Relationship between Data Rate and Harmonics

- Given a bit rate R [bps], the time required to send one bit,

$$T_{bit} = \frac{1}{R}$$

- Total Transmission Time for pattern of 8 bits→

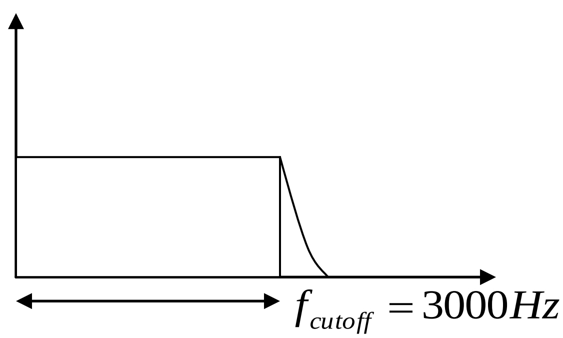
$$T_{8bit} = \frac{8}{R} \qquad \text{.......................................(1)}$$

- The frequency of the 1st Harmonic → $1^{st}\,\text{Harmonic} = \dfrac{Bit\ \ Rate}{8}$ ....(2)

- No. of the Harmonics passed = $\dfrac{Bandwidth}{R/8}$ ……(3)

Example (1):

Consider an ordinary telephone-line (voice-grade line) with cut-off frequency just above 3000Hz is used to send an 8 bit pattern (ASCII) at bit rate of 2400 bps.



Solution:

$$T_{8bit} = \frac{8}{R} = \frac{8}{2400} = 3.33ms$$

$$1^{st}\,\text{Harmonic} = \frac{R}{8} = \frac{2400}{8} = 300Hz$$

The restriction of cut-off frequency (3000Hz) means that the highest No. of harmonics passed through is roughly:

$$\frac{Bandwidth}{R/8} = \frac{3000}{300} = 10$$

(i.e. cut-off is not sharp)

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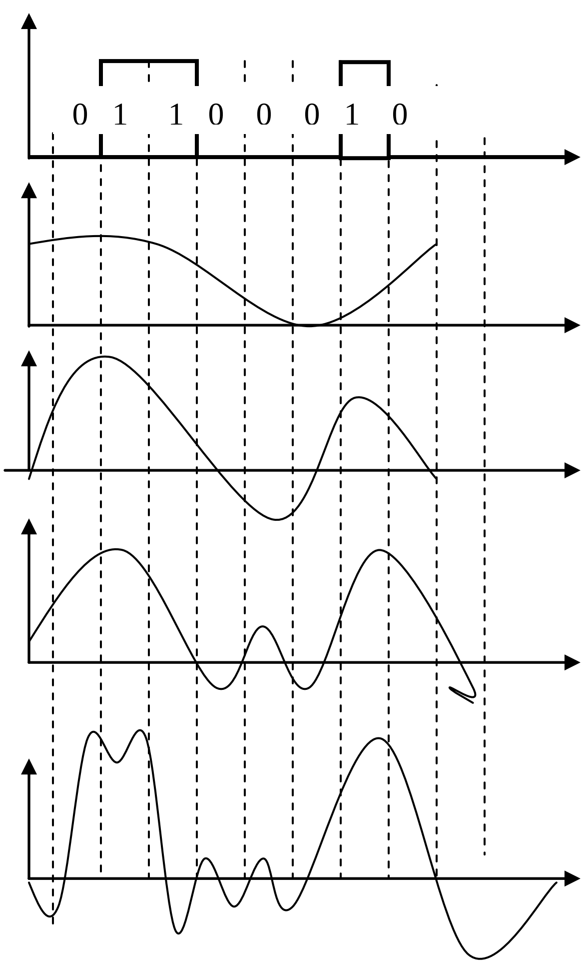
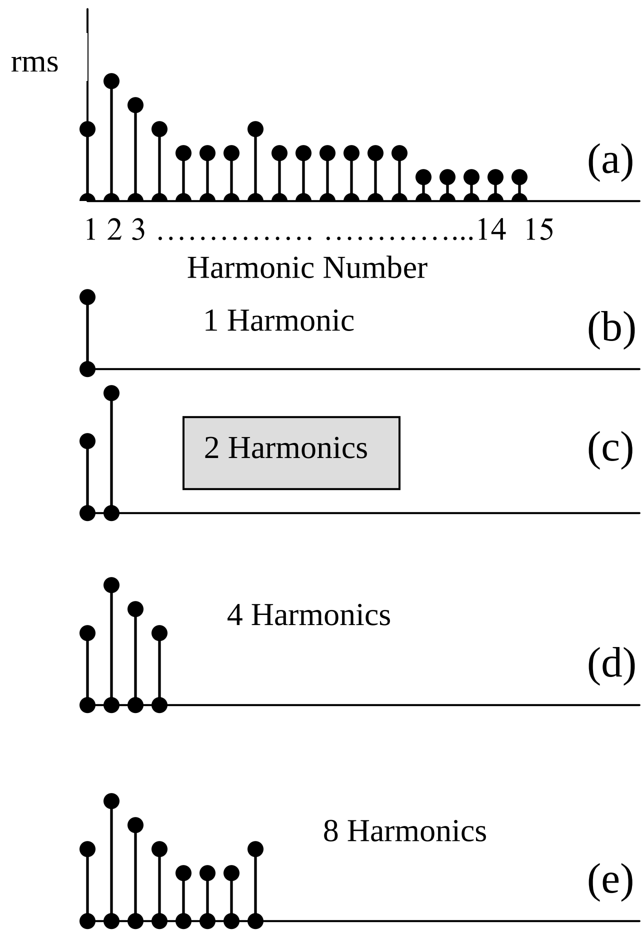
R [bps]	**T (ms)**	**1st Harmonic (Hz)**	**# Harmonics Sent**
300	26.67	37.5	80
600	13.33	75	40
1200	6.67	150	20
2400	3.33	300	10
4800	1.67	600	5
→ 9600	0.83	1200	2
19200	0.42	2400	1
38400	0.21	4800	0

