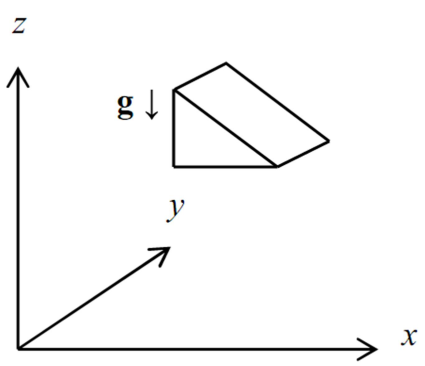
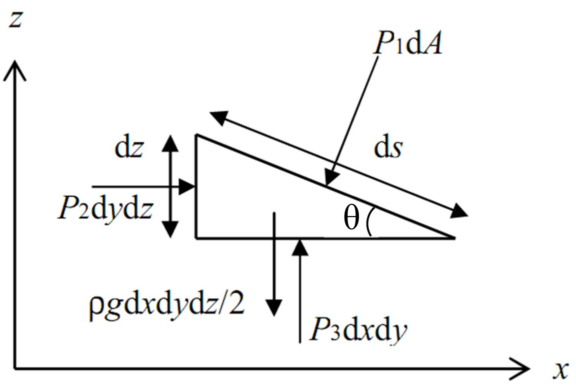
Chapter 2- Fluid Statics

Pressure acting on a point

It can be proven that the pressures acting on a point at rest, has the same value in all directions. Let us assume a particle of a fluid at rest, with free body diagram shown in figure.





$$dA = ds \cdot dy = dy \cdot dz/\sin\theta$$

$$\sum \mathbf{F} = 0$$

$$F_x = P_2 \mathrm{d}y\mathrm{d}z - P_1 \mathrm{d}A \sin\theta = 0$$

$$P_2 \mathrm{d}y\mathrm{d}z = P_1 \mathrm{d}y \frac{\mathrm{d}z}{\sin\theta} \sin\theta$$

$$P_2 = P_1$$

$$F_z = P_3 \mathrm{d}y\mathrm{d}x = \frac{1}{2}\rho g \mathrm{d}x\mathrm{d}y\mathrm{d}z + P_1 \mathrm{d}y \frac{\mathrm{d}x}{\cos\theta} \cos\theta$$

$$P_3 = P_1 + \frac{1}{2}\rho g \mathrm{d}z$$

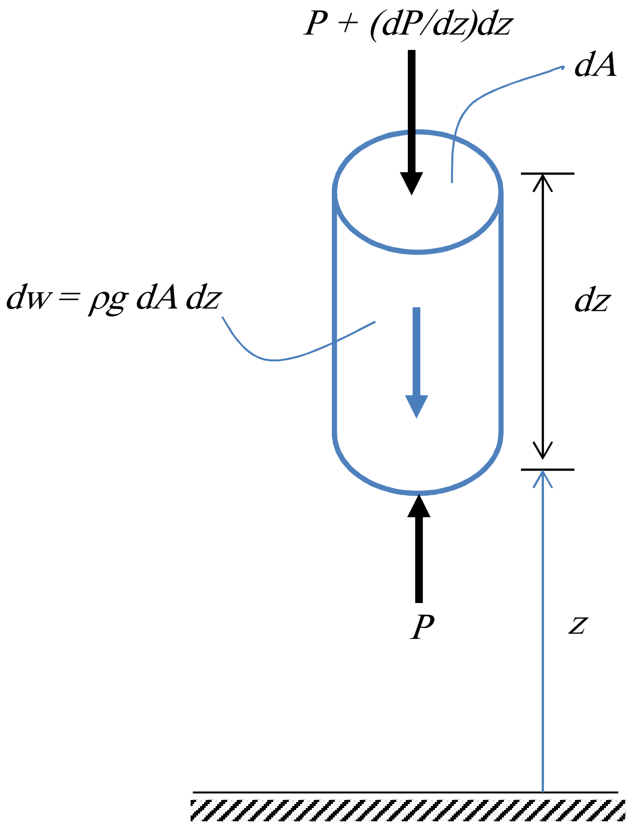
$$\mathrm{d}z \to 0,\ P_3 = P_1$$

$$\therefore P_1 = P_2 = P_3$$

Pressure variation with depth

Assuming a small element with a cross sectional area dA and length dz. The upward acting pressure is P and the downward acting pressure is $P + \frac{dP}{dz}dz$.

