Chapter 3 Part-2

Physical Layer

Transmission Impairments

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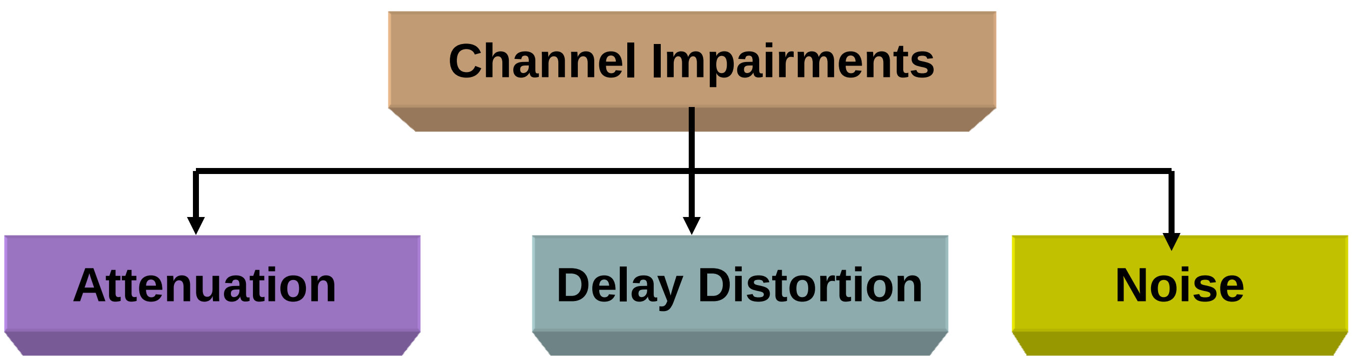
Transmission Impairments

- When Signals travel through transmission media (not perfect), the imperfection causes impairment in the signal.

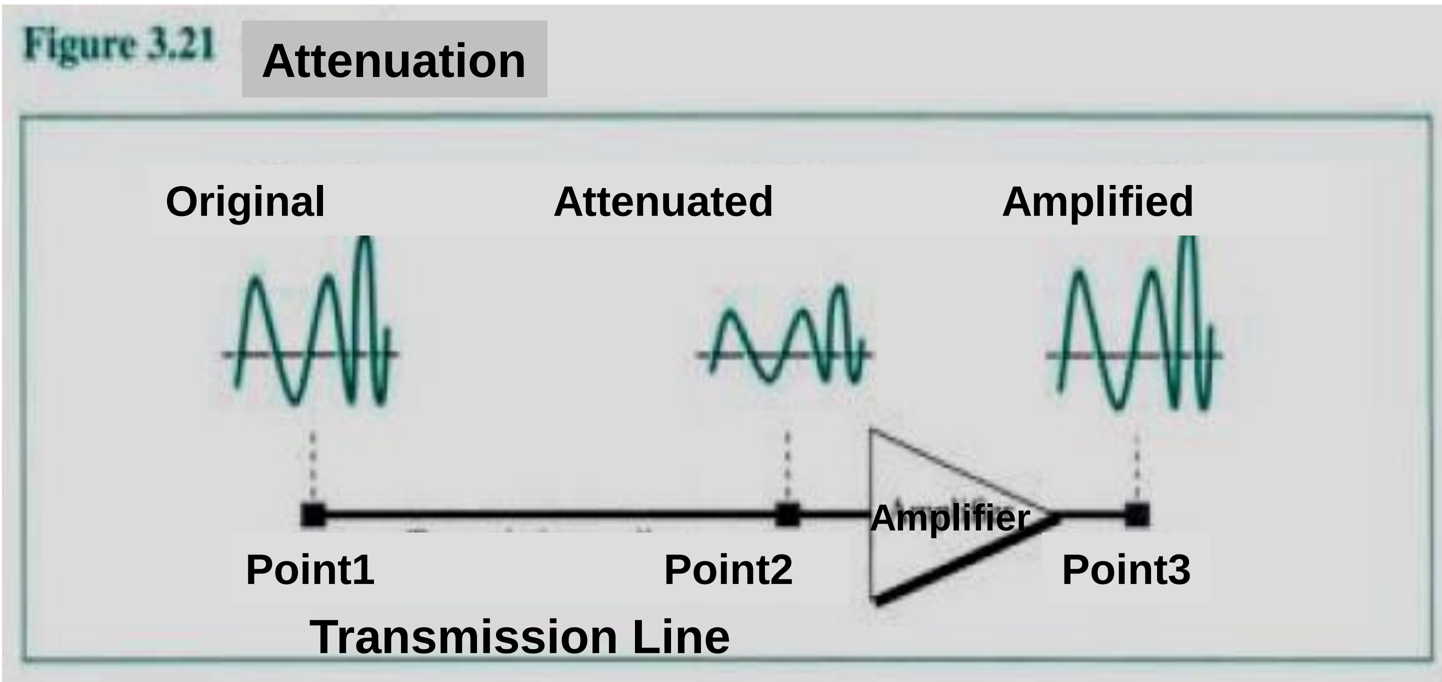
Figure 3.12 Signal Corruption

- **Three types** of impairments occur as follows:



1- **Attenuation**: (Loss of Energy ≡ Reduction in Signal Strength)

- When a signal travels through a medium, it loses some of its energy (because of wire resistance)→ a wire becomes warm (or hot).



- The Strength of Signal falls off with **distance** leads **reduction in signal strength** (attenuation) or called (loss) in dB/m or dB/km in **guided media**.

❑ **Decibel** (dB) measures the relative strengths of two signals or a signal at two different points.

❑ Note that **dB is (-*ve*) means Loss** (signal attenuation) and **(+*ve*)** value means **Signal Gain** (signal is amplified).

$$\boxed{G_{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\frac{P_{out}}{P_{in}}}$$

← Received Signal

← Transmitted Signal

Where P_1 and P_2 the powers of a signal at Point1 & Point2.

$$\boxed{L_{dB} = -10\log_{10}\frac{P_{out}}{P_{in}} = 10\log_{10}\frac{P_{in}}{P_{out}}}$$

$$\boxed{L_{dB} = -10\log_{10}\frac{P_{out}}{P_{in}} = -10\log_{10}\frac{V_{out}^2/R}{V_{in}^2/R} = -20\log\frac{V_{out}}{V_{in}}}$$

Example 1 Assume a signal travels through an amplifier and its power is increased ten times. This means that $P_2=10xP_1$. In this case, the **Amplification (Power Gain)** can be calculated as

$$\mathbf{10\log_{10}(P_2/P_1) = 10\log_{10}(10P_1/P_1) = 10\log_{10}(10) = 10dB}$$

Example 2

Assume a signal travels through a transmission medium and its power is reduced to half. Show the **Loss of Power (attenuation)** in dB.

$$\mathbf{10\log_{10}(P_2/P_1) = 10\log_{10}(0.5P_1/P_1) = 10\log_{10}(0.5) = -3dB}$$

- **Note: Engineers knows that -3dB or a loss of 3dB is equivalent to losing of the half power.**

Examples: Decibel Values

Power Ratio	….. 10^{-3}	10^{-2}	10^{-1}	$\mathbf{10^{0}}$	10^{1}	10^{2}	10^{3} ….
dB	-30dB	-20dB	-10dB	**0dB**	10dB	20dB	30dB

Exercise: If a signal with a power level of 10 mW is inserted onto a transmission line and the power measured power some distance away is 5mW, what is the loss in dB?

Decibel Values: [dBW or dB] and [dBm]

$$\boxed{Power_{dBW} = 10\log_{10}\frac{Power_w}{1\ W}}$$

$$\boxed{Power_{dBm} = 10\log_{10}\frac{Power_w}{1\ mW}}$$

Examples:

$$Power_{dBW} = 10\log_{10}\frac{10\ watt}{1\ W} = 10dBw$$

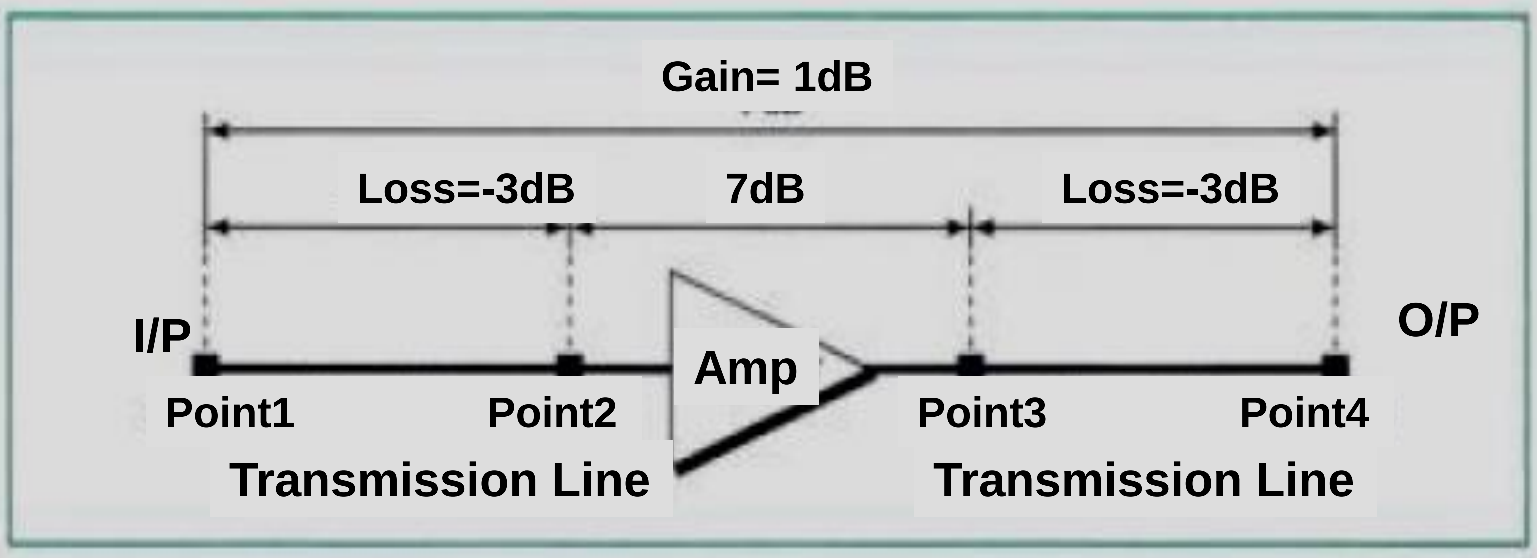
$$Power_{dBm} = 10\log_{10}\frac{10\ watt}{1\ mW} = 10\log 10^{+4} = 40dBm$$

❖ **Note that**: $\boxed{0dBW = +30dBm}$ $\boxed{0dBm = -30dBW}$

Power in [W]	0.001	0.01	0.1	1W	10	100	1000
dB	-30dB	-20dB	-10dB	0dB	10dB	20dB	30dB
dBm	0dBm	10dBm	20dBm	$+30$dBm	40dBm	50dBm	60dBm

$$\boxed{(\ldots)_{dBm} = (\ldots)_{dBW} + 30dBm}$$

Example 14 If $P_{in} = 4mW$ What is the output power at Point 4?



$$G_{dB}(or\ \ L_{dB}) = -3dB + 7dB - 3dB = +1dB \quad \longleftarrow \quad (+ve) \text{ Signal Gain at Point 4}$$

$$+1dB = 10\log(P_{out} / P_{in}) = 10\log(P_{out} / 4mW)$$

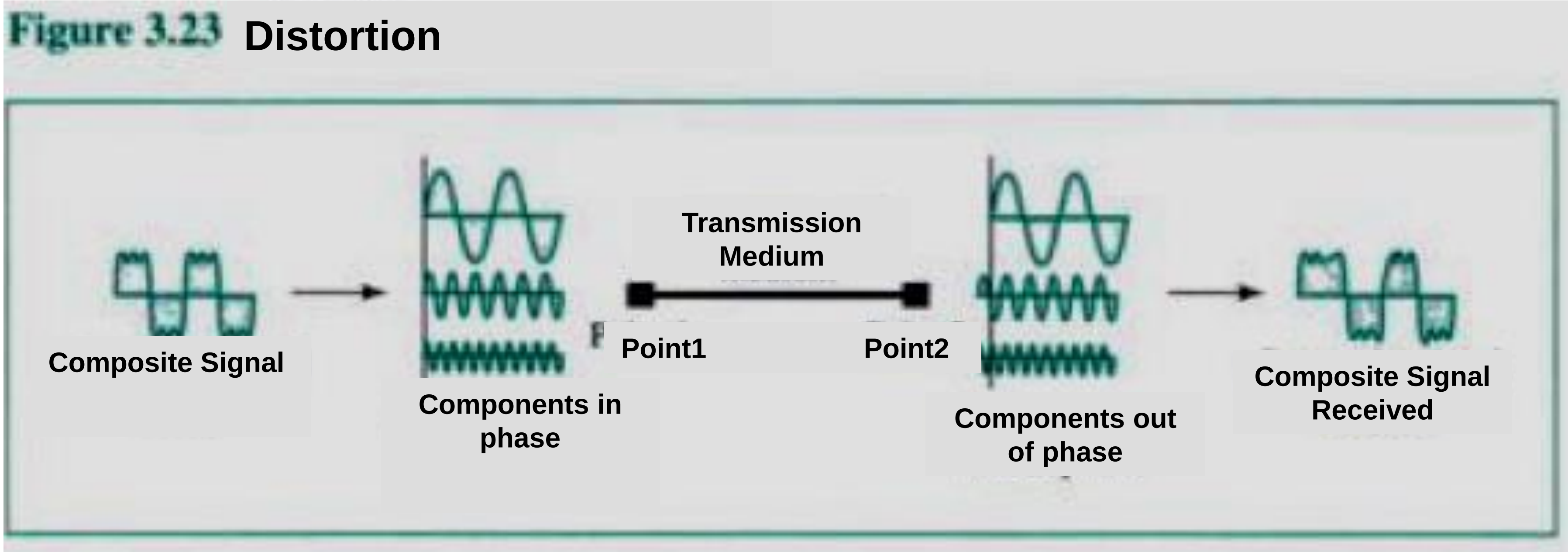
$$P_{out} = 4mW \times 10^{0.1} = 4 \times 10^{-3} \times 1.2589 = 5.0357mW$$

❑ **Exercise:** Find dBW and dBm values for the following powers: 1W, 3W, 5W, 100W, 0.3W, , 0.3mW, 3mW, 10mW, 60mW, 100mW.

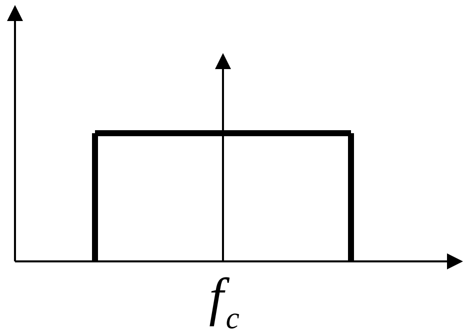
❑ **Exercise**: If power is -90dBm what is the power in Watt?

2- Delay Distortion:

- Signal changes its shape because of signal propagation speed



- For **band-limited signal,** the velocity is **higher near the center frequency** and fall off toward the two end of the band.



- Frequency Components arrive at RX at <u>different times</u> resulting <u>phase shifts</u> between different frequencies → **is called Delay Distortion.**

Solving this problem by Equalization filter

Equalization Filter: is used to reduce delay distortion.

Example: The attenuation of coaxial cable at frequency of f MHz is given by

$$\boxed{A(f) = 11\text{x}\ f^{\,0.6}\ \text{dB/km}}$$

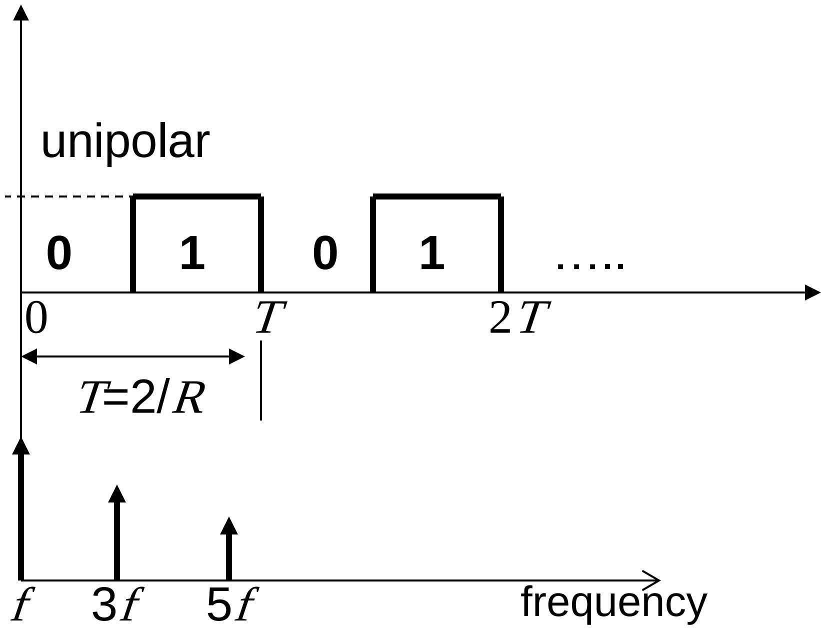
If bits are transmitted in unipolar modulation at rate of R [bps], then max. freq. in the signal occurs when the bits **0s and 1s alternate**. In this case, the signal is square wave with period T=2/R. Assume $\boldsymbol{R}$**=1Mbps**.