Example (2): When R=9600bps is used to transmit the pattern sequence of 8 bits (**01100010**)← ASCII character "b". Show the required harmonics to be sent.





Note:
Signals in (b) - (e) are approximations to the Original Signal.
(b) less quality (c) & (d) good quality (signal representation) (e) higher quality

Note:
When Data rate higher than 38.4kbps→means There is no hope at all for sending binary signals (i.e. limited bandwidth)

Exercise:

Given bit rate 2000bps and pattern sequence of 8 bits 10111101 is sent with different medium bandwidths. Change the bandwidth: 500Hz, 900Hz, 1300Hz, 1700Hz, 2500Hz and 4000Hz, then find which bandwidth provides good quality for given pattern.

Sketch the expected harmonics sent for each bandwidth.

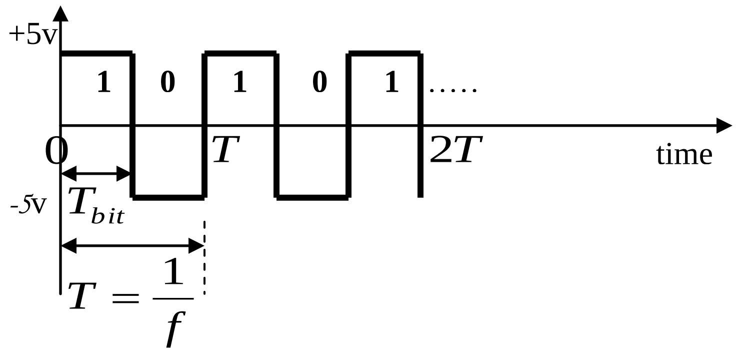
Hint: Assume $B_{max}=2R$ $B_{min}=R/2$

Note: High quality (best), good quality, poor quality, very poor quality (worst)→ depends on No. of Harmonics sent.

Increase No. of Harmonic sent → means increase signal quality (signal representation).

Data Rate vs. Bandwidth and Harmonics

Example (1): A square-wave signal of 1MHz is used to transmit a data bit sequence 10101010….etc. as follows:



Find:

1- The bit rate (R) in bps.

2- The required medium bandwidth (B_w) to send this sequence.

Solution:

$T = \frac{1}{f}$ then the required bandwidth in (Hz), $B_w = f = 1\text{MHz}$

$T_{bit} = \frac{1}{R}$ $T_{bit} = \frac{T}{2} = \frac{1}{2f}$

Hence, the bit rate (data rate) $R = 2f = 2B_w = 2 \times 1\times10^6 = 2$ Mbps.