The force balance gives:



$$PdA - \left(P + \frac{dP}{dz}dz\right)dA - \rho g dA dz = 0$$

$$\frac{dP}{dz}dzdA = -\rho g dA dz$$

$$\therefore dP = -\rho g dz$$

$$P = -\rho g \int dz$$

$$P = -\rho g z + c$$

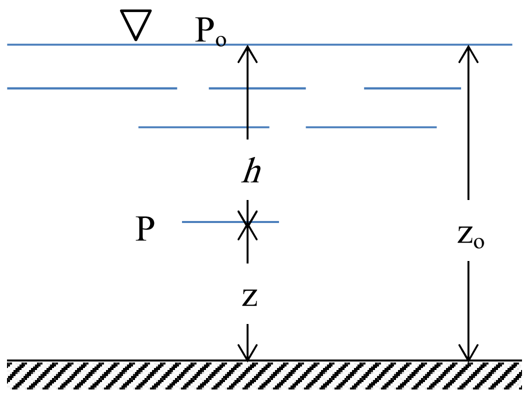
To find the constant c, we need a pressure value at a known elevation.

At $z = z_o$, $P = P_o$

$$\therefore c = P_o + \rho g z_o$$

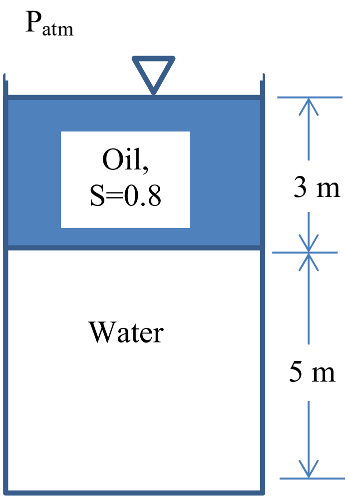
$$\therefore P = P_o + \rho g (z_o - z)$$

$$P = P_o + \rho g\, h$$

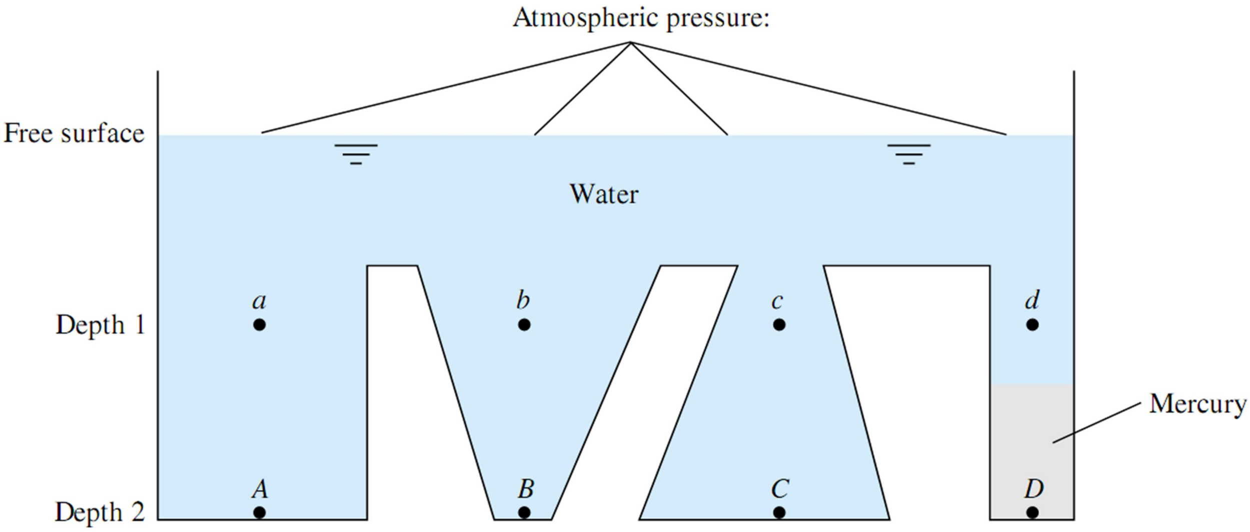


Example 2.1 Determine the pressure of sea water at 10 m under sea level. Given the sea water density as 1020 kg/m^3. Consider the value of atmospheric pressure as 101.3 kPa.

Example 2.2 Determine the pressure at the base of the tank shown in figure below.



Note: Pressure doesn't vary horizontally, provided that the fluid is connected. To illustrate this statement, we may refer to the figure below.



Points *a*, *b*, *c*, and *d* are at equal depths in water and therefore have identical pressures. Points *A*, *B*, and *C* are also at equal depths in water and have identical pressures higher than *a*, *b*, *c*, and *d*. Point *D* has a different pressure from *A*, *B*, and *C* because it is not connected to them by a water path.

<u>*Example 2.3:*</u> For the closed tank shown in figure, the pressure at point *A* is 95 kPa absolute, what is the absolute pressure at point *B*?