Find the attenuation of coaxial cable at different frequency components of the square wave: f=1/T=R/2, 3f=3/T=3R/2 , 5f=5/T=5R/2…etc

R=1Mbps

$$A(f) = 11\text{x}\ (R/2)^{0.6}\ = 7\ \text{dB/km}$$

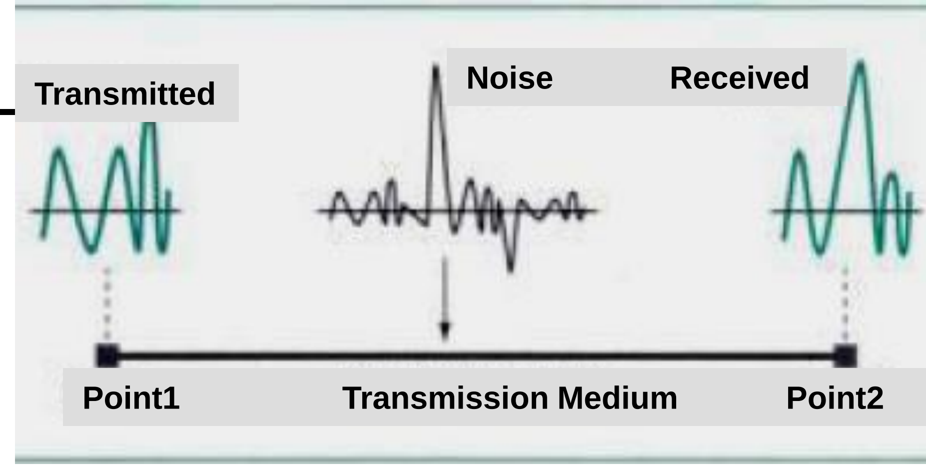
$$A(5f) = 11\text{x}\ (5R/2)^{0.6}\ = 19\ \text{dB/km}$$



If coaxial cable has a length of 1km, the two attenuations **differ by 12dB**.

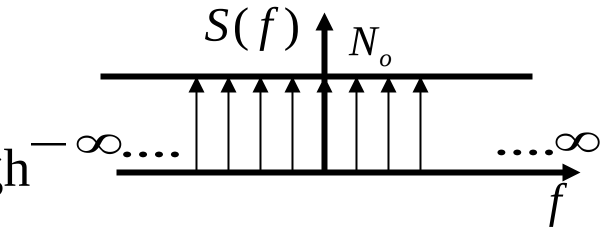
→ This range must be compensated by **Equalizer Filter**.

3- The Noise: Four Types



(1) Thermal Noise
(2) Inter-Modulation Noise (Induced Noise)
(3) Crosstalk
(4) Impulse Noise

(1) Thermal Noise (Gaussian Noise):



- Electrons induce a randomly varying current through a resistance.
- It is called **White Noise**, i.e. it has a **Flat Power spectral density** $S(f)$ over a wide rang of frequencies.

❑ In any device (conductor), the amount of Thermal Noise in a Bandwidth of 1 Hz

$$\boxed{N_o = kT \qquad \text{in (W/Hz)}}$$

$$\boxed{C^o = (F^o - 32)/1.8}$$

$$\boxed{K = C^o + 273}$$

N_o : Noise power density in watts per 1Hz of bandwidth

k : Boltzmann's constant =1.38x10^{-23}J/K ← K= Kelvin

T : Temperature (in Kelvin) absolute temperature,

Example 1: A room temperature is specified as T=17°C (or 290 K). Compute the thermal noise power.

Solution:

$$N_o = (1.38\times10^{-23})\times 290 = 4\times10^{-21}(W/Hz) = -204 dBW/Hz$$

❑ **Thermal Noise in watts present in a Bandwidth B (Hz) can be:**

$\boxed{N = kTB}$ in [Watt] **or** $\boxed{N = 10\log k + 10\log T + 10\log B}$

$\boxed{N = -228.6 dBW + 10\log T + 10\log B}$ in [dBW]

Example 2:

Given a receiver with effective noise temperature of 294K and 10MHz bandwidth, compute the thermal noise level at the receiver output.

$$N = -228.6 dBW + 10\log(294) + 10\log 10^7$$

$$= -228.6 + 24.7 + 70 = -133.9 dBW$$

(2) Inter-Modulation Noise:

❑ When mixing of signals at different frequencies f_1 and f_2 , it may produce energy at frequency ($f_1 + f_2$).

(3) Crosstalk (Interference):

❑ It is unwanted coupling between signal paths (i.e. Signals-Cross) (e.g. in twisted pairs)

(4) Impulsive Noise:

❑ High amplitude noise bursts interrupt the long interval of the signal.
Example: Noise from lighting, transients caused high voltage switching.

Note: Typical transmission lines BER=10^{-6} or 10^{-7}

For **optical transmission channel** BER≤10^{-9}

Thermal Noise (WG)

Shot Noise: Discrete electrons generating ripple-current in the receiver

Both cause Transmission errors→ BER

Receiver Sensitivity: **BER= Bit-Error Rate**

- ❑ It is performance measurement of digital receiver.

- ❑ In practice, there are **several ways** to measure the rate of error occurrence in digital data stream. Like **BER , SNR**, **E_b/N_o, P_e**….etc.

- ❑ **BER**: is a division of no. of errors (N_e) occurring over a certain time interval (t) by the total no. of pulses (ones & zeros) (N_t) transmitted during this interval.

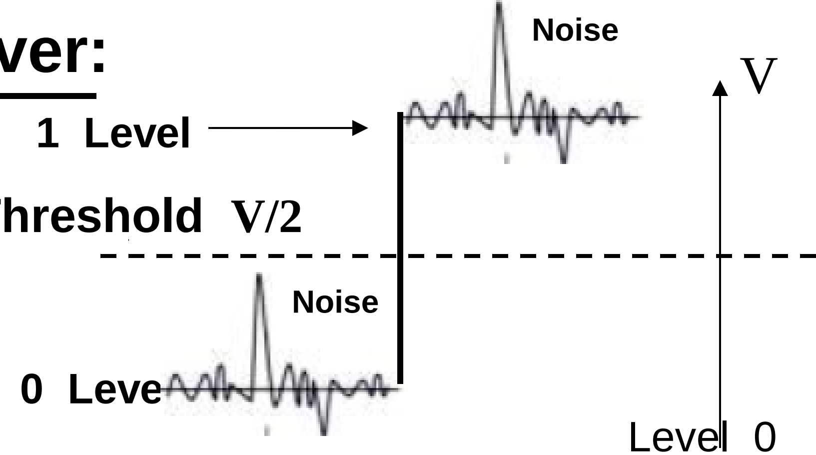
$$\boxed{BER = \frac{N_e}{N_t} = \frac{N_e}{R.\ t}}$$

$$\boxed{R = \frac{1}{T} = Bit\ \ Rate}$$

<u>Notes</u>:

- ➢BER=10^{-6} → i.e. average one error occurs for every million bits sent
- ➢This BER depends on S/N ratio (SNR) at the receiver, so that at RX, **BER and Noise** must be at the lower-limits.
- ➢**BER**=Probability that a bit is incorrectly detected by the receiver.

❑ Calculation of BER at the Receiver:



- $BER = P_e$ = **Probability of Error** (due to Additive Noise)

$$P_e = \frac{1}{2}\left(1 - erf\left(\frac{V}{2\sqrt{2}\sigma}\right)\right)$$

$$erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}dy$$

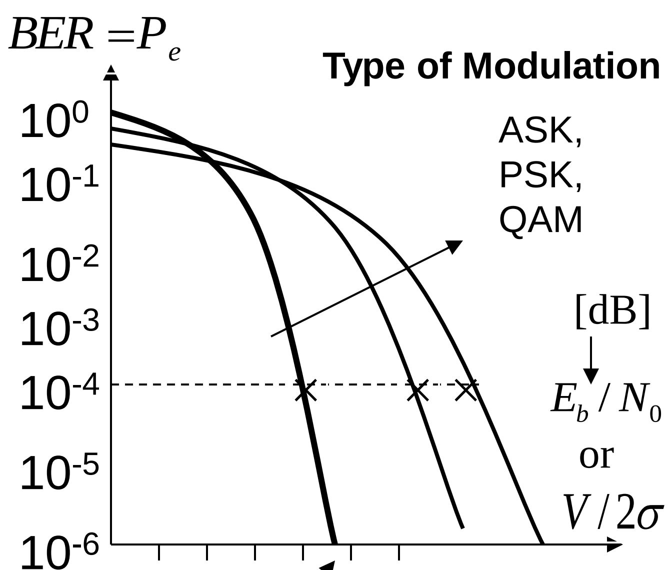
Error-function

- $(S/N)_{dB}$ **=Signal-to-Noise Ratio (in dB)**

$$\boxed{(S/N)_{dB} = 20\log\frac{V}{2\sigma}} \qquad \boxed{\frac{S}{N} = \frac{V}{2\sigma}}$$

$\frac{V}{\sigma} =$ Ratio of Signal Amplitude V to the Standard deviation σ of the noise

$$\sigma = r.m.s.\ noise$$
$$V/\sigma = Peak\ \ Signal - to - r.m.s\ noise\ ratio$$



$$e.g.\ P_e = 10^{-6} \rightarrow \frac{V}{2\sigma} = \underline{4.75} \rightarrow \frac{V}{\sigma} = 9.5$$

The Expression E_b / N_0

SNR_{dB}=10 $\log_{10}$ (Signal/Noise)

From Shannon Theory → **SNR** is used to determine the **Data Bit Rate** and **Error Rate (BER) where SNR is** standard measure for digital communication system performance.

