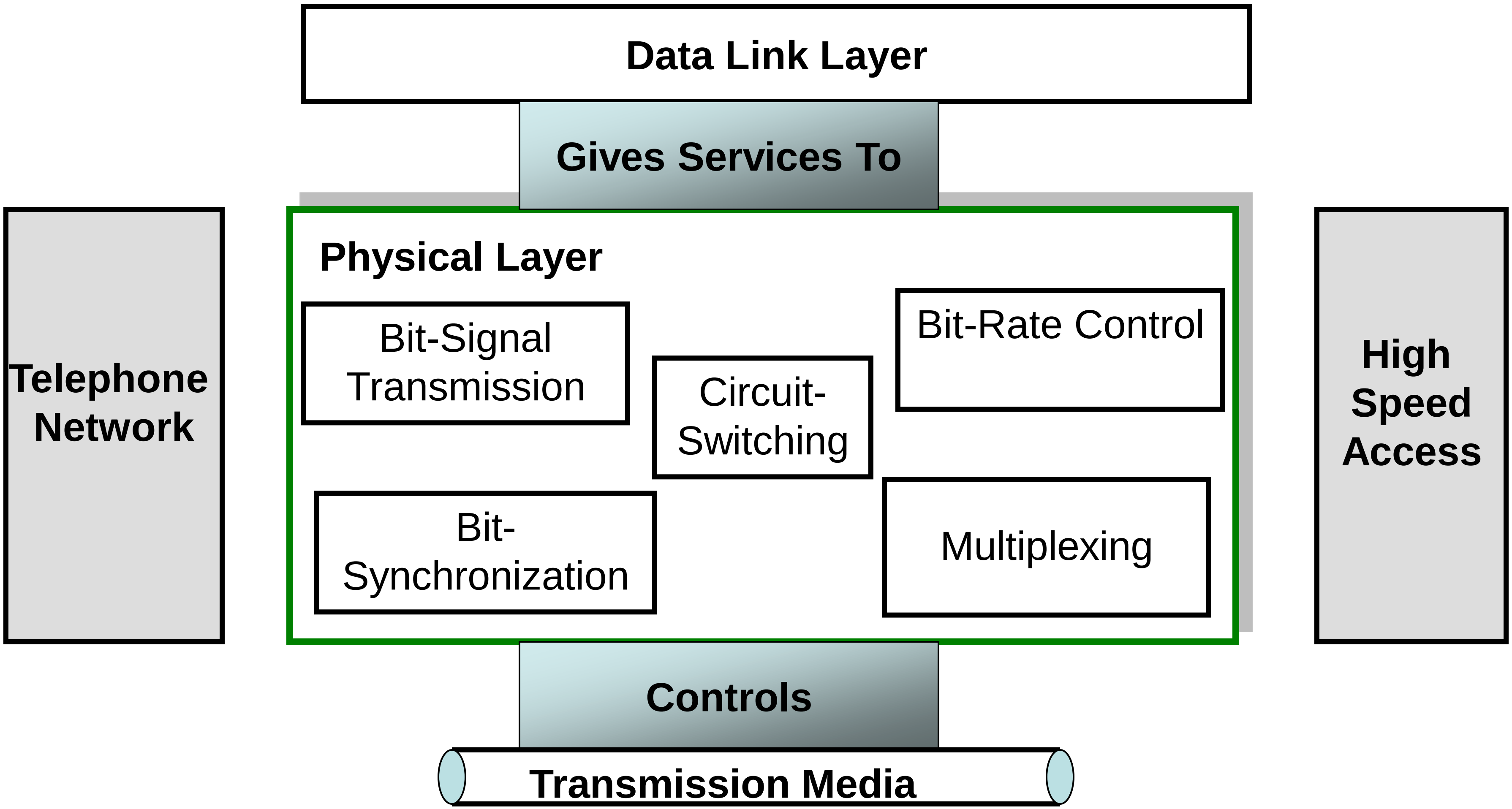
Chapter 3
Physical Layer

An Introduction to Digital Transmission

Physical Layer



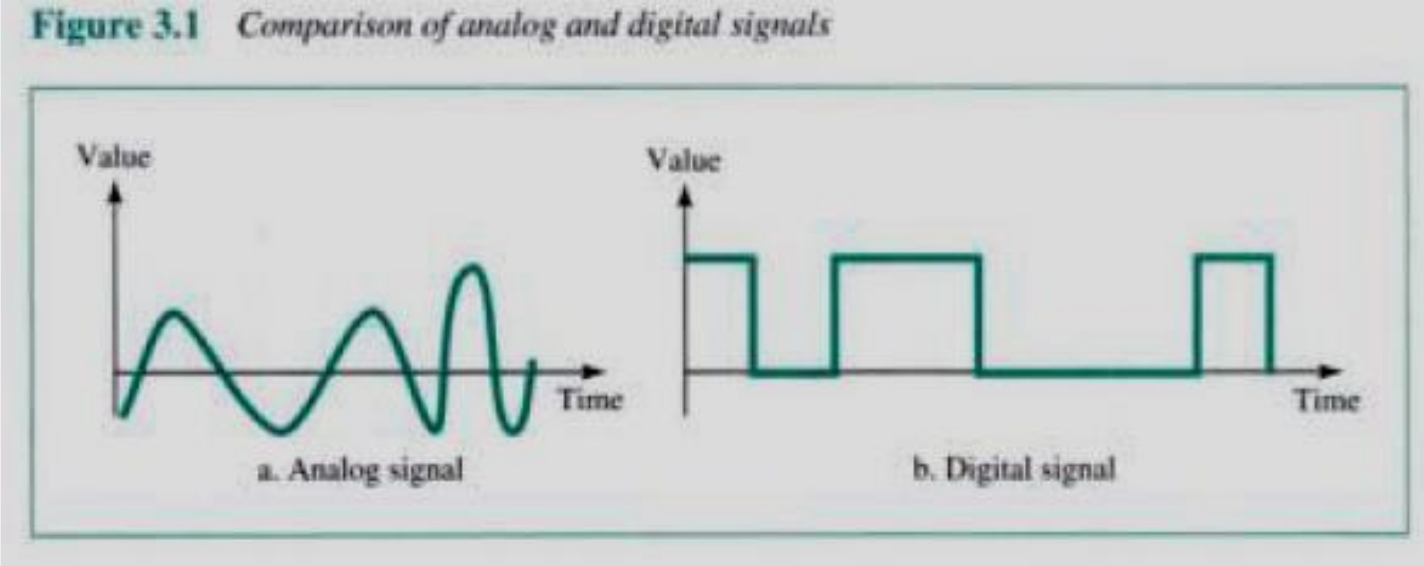
Physical Layer Services

Duties of Physical Layer

- Bit-Signal Transformation
- Bit-Signal Synchronization
- Bit-Rate Control
- Multiplexing (TDM, FDM, CDM, WDM)
- Circuit-Switching

