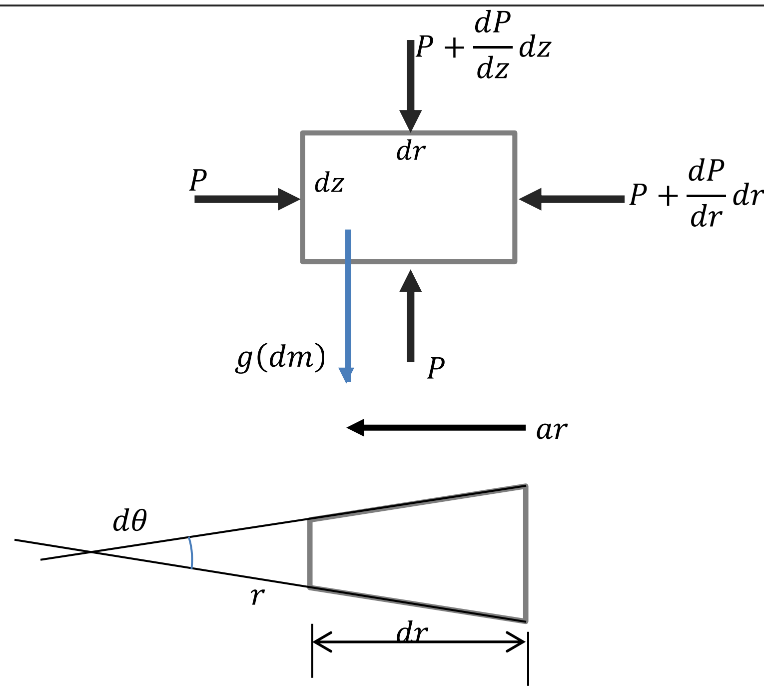
Assumptions:

- No pressure variation with θ direction
- The horizontal rotation will not alter the pressure distribution in the vertical direction (i.e. the pressure equals to $P = \gamma h$).

Applying Newton's 2nd low in r-direction:

$-ma_r = \sum F_r$





$$-dm\, a_r = \sum F_r$$

$$-\rho r d\theta dr dz\, a_r = P r d\theta dz - \left(P + \frac{\partial P}{\partial r} dr\right) dz\, r\, d\theta$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, a_r$$

$$a_r = r\omega^2$$

$$\therefore \frac{\partial P}{\partial r} = \rho\, r\omega^2 \qquad (1)$$

$$-ma_z = \sum F_z = 0\,, \quad a_z = 0$$

$$P\, rd\theta\, dr - \left(P + \frac{\partial P}{\partial z} dz\right) r\, dr\, d\theta - \rho\, r\, dr\, d\theta\, dz\, g = 0$$

$$\therefore \frac{\partial P}{\partial z} = -\rho\, g \qquad (2)$$

But, $dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$

$$\boldsymbol{dP = \rho\, r\, \omega^2 dr - \rho g\, dz} \qquad (3)$$

on the free surface, dP = 0 .

$$\omega^2\left(\frac{r_2^2}{2}-\frac{r_1^2}{2}\right) = g(z_2 - z_1)$$

If we put point 1 at the z-axis so that $r_1 = 0$

$$\omega^2 \frac{r_2^2}{2} = g(z_2 - z_1)$$ Equation of Parabola.

<u>*Example 5.1*</u>

The tank shown in Fig. 1a is accelerated to the right. Calculate the acceleration ax needed to cause the free surface shown in Fig. 1b to touch point A. Calculate also the pressure at point B.

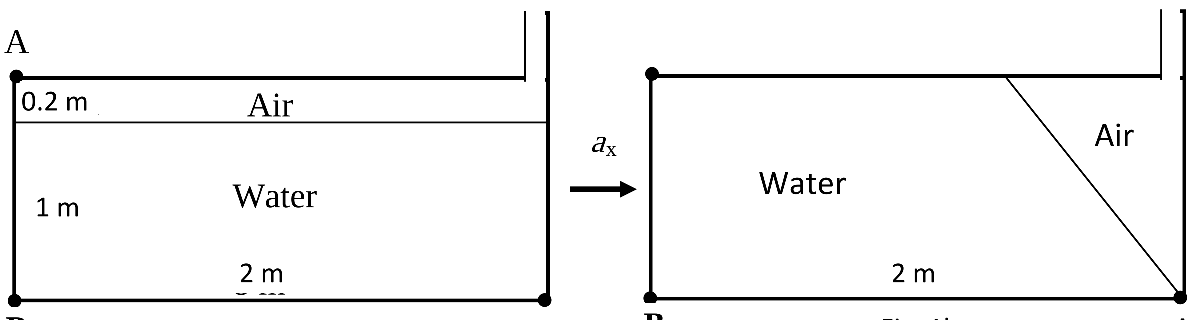
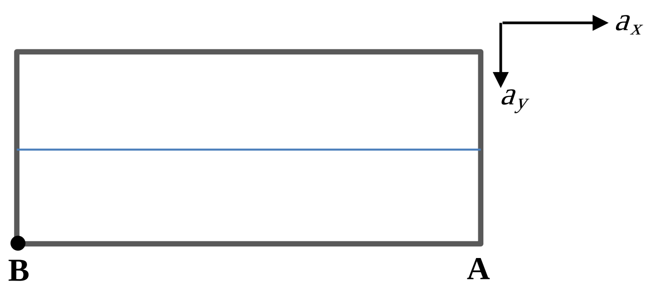


Fig. 1a Fig. 1b

Example 5.2

A closed box with horizontal base of 6x6 m and height of 2 m is half filled with water. It is given a_x =g/2 and a_y=-g/4. Find the pressure at point b as shown.

`



<u>*Example 5.3*</u>

A water-filled cylinder is rotating about its center line. Calculate the rotational speed that is necessary for the water to just touch the origin and the pressures at A and B.