E_b / N_0 = Signal-Energy per Bit to Noise-power per Hz

❖ Now Consider a Signal, digital or analog, contains digital data transmitted at a certain bit rate R [bps]. Recalling **1Watt=1J/s**, Energy per Bit is given by $\boxed{E_b = ST_b}$ where:

$$S = Signal\ \ Power\ \ [watt]\ \&\ \ T_b = 1/R$$

Thus,

$$\boxed{\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{kTR}}$$

OR

$$\boxed{\left(\frac{E_b}{N_0}\right)_{dB} = S_{dB} - 10\log R - \underbrace{10\log k}_{-228.6\ dBW} - 10\log T}$$

in dB Notation

Example:

Consider BPSK with E_b/N_o=8.4dB is required for a bit error rate BER=10^{-4}. If the effective noise temperature is 290°K (room temperature) and the data rate is 2400bps, what the received signal level is required?

Solution:

$$\boxed{\frac{E_b}{N_0}=\frac{S/R}{N_0}=\frac{S}{kTR}}$$

$$\boxed{\left(\frac{E_b}{N_0}\right)_{dB}=S_{dB}-10\log R-10\log k-10\log T}$$

$$8.4=S_{dB}-10\log 2400+228.6dB-10\log 290$$

$$S_{dB}=-161.8dB$$

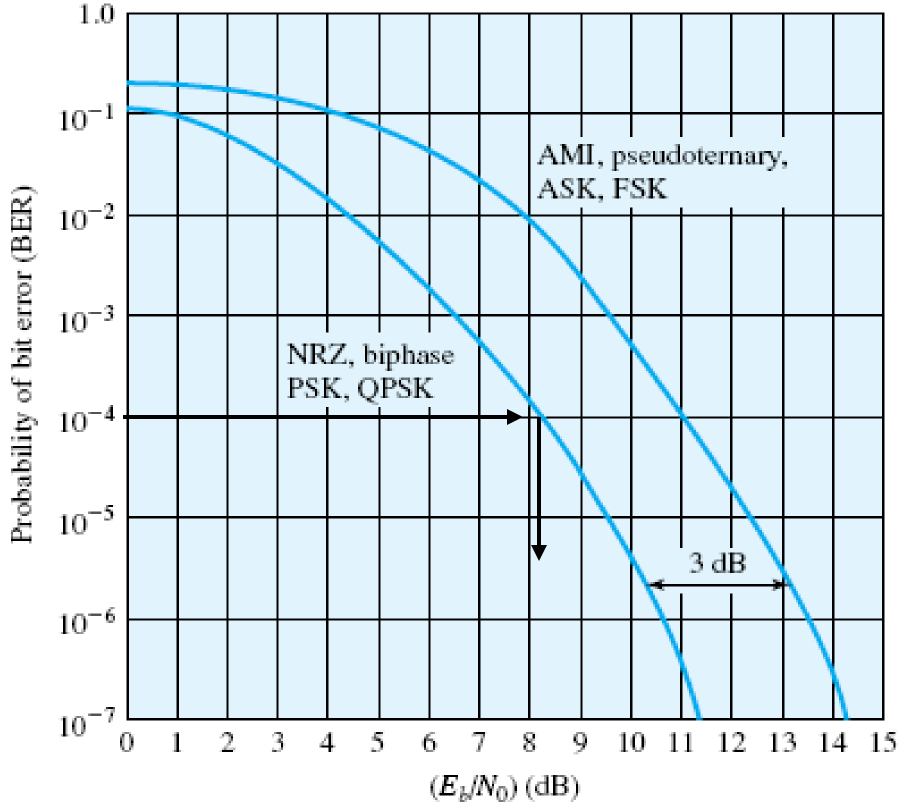
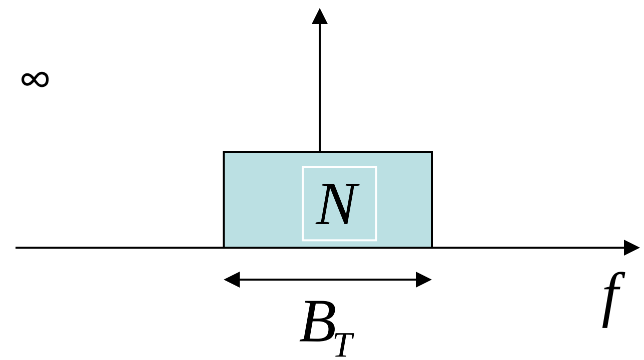


Figure 5.4 Theoretical Bit Error Rate for Various Encoding Schemes

Relationship between E_b / N_0 and $SNR = S / N$

We have: $$\boxed{\frac{E_b}{N_0} = \frac{S/R}{N_0} = \frac{S}{N_0 R}}$$ ← For Bandwidth ∞ ....(1)



N_0 =Noise power in W/Hz.

Let N = Noise with Limited Bandwidth B_T then $N = N_0 B_T$. By Substituting

$$\boxed{\frac{E_b}{N_0} = \frac{S}{N} \times \frac{B_T}{R}}$$ ← For Limited Bandwidth .... (2)

➢Capacity C=B $\log_2$(1+SNR)

Another Formula: E_b / N_0 and C / B using Shannon Theory

$$\boxed{\frac{S}{N} = 2^{C/B} - 1}$$ ....(3)

⇨ From (2) & (3), Assume $B = B_T$ & $R = C$ then

$\eta = C / B$ ←is **called Spectral Efficiency in [bps/Hz]**

$$\boxed{\frac{E_b}{N_0} = \frac{B}{C}\left(2^{C/B} - 1\right)}$$ ....(4)

Example:

Suppose we want to find minimum E_b / N_0 required to achieve a spectral efficiency of 6 bps/Hz.

Solution:

$$\frac{E_b}{N_0} = \frac{B}{C}\left(2^{C/B} - 1\right) =>$$

$$\eta = \frac{C}{B} = 6 bps / Hz$$

Then, the minimum E_b / N_0

$$\frac{E_b}{N_0} = \frac{1}{6}\left(2^6 - 1\right) => 10.5 => 10\log(10.5) = \ 10.21 dB$$

Home Work:

1. What is the thermal noise level of a channel with a bandwidth of 10kHz carrying 1000 watts of power operating at $50^{o}C$?
2. Given an amplifier with an effective noise temperature of $10{,}000^{o}K$ and a 10MHz bandwidth, what is the expected thermal noise level at its output?
3. What is the channel capacity for a teleprinter channel with 300Hz bandwidth and a signal-to-noise of 3 dB, where the noise is white thermal noise?
4. A digital signaling system is required to operate at 9600bps: (a) If a signal element encodes a 4bit word, what is the minimum required bandwidth of the channel? (b) Repeat (a) for the case of 8-bit word.
5. Given a channel with capacity of 20Mbps, the bandwidth of channel is 3 MHz. Assuming white thermal noise, what is the required SNR to achieve this capacity?

Home Work (2):

1. If the received signal level for digital system is -151 dBW and the receiver system effective noise temperature is 1500K, what is E_b / N_0 for a link transmitting 2400bps?