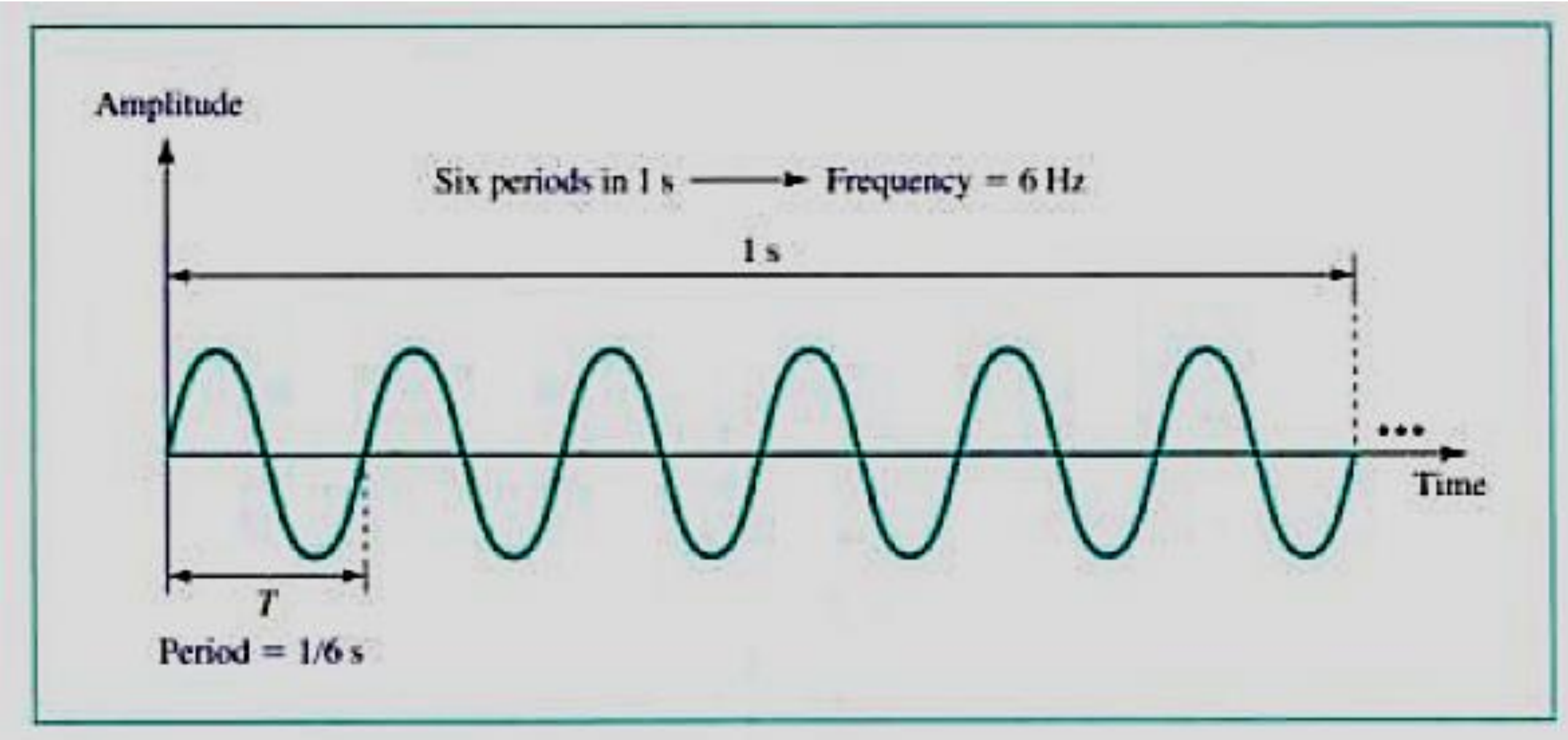
Analog & Digital Signals

❖ Periodic (cycle) & Aperiodic (Pattern)



In data communication, we commonly use **periodic analog signals and aperiodic digital signals.**