Example (2):

Now Show the 1st, 3rd and 5th harmonics for this signal and find the bit rate for the following three Cases:

Case I : f=1MHz B_w=4MHz

Case II : f=2MHz B_w=8MHz

Case III: f=2MHz B_w=4MHz

f = Signal frequency B_w= Medium Bandwidth

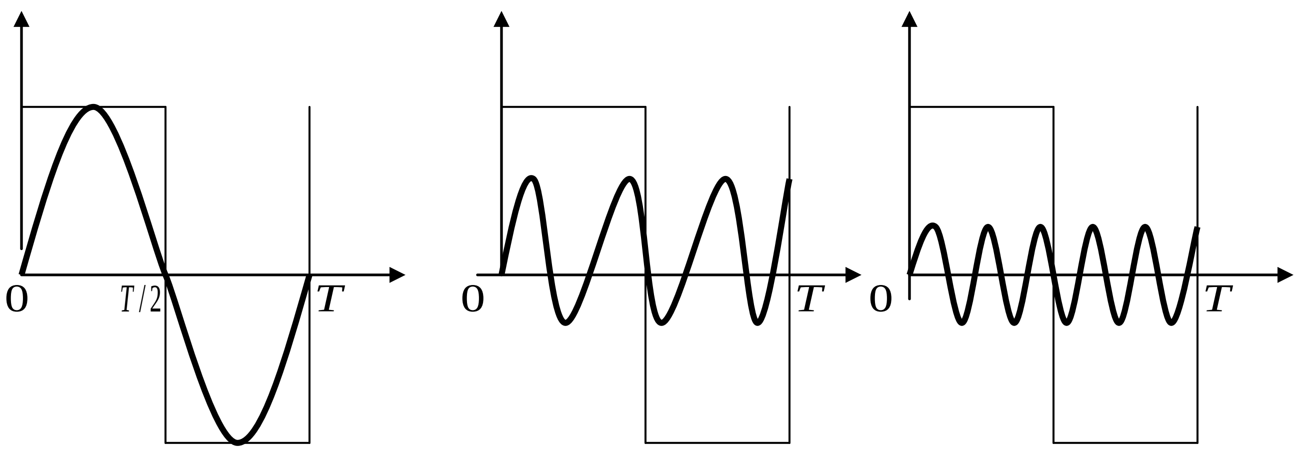
And give your comments on signal quality (representation) for each case.

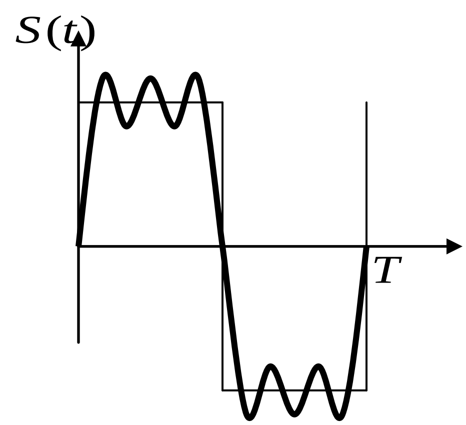
Solution:

The square-wave signal S(t):

$$S(t) = \frac{4A}{\pi}\sum_{K=1}^{M}\frac{1}{K}\sin(2\pi Kft)$$

$$= \underbrace{\frac{4A}{\pi}\sin(2\pi ft)}_{1^{st}\ \text{Harmonic}} + \underbrace{\frac{4A}{3\pi}\sin[2\pi(3f)t]}_{3^{rd}} + \underbrace{\frac{4A}{5\pi}\sin[2\pi(5f)t]}_{5^{th}}....$$





Case I : f=1MHz B_w=4MHz

Using S(t) formula: $B_w = f_h - f_L$

$$B_w = 5f - f = 5\times1\times10^6 - 1\times10^6 = 4\text{MHz}$$

- The Bit Rate:

$$T_{bit} = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2\times1\times10^6} = 0.5\mu s$$

$$R = \frac{1}{T_{bit}} = \frac{1}{0.5\times10^{-6}} = 2\text{Mbps}$$

Case II : f=2MHz B_w=8MHz

We also use frequencies from f to $5f$:

$$B_w = 5f - f = 5\times 2\times 10^6 - 2\times 10^6 = 10 - 2 = 8\text{MHz}$$

- The Bit Rate:

$$T_{bit} = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2\times 2\times 10^6} = 0.25\mu s$$

$$R = \frac{1}{T_{bit}} = \frac{1}{0.25\times 10^{-6}} = 4\text{Mbps}$$

Note: Doubling B_w means doubling data rate.