2. Fill the missing elements in this table of power ratios in [dB]

dB	1	3	6	7	9	10 20 35
Loss						
Gain						

3. If an amplifier has 30dB voltage gain, what voltage ratio does the gain represent?

4. An amplifier has an output of 20W. What is its output in dBW?

More Measurements About Signals:

Other Measurements used in data communications are:

(1) Throughput (2) Propagation speed (3) Time Propagation and (4) Wave length

(1) Throughput : measure how fast data can pass through network. [bps]→ **called Effective Bit Rate**

$$\eta = \frac{T_{Pkt}}{T_{Pkt} + \Delta}$$

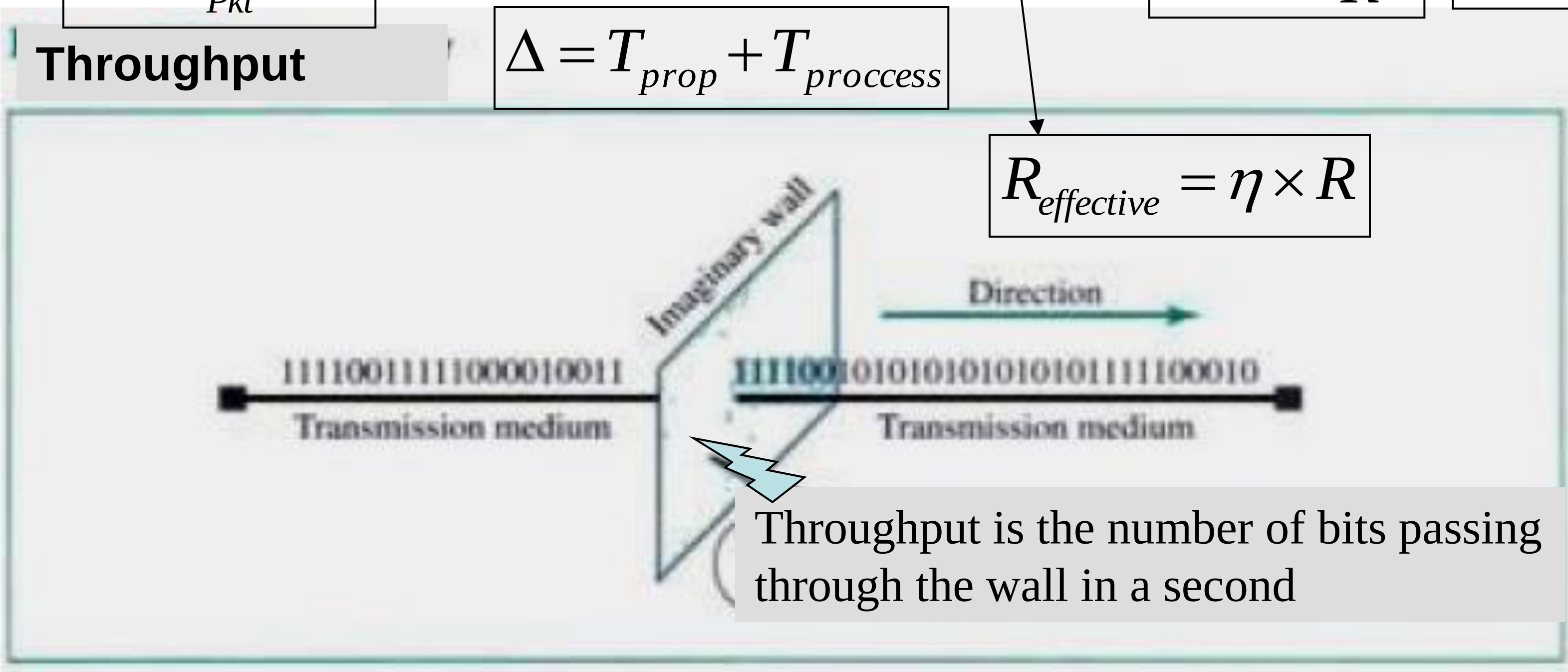
Δ: Total Delay

$$T_{Pkt} = \frac{L_{pkt}}{R}$$

$$T_{prop.} = \frac{d[m]}{v[m/s]}$$

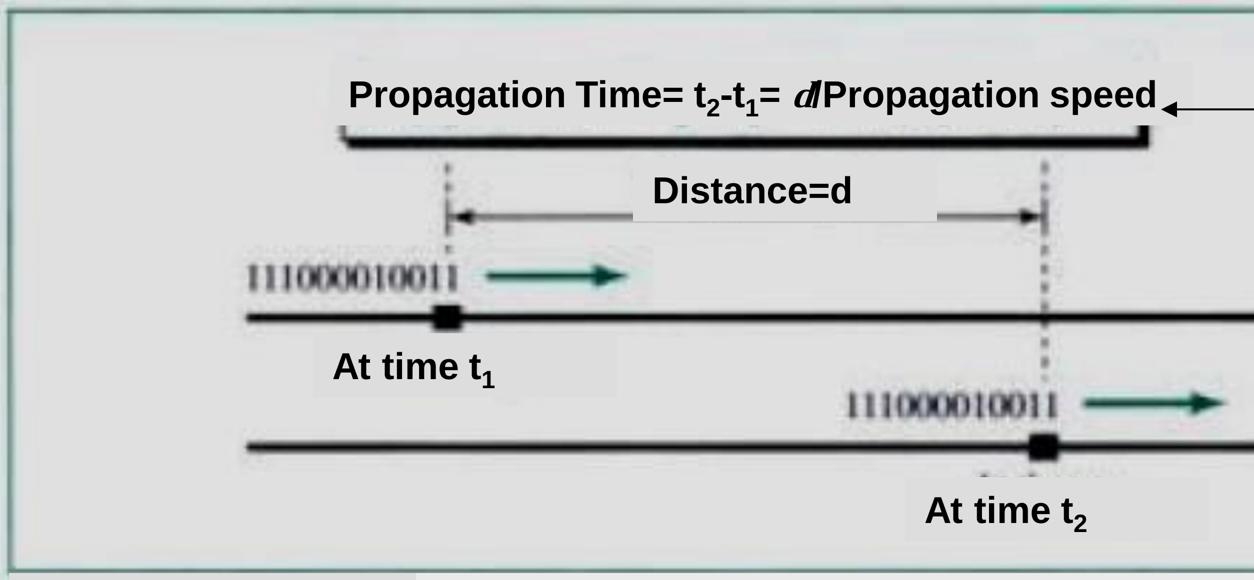
$$\Delta = T_{prop} + T_{proccess}$$

$$R_{effective} = \eta \times R$$



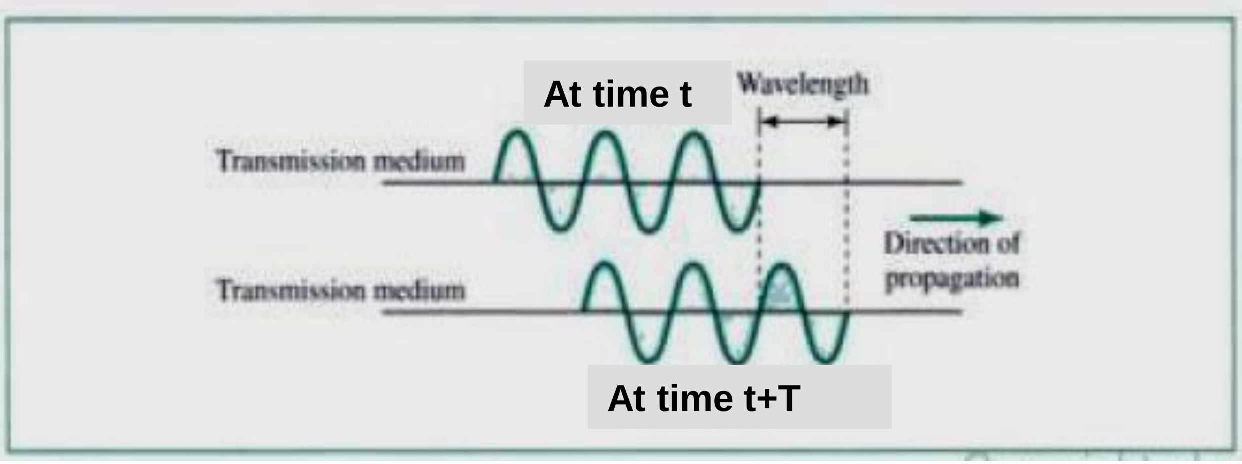
(2) Propagation Speed: measure the distance the signal or bit can travel through medium in one second.-→ [m/s] example: light speed $3x10^8$ m/s

(3) Propagation Time:



$$T_{prop} = \frac{d\ [m]}{v\ [m/s]}$$

(4) Wave length:



$$\lambda = \frac{c}{f}$$

$$c = 3\times 10^8\, m/s$$

The wavelength is normally in **micrometers** (microns) instead of meter.

Example:

The wavelength of Red light (frequency=$4x10^{14}$ Hz) in air is

$$\lambda = \frac{c}{f} = \frac{3\times10^{8}}{4\times10^{14}} = 0.75\times10^{-6} = 0.75\mu m$$

In coaxial or fiber cable, the wavelength is lower (0.5μm) because the propagation speed in cable is less than in air.