❖ Analog Signal: $s(t) = A\sin(2\pi f t + \phi)$



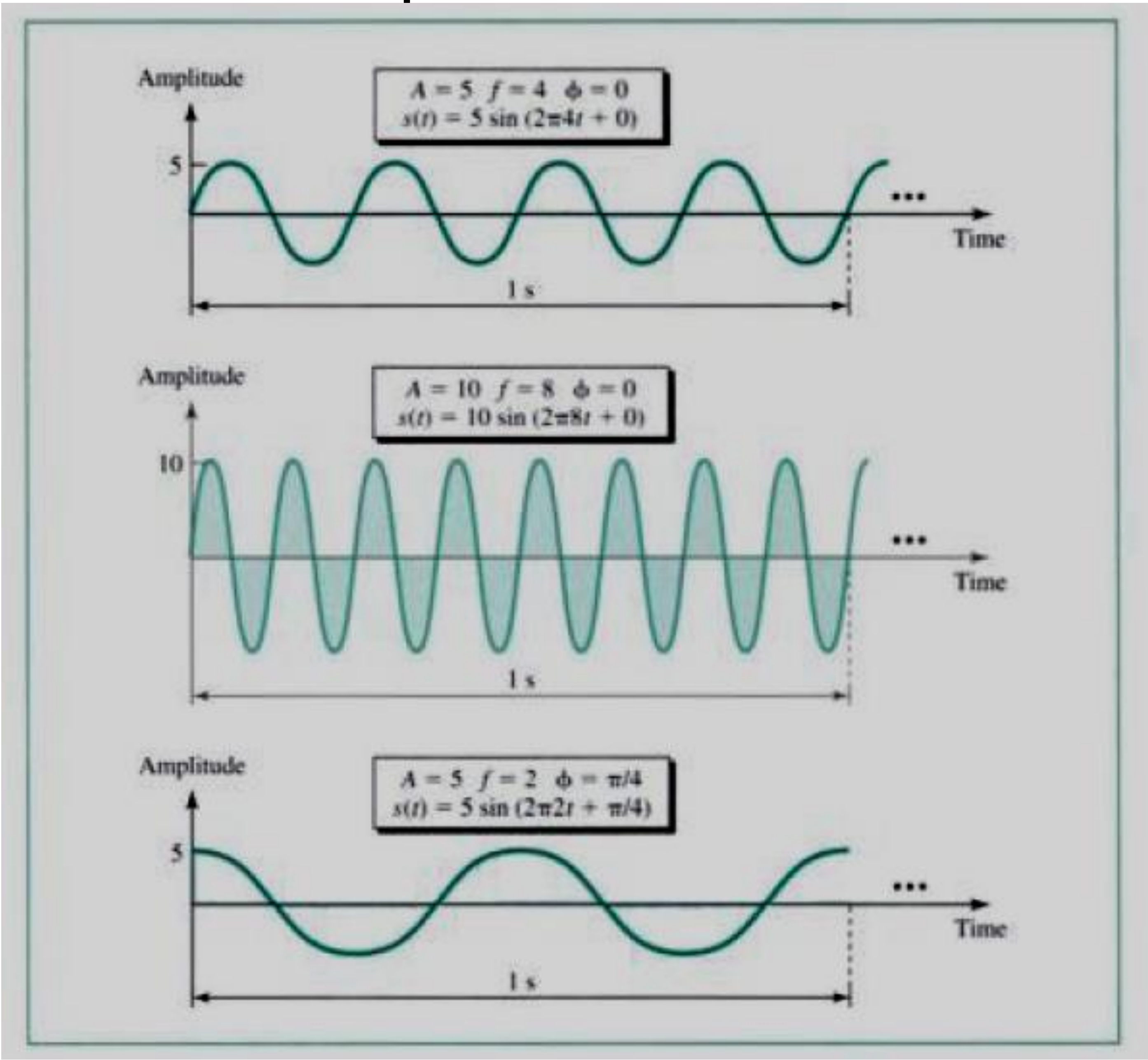
$$f = \frac{1}{T}$$

S(t)= Sin wave signal with f: Frequency *(Hz) T:* Period *(sec)*
A: Amplitude *(volt) φ:* Phase

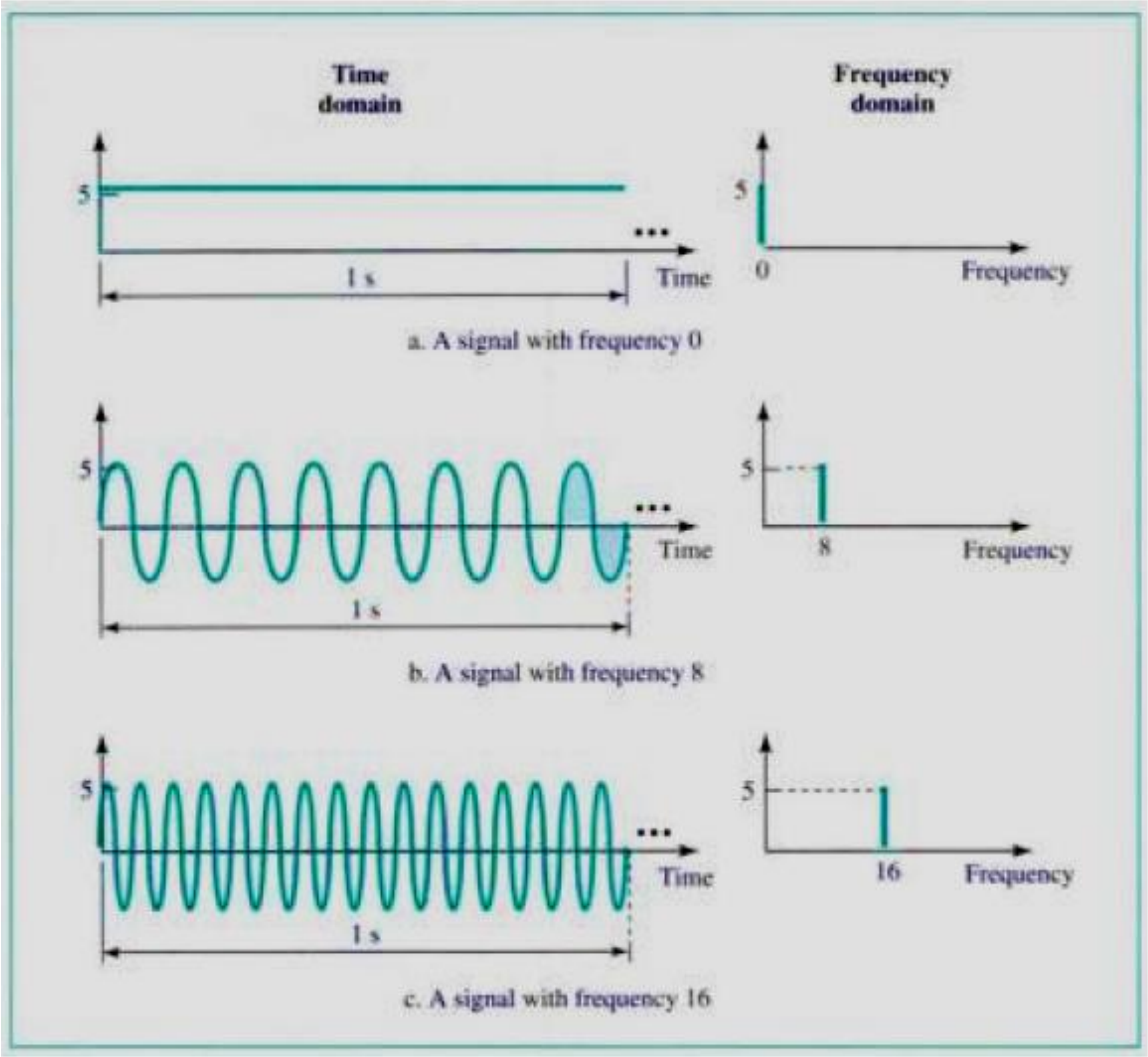
Ex: *f=10 KHz* ➔ *T = 0.01 ms*

Note: *f=0 Hz* (i.e. DC signal) *f= ∞* (infinity)

Examples of Sine Waves:



Time & Frequency Domains:



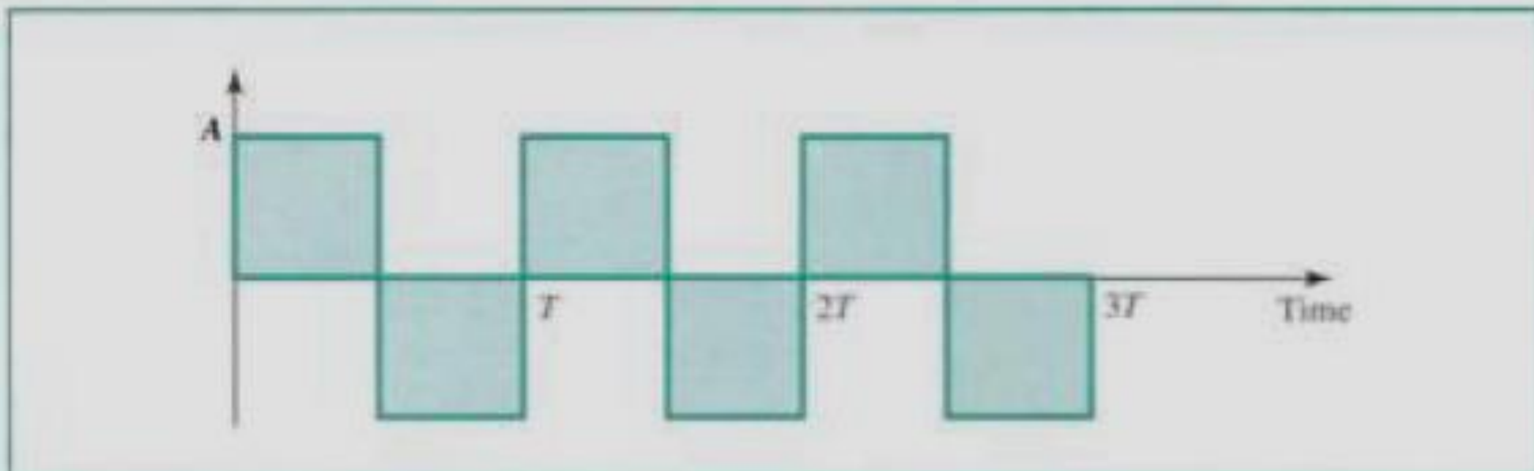
Fourier Analysis: (Composite Signals)