Case III: f=2MHz B_w=4MHz

$$B_w = 3f - f = 3\times 2\times 10^6 - 2\times 10^6 = 6 - 2 = 4\text{MHz}$$

- The Bit Rate:

$$T_{bit} = \frac{T}{2} = \frac{1}{2f} = \frac{1}{2\times 2\times 10^6} = 0.25\mu s$$

$$R = \frac{1}{T_{bit}} = \frac{1}{0.25\times 10^{-6}} = 4\text{Mbps}$$

	f	B_w	Bit Rate (R)	
Case I	1MHz	4 MHz	**2 Mbps**	$B_w=5f-f$
Case II	2MHz	**8 MHz**	**4 Mbps**	$B_w=5f-f$
Case III	2MHz	**4 MHZ**	**4 Mbps**	$B_w=3f-f$

Comments:

- **Case I & Case II** are with more signal quality
- Case III is with less quality (signal representation).

Exercises:

1. A voice has spectrum 300-3300Hz, if a transmission bandwidth is 1000Hz (between 2500 & 1500Hz). What happens to voice frequencies?

2. If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900Hz, what is the bandwidth? Draw spectrum, assume all components have maximum amplitude of 10v.

3. A signal has bandwidth of 150Hz. The highest frequency is 260Hz. What is the lowest frequency? Draw the spectrum and specify the bit rate of this signal if it is used to send a bit sequence of 1010101…..etc.

Relationship between BER and PER

We have: **BER**=Bit Error Rate (independent error bits at Physical Layer)

PER= Packet (Block) Error Rate for CRC used at Data-Link Layer

$$\boxed{\mathrm{PER} = \mathrm{P_B} = 1-(1-\mathrm{P_e})^N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

N = Packet Length (Size) in bits

$$\boxed{\mathrm{PER} = 1-(1-\mathrm{P_e})^N \cong \mathrm{N}\times\mathrm{P_e} \text{ if } \mathrm{N}\times\mathrm{P_e} < 1} \leftarrow \text{Less than 1} \ldots\ldots\ldots\ldots \quad (2)$$

Example (1): Determine the max. block size over a channel when it has BER of 10^{-4} and the probability of block containing error is 10^{-1}.

Solution:

$$\mathrm{P_B} = 1-(1-\mathrm{P_e})^N$$

$$10^{-1} = 1-(1-10^{-4})^N \quad \rightarrow N = ? \quad \rightarrow N \cong 1000\text{bits}$$

OR

$$\mathrm{P_B} = N\times\mathrm{P_e} \rightarrow 0.1 = N\times 10^{-4} \quad \rightarrow N = 1000\text{bits}$$ ←Packet or Block size

Example (2): Consider SNR=18.6dB for $P_e=10^{-5}$. If the signal is sent using telephone -line with bit-rate 1.544Mbps (T1 line). Compute the time required to occur error.

Repeat for SNR=21.6dB for $P_e=10^{-9}$

Solution:

$$\mathrm{BER} = \mathrm{P_e} = 10^{-5} \rightarrow \textit{for}\ \mathrm{SNR} = 18.6\mathrm{dB}$$

$$1\text{-} \quad \mathrm{P_e} = \frac{N_e}{N_t} = \frac{N_e}{R.t} \Rightarrow 10^{-5} = \frac{1}{1.544\times10^6\times t} \rightarrow \mathrm{t} = 0.065\mathrm{sec} = 65\mathrm{msec}$$

$$2\text{-} \quad \text{For SNR} = 21.6\mathrm{dB} \Rightarrow \mathrm{P_e} = 10^{-9}$$

$$10^{-9} = \frac{1}{1.544\times10^6\times t} \rightarrow \mathrm{t} = 650\mathrm{sec} = 11\text{minutes}$$ ← is more tolerable

<u>**Note**</u>: If SNR increases by 3dB, BER then decreases by 10^{-4}