Home Work (3):

1. A TV channel has a bandwidth of 6MHz. If we send a digital signal using one channel, what are the data rates if we use one harmonic; three harmonics, and five harmonics?
2. A signal travels from point A to point B. At point A, the signal power is 100W. At point B, the power is 90 W. What is the attenuation in decibel?
3. The attenuation of a signal is -10dB. What is the final signal power if it was originally 5 W?
4. A signal has passed through three cascaded amplifiers, each with 4dB gain. What is the total gain? How much is the signal amplified?
5. If the throughput at the connection between a device and the transmission medium is 5 Kbps, how long does it take to send 100,000bits out of this device?
6. The light of the sun takes approximately 8 minutes to reach the earth. What is the distance between the sun and earth?
7. A signal has wavelength of 1μm in air. How far can the front of wave travel during five periods?
8. A line has a signal-to-noise ratio of 1000 and a bandwidth of 4000KHz. What is the maximum data rate supported by this line?
9. We measure the performance of a telephone line (4KHz of bandwidth). When the signal is 10V, the noise is 5mV. What is the maximum data rate supported by this telephone line?

Home Work (4): Chapter 3 Solve: Q.3.13- Q.3.26 Stallings

3.13 a. Suppose that a digitized TV picture is to be transmitted from a source that uses a matrix of 480×500 picture elements (pixels), where each pixel can take on one of 32 intensity values. Assume that 30 pictures are sent per second. (This digital source is roughly equivalent to broadcast TV standards that have been adopted.) Find the source rate R (bps).

b. Assume that the TV picture is to be transmitted over a channel with 4.5-MHz bandwidth and a 35-dB signal-to-noise ratio. Find the capacity of the channel (bps).

c. Discuss how the parameters given in part (a) could be modified to allow transmission of color TV signals without increasing the required value for R.

3.14 Given an amplifier with an effective noise temperature of 10,000 K and a 10-MHz bandwidth, what thermal noise level, in dBW, may we expect at its output?

3.15 What is the channel capacity for a teleprinter channel with a 300-Hz bandwidth and a signal-to-noise ratio of 3 dB, where the noise is white thermal noise?

3.16 A digital signaling system is required to operate at 9600 bps.

a. If a signal element encodes a 4-bit word, what is the minimum required bandwidth of the channel?

b. Repeat part (a) for the case of 8-bit words.

3.17 What is the thermal noise level of a channel with a bandwidth of 10 kHz carrying 1000 watts of power operating at 50°C?

3.18 Given the narrow (usable) audio bandwidth of a telephone transmission facility, a nominal SNR of 56dB (400,000), and a certain level of distortion,

a. What is the theoretical maximum channel capacity (kbps) of traditional telephone lines?

b. What can we say about the actual maximum channel capacity?

3.19 Study the works of Shannon and Nyquist on channel capacity. Each places an upper limit on the bit rate of a channel based on two different approaches. How are the two related?

3.19 Consider a channel with a 1-MHz capacity and an SNR of 63.

a. What is the upper limit to the data rate that the channel can carry?

b. The result of part (a) is the upper limit. However, as a practical matter, better error performance will be achieved at a lower data rate. Assume we choose a data rate of 2/3 the maximum theoretical limit. How many signal levels are needed to achieve this data rate?

3.20 Given the narrow (usable) audio bandwidth of a telephone transmission facility, a nominal SNR_{dB} of 56dB (400,000), and a distortion level of $<0.2\%$,

a. What is the theoretical maximum channel capacity (kbps) of traditional telephone lines?

b. What is the actual maximum channel capacity?

3.21 Given a channel with an intended capacity of 20 Mbps, the bandwidth of the channel is 3 MHz. Assuming white thermal noise, what signal-to-noise ratio is required to achieve this capacity?

3.22 The square wave of Figure 3.7c, with $T = 1$ ms, is passed through a lowpass filter that passes frequencies up to 8 kHz with no attenuation.

a. Find the power in the output waveform.

b. Assuming that at the filter input there is a thermal noise voltage with $N_0 = 0.1\ \mu$Watt/Hz, find the output signal to noise ratio in dB.

3.23 If the received signal level for a particular digital system is -151 dBW and the receiver system effective noise temperature is 1500 K, what is E_b/N_0 for a link transmitting 2400 bps?

3.24 Fill in the missing elements in the following table of approximate power ratios for various dB levels.

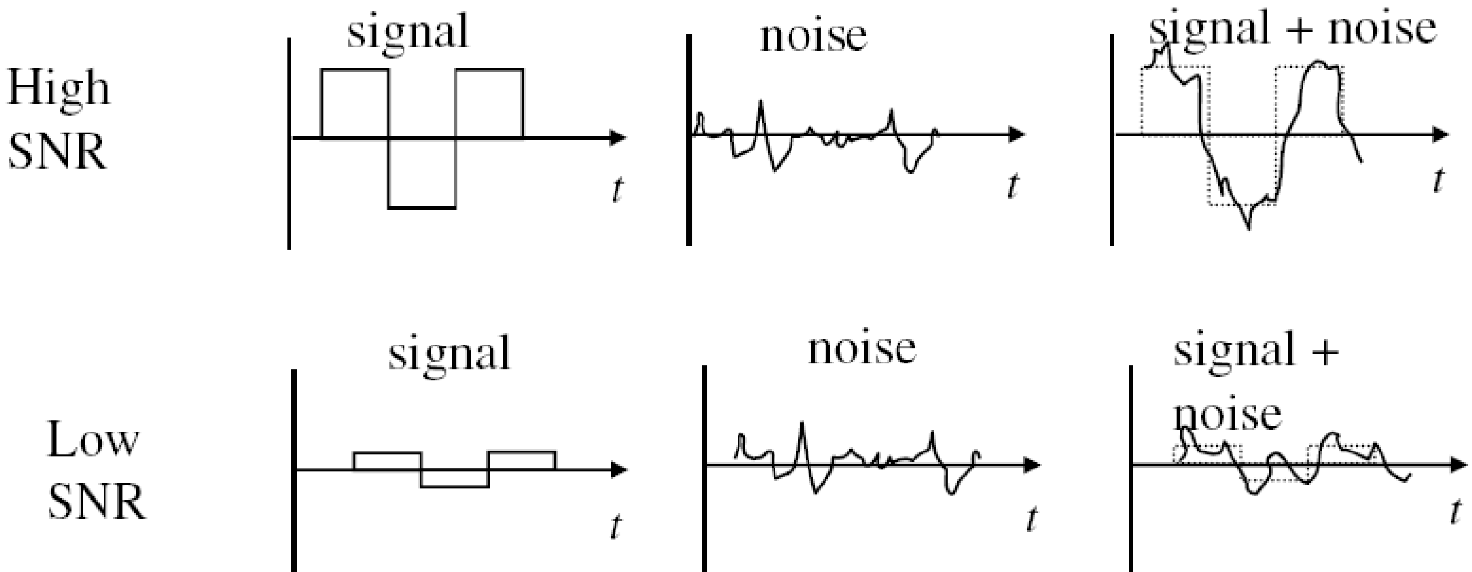
Decibels	1	2	3	4	5	6	7	8	9	10
Losses			0.5							0.1
Gains			2						10	

3.25 If an amplifier has a 30-dB voltage gain, what voltage ratio does the gain represent?

3.26 An amplifier has an output of 20 W. What is its output in dBW?

Signal-to-Noise Ratio

- Definition of SNR



$$\text{SNR} = \frac{\text{Average Signal Power}}{\text{Average Noise Power}} = \frac{A^2}{\sigma^2} \qquad \text{SNR (dB)} = 10 \log_{10} \text{SNR}$$

A=noise free sample voltage at the receiver

σ^2=the total noise power at the detector=(N_0)(NBW)

NBW=noise bandwidth

N_0=Power of white noise per Hertz

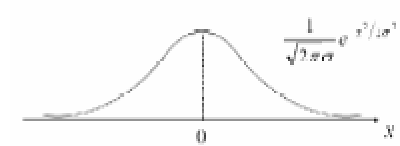
Error Performance

- Signal Detection: A decision of which signal was transmitted is made by comparing the measurement (at the appropriate time) to a threshold located halfway between these nominal voltages that represent “0” and “1”.
- Error performance depends on the nominal distance between the voltages and the amount of fluctuation in the measurements caused by noise.
- In absence of noise, the measurement of the positive pulse would be A and that of negative pulse would be -A. Because of noise, these samples would be $\mp A+n$ where n is the random noise amplitude.
- The error performance analysis in communication circuits is typically based on white Gaussian noise.