$$s(t) = \underbrace{A_1 \sin(2\pi f_1 t + \phi_1)}_{\text{Fundamental frequency}} + \underbrace{A_2 \sin(2\pi f_2 t + \phi_2) + A_3 \sin(2\pi f_3 t + \phi_3)}_{\text{Harmonics (freq. components)}} + \cdots$$

Fundamental frequency Harmonics (freq. components)

<u>Example:</u> Square Wave

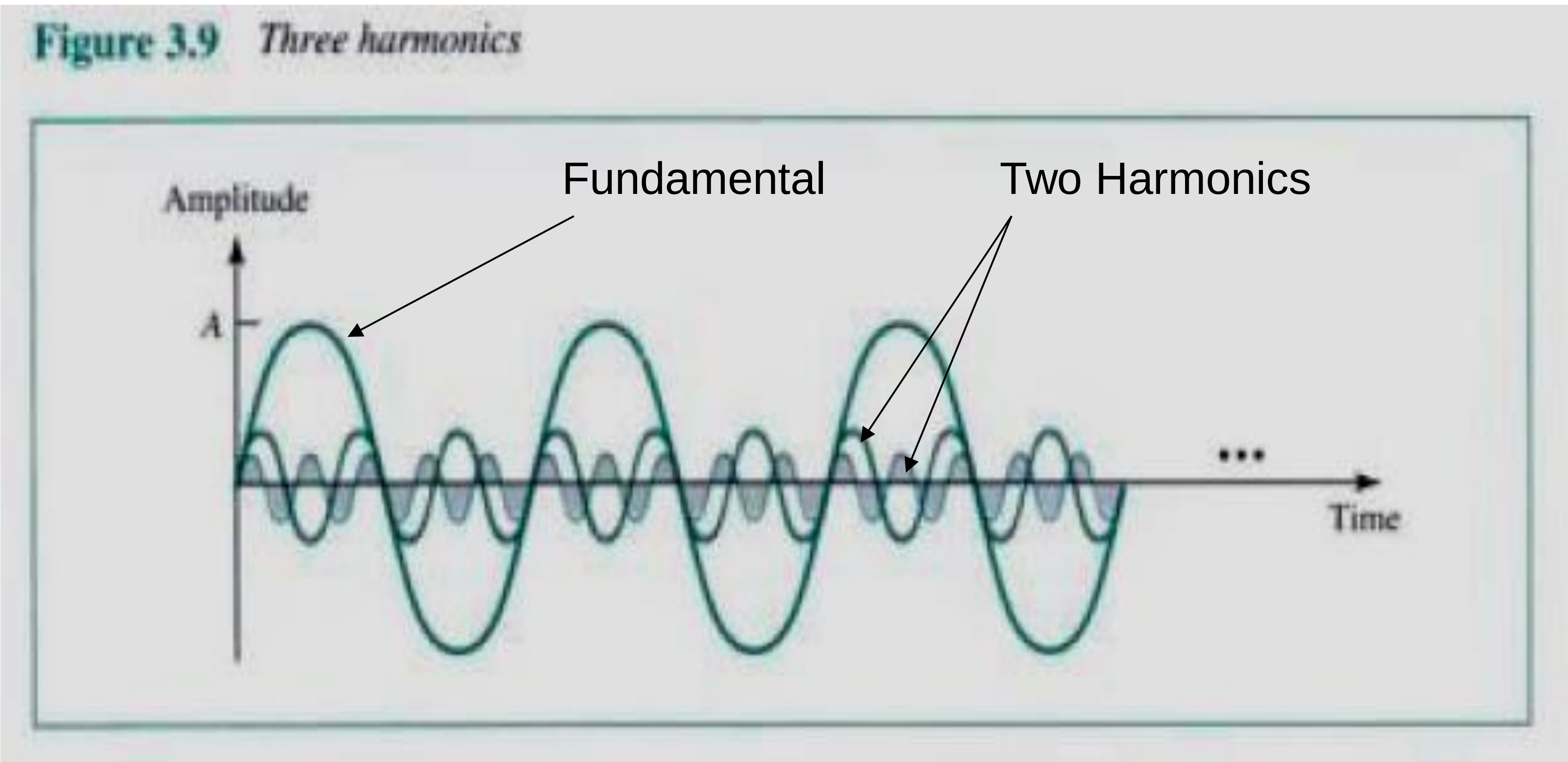
Figure 3.8 *Square wave*



$$s(t) = \frac{4A}{\pi}\sin 2\pi f t + \frac{4A}{3\pi}\sin[2\pi(3f)t] + \frac{4A}{5\pi}\sin[2\pi(5f)t] + \cdots$$

Fundamental Frequency & Harmonics:

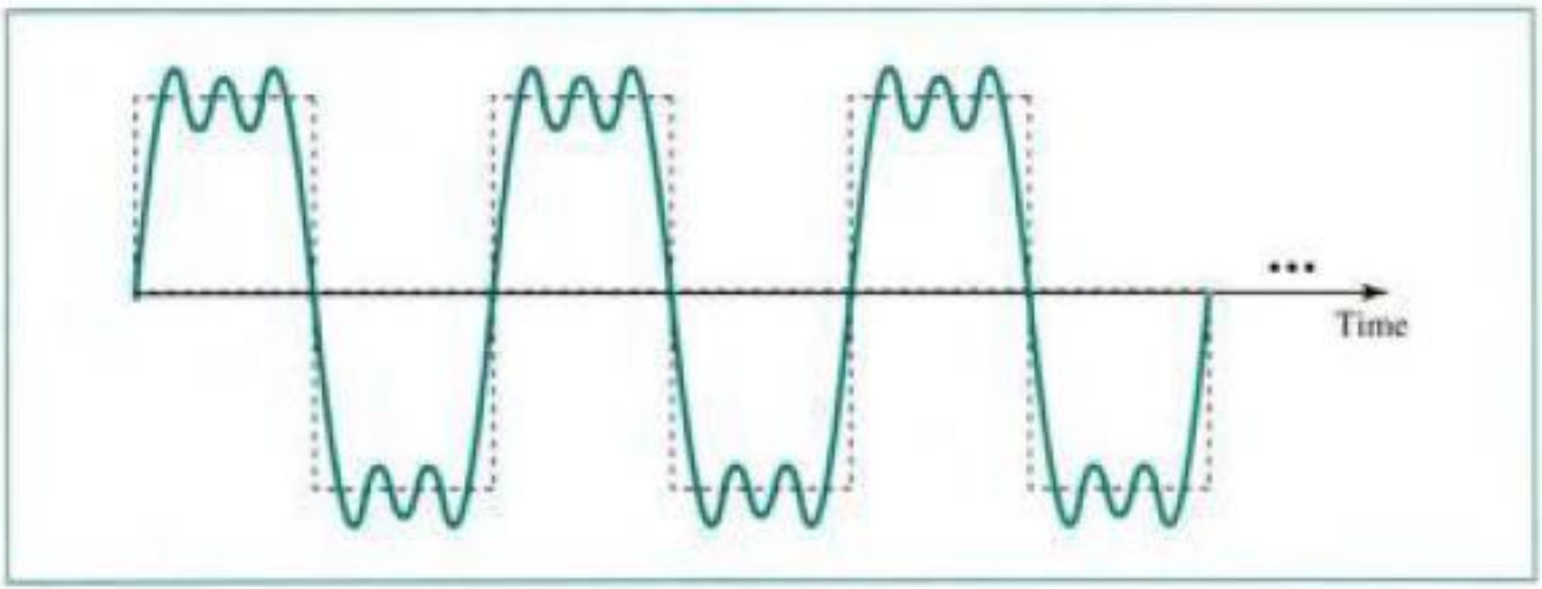
Example: If a square wave has f=5kHz then the component frequencies are 5000 (f), 15000 (3f), 25000 (5f),….so on



Composite Signal: By Adding harmonics

Figure 3.10 *Adding first three harmonics*

(1) Time Domain

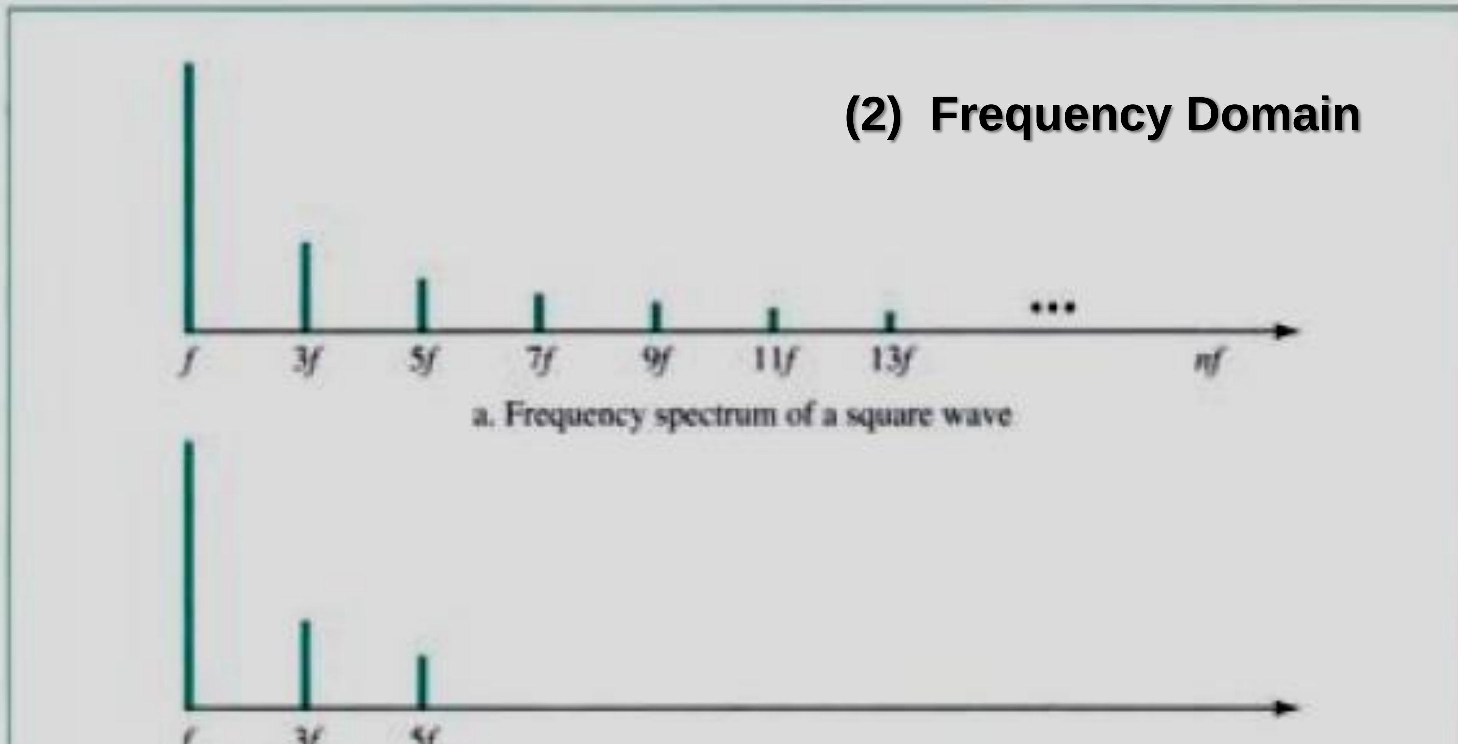


❖ If we need signal closer to square wave, we need to add more harmonics → **for more exact (accurate)**

Frequency Spectrum:

It describes signal by its components in frequency domain

Figure 3.11 *Frequency spectrum comparison*



a. Frequency spectrum of a square wave

b. Frequency spectrum of an approximation with only three harmonics

Signal Corruption: Transmit signal in media (medium)

Figure 3.12 *Signal corruption*

$$B = f_{high} - f_{low}$$

Bandwidth: The range of frequencies that a medium can pass without losing ½ of the power contained in that signal.

The bandwidth is a property of a medium: It is the difference between the highest and the lowest frequencies that the medium can satisfactorily pass.

Ex: If a medium can pass frequencies between 1KHz and 5kHz, then the bandwidth → B = 5000-1000= 4000= 4KHz

Note: If the Medium Bandwidth (B_w) **does not** match the spectrum of a signal (signal bandwidth), **some of frequencies are lost.**

Example (1):

If square wave with infinity bandwidth (frequency spectrum), so there is **no medium** with infinity bandwidth (∞).

Here, We use Bandwidth → for Medium or for Signal

Note:

If Signal Bandwidth << Medium Bandwidth → waste in Medium Bandwidth.

If **Signal Bandwidth > or >> Medium Bandwidth → loss power**

❖ Loss in frequencies ➔ means Loss in total signal power →**Attenuation** in signal

Example (2):

Consider Voice normally with spectrum 300-3300 Hz. Transmission line is between 1500 and 2500Hz. Explain the status of received voice at the RX.

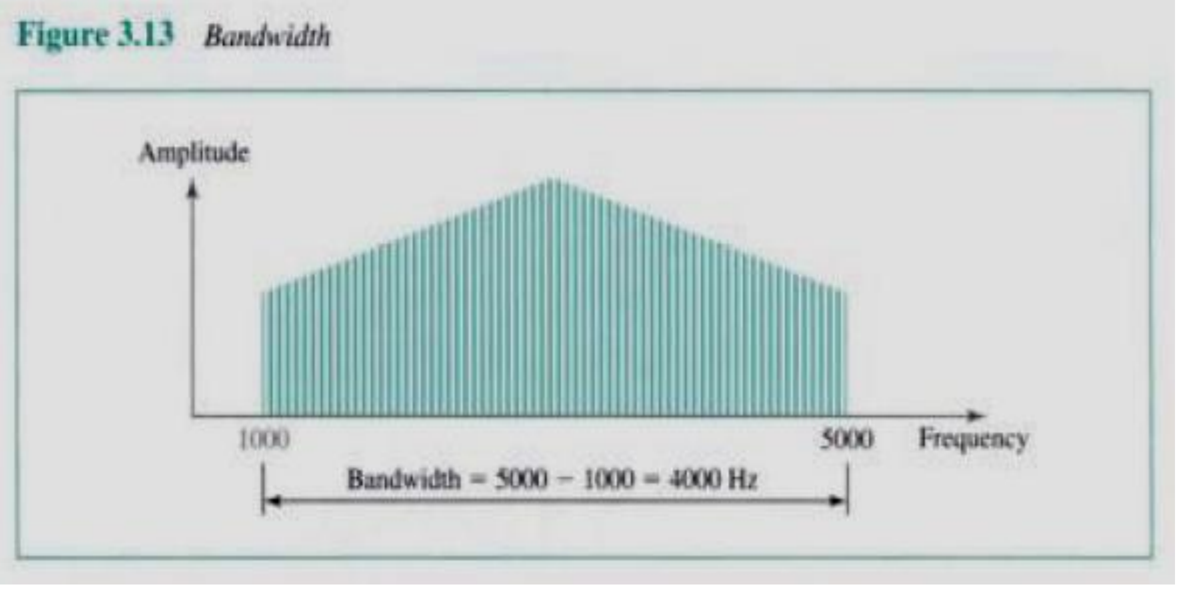
Sol.:

Medium Bandwidth (B_w)= 2500-1500 = 1000 Hz

Voice Bandwidth = 3300-300 = 3000Hz

Since **Voice Bandwidth > Medium Bandwidth,**

At RX, some frequencies are lost in the voice and we cannot recognize it. → **This is called Attenuation in signal.**



Example 3

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is the bandwidth? Draw the spectrum, assuming all components have a maximum amplitude of 10 V.

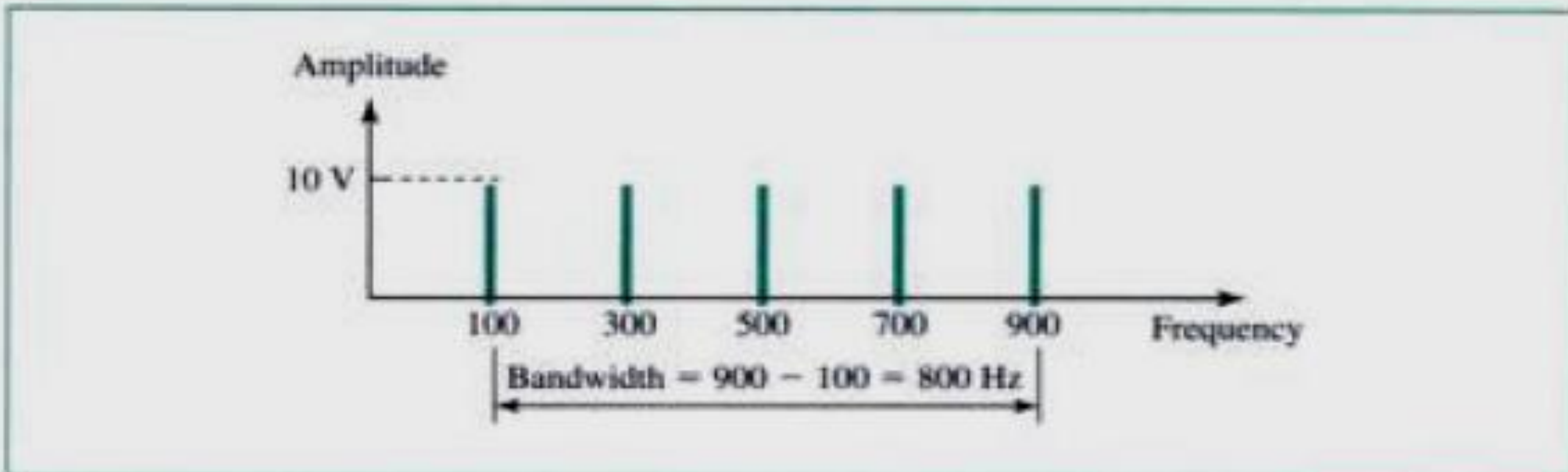
Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l = 900 - 100 = 800 \text{ Hz}$$

The spectrum has only five spikes, at 100, 300, 500, 700, and 900 (see Fig. 3.14).

Figure 3.14 *Example 3*



Example 4

A signal has a bandwidth of 20 Hz. The highest frequency is 60 Hz. What is the lowest frequency? Draw the spectrum if the signal contains all integral frequencies of the same amplitude.