Error Probabilities

- We now compute the probability of error for a polar signal. The amplitude n of the noise is Gaussian distributed. It ranges from $-\infty$ to ∞ according Gaussian PDF.
- When "0" is transmitted, the sample value of the received pulse is -A+n. If n>A, the sample value is positive and the digit will be detected wrongly as 1. If P(error|0) is the probability of error given that 0 is transmitted, then,

$$\begin{aligned} P(\text{error}|0) = Prob(n < -A) &= \frac{1}{\sigma\sqrt{2\pi}}\int_A^\infty e^{-n^2/2\sigma^2}dn \\ &= \frac{1}{\sqrt{2\pi}}\int_{A/\sigma}^\infty e^{-x^2/2}dx \\ &= Q\left(\frac{A}{\sigma}\right) \end{aligned} \qquad Q(y) = \frac{1}{\sqrt{2\pi}}\int_y^\infty e^{-x^2/2}dx$$



- Probability of error for a polar signal

$$P(\text{error}) = \frac{1}{2}\left[P(\text{error}|0) + P(\text{error}|1)\right] = Q\left(\frac{A}{\sigma}\right)$$

Error Performance

- Polar Signaling

$$P(\text{error}) = Q\left(\frac{A}{\sigma}\right)$$ Power=A^2

A → Peak amplitude (Volts)
σ → noise rms amplitude (Vo
σ^2=total noise power

- On-Off Signaling

$$P(\text{error}) = Q\left(\frac{A}{2\sigma}\right)$$ Power=$A^2/2$

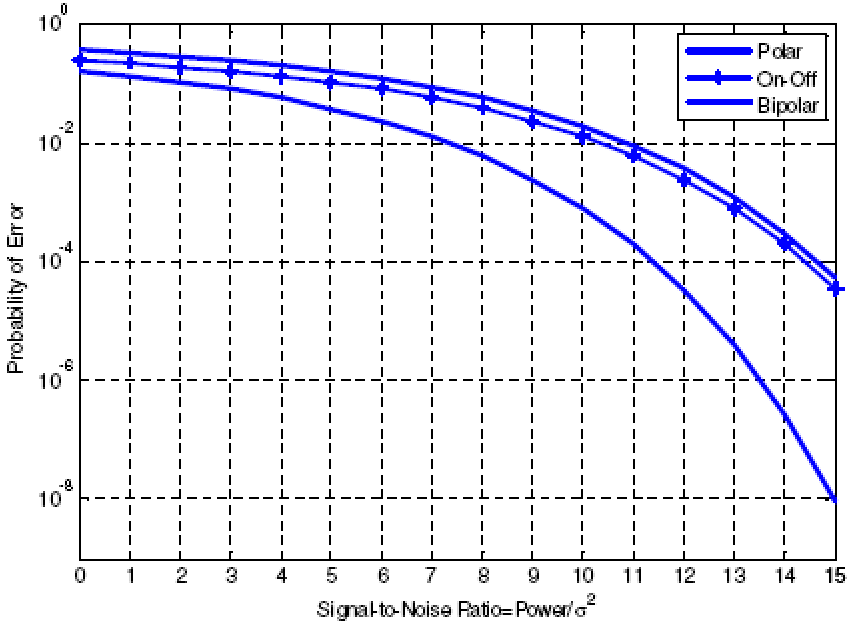
- Bipolar Signaling

$$P(\text{error}) = 1.5Q\left(\frac{A}{2\sigma}\right)$$

Power=$A^2/2$

SNR=Power/σ^2

SNR=Energy/N_0



Performance Monitoring

- Redundancy Checks
  - **Parity Bits** are inserted into DS3 and DS4 signals for the purpose of monitoring the channel error rate.
  - The following equation relates the parity error rate (PER) to the channel probability of error or bit error rate (BER)

$$\text{PER} = \sum_{i=1}^{N} \binom{N}{i} p^i (1-p)^{N-i} \quad \text{(i odd)}$$

N=number of bits over which parity is generated
p=BER assuming independent errors

  - **Cyclic redundancy check (CRC) codes** are also incorporated into a number of transmission systems as a means of monitoring BERs and validating framing acquisition.
  - Examples of CRC use: Extended superframe (ESF) on T1 lines

$$\text{CRCER} = 1 - (1-p)^N$$

N=length of CRC field (including CRC bits)
p=BER assuming independent errors

Relationship between BER and PER

We have: **BER**=Bit Error Rate (independent error bits at Physical Layer)

PER= Packet (Block) Error Rate for CRC used at Data-Link Layer

$$\boxed{\mathrm{PER} = \mathrm{P_B} = 1-(1-\mathrm{P_e})^N} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

N = Packet Length (Size) in bits

$$\boxed{\mathrm{PER} = 1-(1-\mathrm{P_e})^N \cong \mathrm{N}\times\mathrm{P_e} \text{ if } \mathrm{N}\times\mathrm{P_e} < 1} \leftarrow \text{Less than 1} \ldots\ldots\ldots\ldots \quad (2)$$

Example (1): Determine the max. block size over a channel when it has BER of 10^{-4} and the probability of block containing error is 10^{-1}.

Solution:

$$\mathrm{P_B} = 1-(1-\mathrm{P_e})^N$$

$$10^{-1} = 1-(1-10^{-4})^N \quad \to N = ? \quad \to N \cong 1000\text{bits}$$

OR

$$\mathrm{P_B} = N\times\mathrm{P_e} \to 0.1 = N\times10^{-4} \quad \to N = 1000\text{bits}$$ ←Packet or Block size

Example (2): Consider SNR=18.6dB for $P_e=10^{-5}$. If the signal is sent using telephone -line with bit-rate 1.544Mbps (T1 line). Compute the time required to occur error.

Repeat for SNR=21.6dB for $P_e=10^{-9}$

Solution:

$$\mathrm{BER} = \mathrm{P_e} = 10^{-5} \rightarrow \textit{for } \mathrm{SNR} = 18.6\mathrm{dB}$$

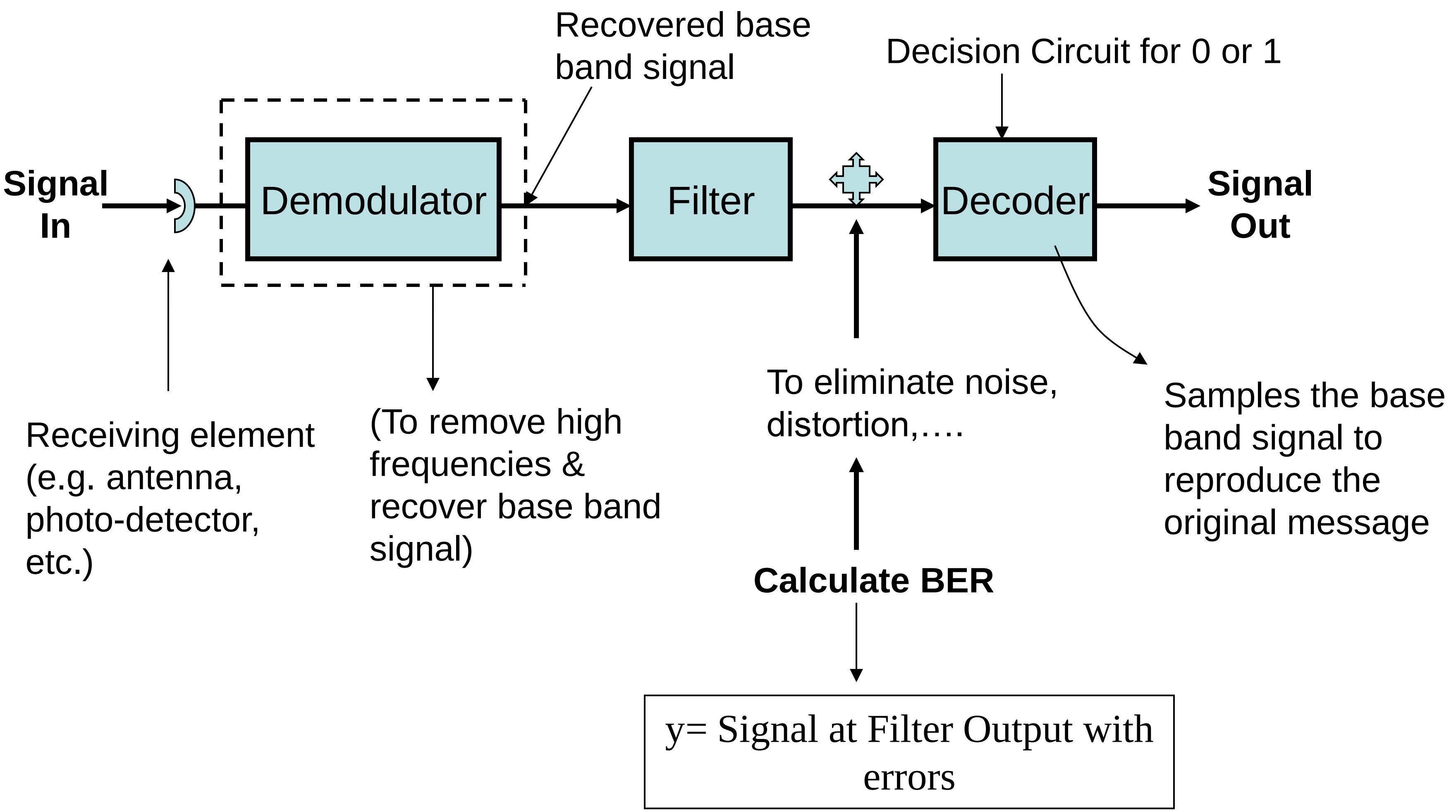
1- $$\mathrm{P_e} = \frac{N_e}{N_t} = \frac{N_e}{R.t} \Rightarrow 10^{-5} = \frac{1}{1.544\times10^6\times t} \rightarrow \mathrm{t} = 0.065\mathrm{sec} = 65\mathrm{msec}$$