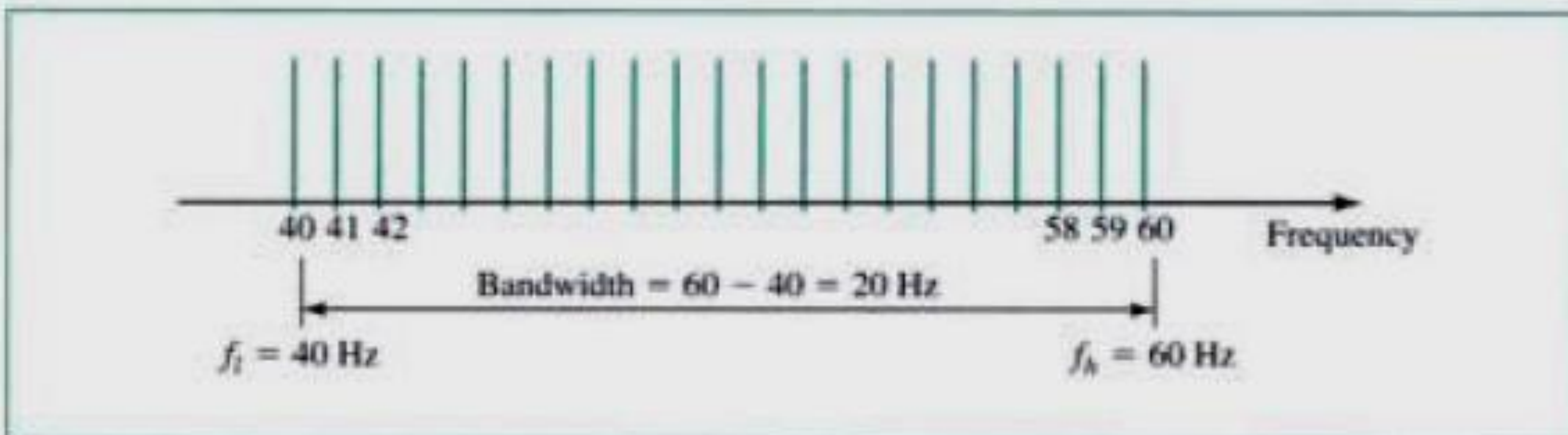
Solution

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l$$
$$20 = 60 - f_l$$
$$f_l = 60 - 20 = 40 \text{ Hz}$$

The spectrum contains all integral frequencies. We show this by a series of spikes (see Fig. 3.15).

Figure 3.15 *Example 4*



Example 5

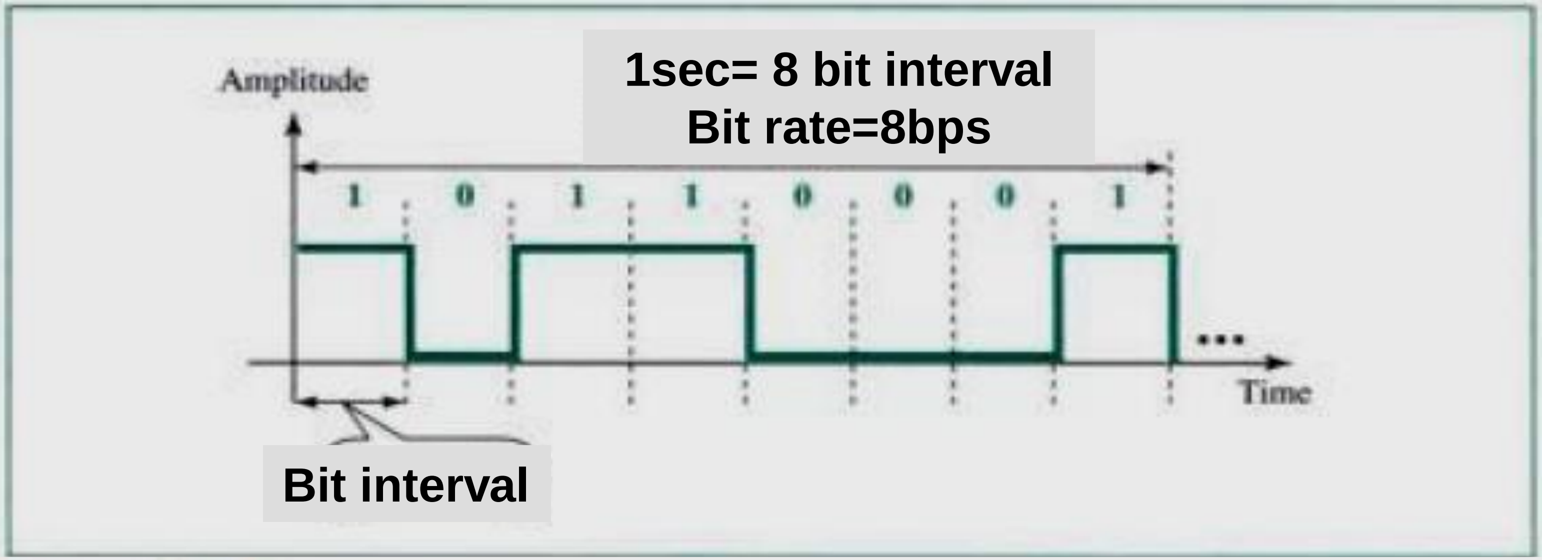
A signal has a spectrum with frequencies between 1000Hz and 2000Hz (bandwidth =1000Hz). A medium can pass frequencies from 3000Hz to 4000Hz (a bandwidth of 1000Hz). Can this signal faithfully pass through this medium.

Solution

The answer is definitely No. Although signal can have the same bandwidth (1000Hz), the range does not overlap. The medium can only pass the frequencies between 3000Hz and 4000Hz. And the signal is totally lost.

Digital Signals

Figure 3.17 Bit Interval and Bit Rate



Example 6

A digital signal has a bit rate of 2000 bps. What is the duration of each bit (bit interval)?

Solution

The bit interval is the inverse of the bit rate.

$$\text{Bit interval} = \frac{1}{\text{bit rate}} = \frac{1}{2000} = 0.000500 \text{ s} = 0.000500 \times 10^6 \ \mu\text{s} = 500 \ \mu\text{s}$$

Digital Signal as a Composite Analog Signal

Digital Signal Through a Wide-Bandwidth Medium

We can send a digital signal through it. But of course, some of frequencies are blocked by the medium (with less attenuation)

Digital Signal Through a Band-Limited Medium

Yes, we can send a digital signal through it. e.g. Using band-limited telephone line to Internet.

- But, what is the minimum required bandwidth (*B*) in (Hz) if we want to send *R* (bps)?

Nyquist Theorem

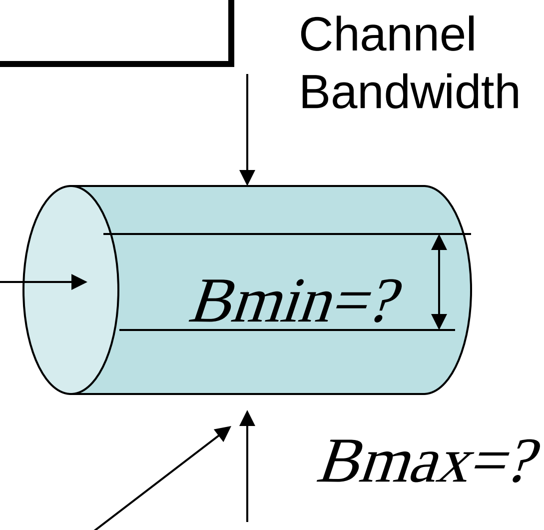
Shannon Capacity

Digital Bandwidth [bps] vs. **Analog Bandwidth** [Hz]

- If the data rate of digital signal is *R* [bps], then a **very good signal representation** can be achieved with a bandwidth *2R*.

- **Example**: R=2000 bps then $B=2R$ =2x2000=4000Hz

- Bandwidth Requirements: $R/2 \leq B \leq 2R$



- **Ex:** Consider R=2000 bps what is the *min.* and *max.* bandwidths required?

  $Bmin = R/2$ = 2000/2=1000Hz (worst)

  $Bmax = 2R$ = 2x2000=4000Hz (Best representation)

❑ Bandwidth Requirements: $R/2 \leq B \leq 2R$

To improve the shape of signal (quality) for better communication we need to add some more harmonics:

❑ To send R (bps) through analog channel, then

$B=R/2$ (using only one harmonic)

$B=R/2+3R/2=2R$ (Adding 3rd harmonic) → (best)

$B=R/2+3R/2+5R/2=9R/2$ (Adding 3rd & 5th harmonic)

…….so on

❑ *For only One Harmonic:* $B=R/2$ → *Min. Bandwidth*

❑ *For More Harmonic:* $B \geq R/2$ → *Max. Bandwidth* → $2R$

- **Example:** How much bandwidth we need to send *R* bps?

  What is the *Min.* and *Max.* bandwidths required?

Bit Rate R	**Harmonic 1**	**Harmonics 1,3**	**Harmonics 1,3,5**	**Harmonics 1,3,5,7**
1 kbps	*B*=0.5 kHz	*B*=2 kHz	*B*=4.5 kHz	*B*=8 kHz
10 kbps	*B*=5 kHz	*B*=20 kHz	*B*=45 kHz	*B*=80 kHz
100 kbps	*B*=50 kHz	*B*=200 kHz	*B*=450 kHz	*B*=800 kHz

Min. Bandwidth (↑ Harmonic 1)

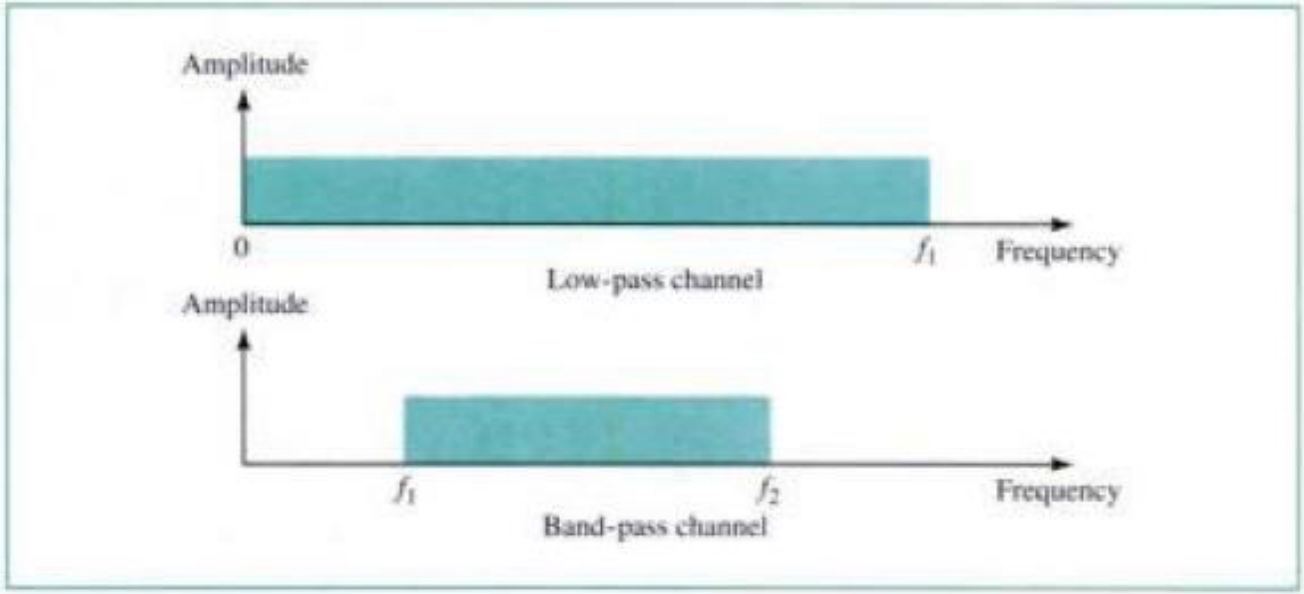
Max Bandwidth (↑ Harmonics 1,3,5,7)

- **Note: If we double *R* we need to double the Bandwidth *B***

  e.g. R=1kbps *B*=500Hz

  R=2kbps *B*=1000Hz …..*so on*

Low-pass and Band-pass Channels



Digital transmission needs a low-pass channel.

Example: It is used Only for **point-to-point** or shared between several devices in time (not in frequency)

Analog transmission can use a band-pass channel.

Example: It is used for analog cellular telephone, each user needs 30 kHz

To Calculate Data Rate: Using Two Formulas

1- **Noiseless Channel**: **Nyquist Bit Rate**

$$R = 2 \times B \times \log_2 L$$

where *R = Bit Rate [bps]*

B= Bandwidth [Hz]

L= no. of signal levels to represent data

- **Example:**

  Consider noiseless channel of 3kHz, transmitting signal of 2 levels.

$$R = 2 \times B \times \log_2 L = 2 \times 3000 \times \log_2 2 = 6000 Hz$$

- Repeat for transmitting signal of 4 levels. Comment on your result.

2- **Noisy Channel**: (**Shannon Capacity) 1948**

$$C\uparrow = B\uparrow \times \log_2\left(1 + SNR\uparrow\right)$$

Maximum or Highest Bit Rate [bps]

where C = *Channel Capacity [bps]*

B = *Bandwidth [Hz]*

SNR = *Signal-to-Noise Ratio*

Shannon Theory:

❖ Every Transmission Channel can transmit bits **reliably** provided that:

"Transmission Rate (R) does not exceed Channel Capacity (C)"

❑ **Example (1):**

Consider extremely noisy channel with SNR=0 (i.e. very strong noise)

$$C = B \times \log_2\left(1 + 0\right) = B \times 0 = 0$$

→So we cannot receive data

❑ **Example (2):**

Telephone line bandwidth 3kHz (300-3300Hz), SNR=3162 (35dB)

$$C = 3000 \times \log_2\left(1 + 3162\right) = 3000 \times 11.62 = 34.86kbps$$

Using Both Limits: Nyquist Bit Rate & Shannon Capacity

❑ **In practice**, we need to use **both methods** to find what bandwidth of what signal level we need.

Example (3): A channel with 1MHz bandwidth, SNR=18 dB what is the appropriate bit rate and signal level?

Soln:

$$C = B\times\log_2(1+SNR) = 10^6\times\log_2\left(1+10^{18/10}\right)$$

$$= 10^6\times\log_2(1+63) = 10^6\times\log_2 2^6 = 6\text{Mbps}$$

But ***R*** **must not exceed *C* (Shannon Theory)** $R < C$

For better performance, we choose: Bit Rate (R) → 4Mbps < C

$$4\text{Mbps} = 2\times B\times\log_2 L$$

$$4\times10^6 = 2\times10^6\times\log_2 L$$

→ Find L? → $L=4$

Note: $\log_2 X \equiv \dfrac{\log_{10} X}{\log_{10} 2} = 3.32\times\log_{10} X$