2- $$\text{For SNR} = 21.6\mathrm{dB} \Rightarrow \mathrm{P_e} = 10^{-9}$$

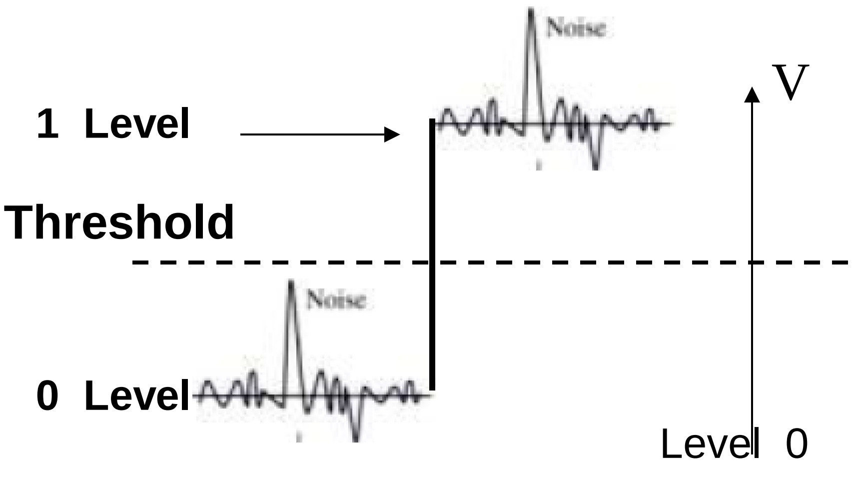
$$10^{-9} = \frac{1}{1.544\times10^6\times t} \rightarrow \mathrm{t} = 650\mathrm{sec} = 11\mathrm{minutes}$$ ← is more tolerable

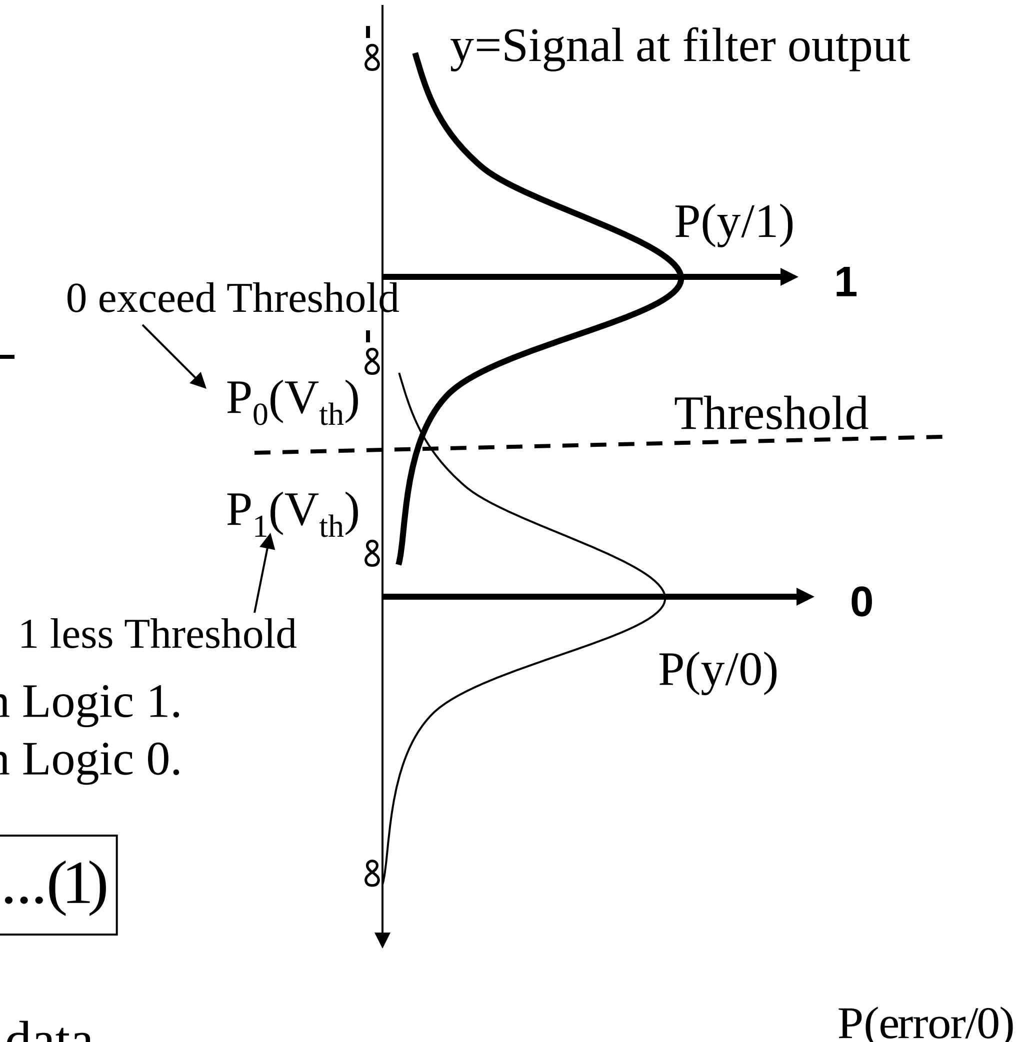
Note: If SNR increases by 3dB, BER then decreases by 10^{-4}

BER at the Receiver



To calculate BER, the probability distribution of the signal at the **decoder input.**





P(y/1)= Probability of Output y for given Logic 1.

P(y/0)= Probability of Output y for given Logic 0.

$$\boxed{P_e = a \times P_1(V_{th}) + b \times P_0(V_{th}) \ ....(1)}$$

a and *b* are priori distribution of the data.

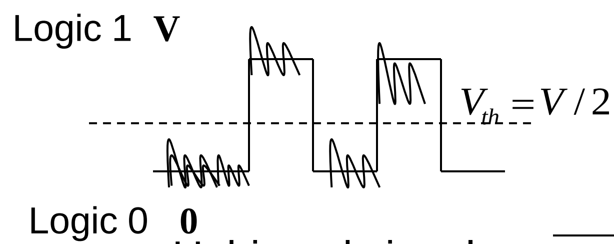
Note: For unbiased data ***a*=*b*=0.5** (equal probability) of 1 and 0 occurrence

We have <u>Two</u> steps:

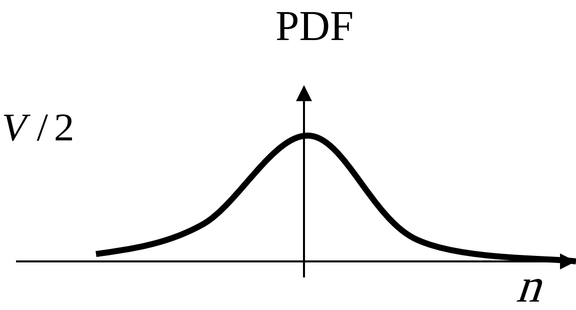
1. **The Noise Statistics:** Assume *n* be a noise amplitude defined by Gaussian Probability Density Function (PDF) distribution with zero-mean value (dc=0).

$$f(n)=\frac{1}{\sigma\sqrt{2\pi}}e^{(-n^2/2\sigma^2)}$$

$\sigma^2 = \text{Total Noise Power}$



Unbiased signal



2. **To compute P_e:** Assume Unbiased signal (data), assume all pulses has **V** amplitude

$$P_0(V_{th})=\int_{-\infty}^{V/2}P(y/0)dy=\int_{-\infty}^{V/2}f_0(y)dy=\int_{-\infty}^{V/2}\frac{1}{\sigma\sqrt{2\pi}}e^{(-v^2/2\sigma^2)}dv$$

Logic 0 **0**

$$P_1(V_{th})=\int_{V/2}^{\infty}P(y/1)dy=\int_{V/2}^{\infty}f_1(y)dy=\int_{V/2}^{\infty}\frac{1}{\sigma\sqrt{2\pi}}e^{(-(v-V)^2/2\sigma^2)}dv$$

Logic 1 **V**

$$\mathrm{P_e}=\frac{1}{2}\left[\mathrm{P}_1(V_{th})+\mathrm{P}_0(V_{th})\right]=\frac{1}{2}\left[\mathrm{P(error/1)}+\mathrm{P(error/0)}\right]\ ....(2)\longrightarrow \boxed{\mathrm{P_e}=Q\left(\frac{V}{2\sigma}\right)}$$