Flexural Analysis & Design of S.R.R.B.

The beam designed for moment involves the following steps ;

1- Choice of beam cross-section.
2- Choice of reinforcing steel at points of max. moment.

In general beam size will depend on percentage of steel reinforcement selected. A $\frac{d}{b}$ ratio may be taken 1, 1.5, 2, 2.5, or 3.0 . under service loads deflection of member must be considered, ACI code 318 in table 9.5.a gives the minimum thickness of flexural member to be controlled deflection.

Table 9.5(a). Minimum thickness of non-prestressed Beams or one-way slabs unless deflections are computed.

	Minimum thickness (h)			
	Simply Supported	One end Continuous	Both end Continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damged by large deflections			
Solid one-way slabs	$l/20$	$l/24$	$l/28$	$l/10$
Beams or ribbed one-way slabs	$l/16$	$l/18.5$	$l/21$	$l/8$

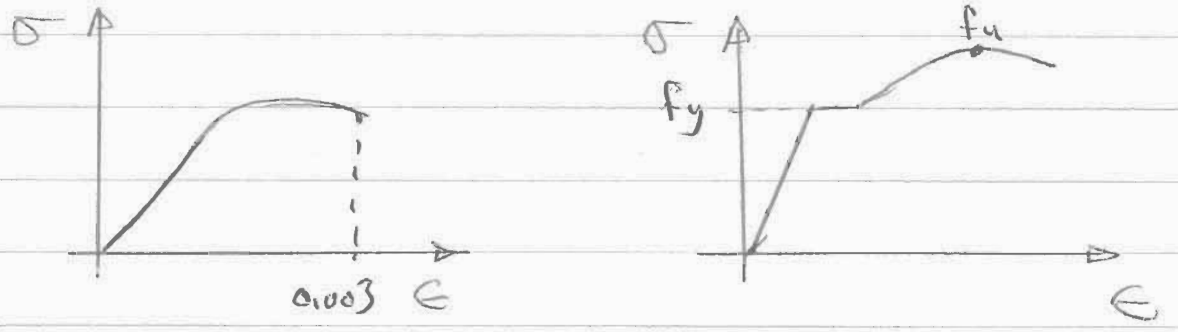
Notes, ($W_c = 2300\ kg/m^3$, $f_y = 420$ MPa)

Design by Ultimate stress Method

1- use ultimate load.

$U = 1.4\, D.L + 1.7\, L.L$

2- Use the ultimate strength of materials.



Concrete　　　　　　　　Steel

(stress - Strain curve)

$\epsilon_c = e_u = 0.003$, $f_s = f_y$

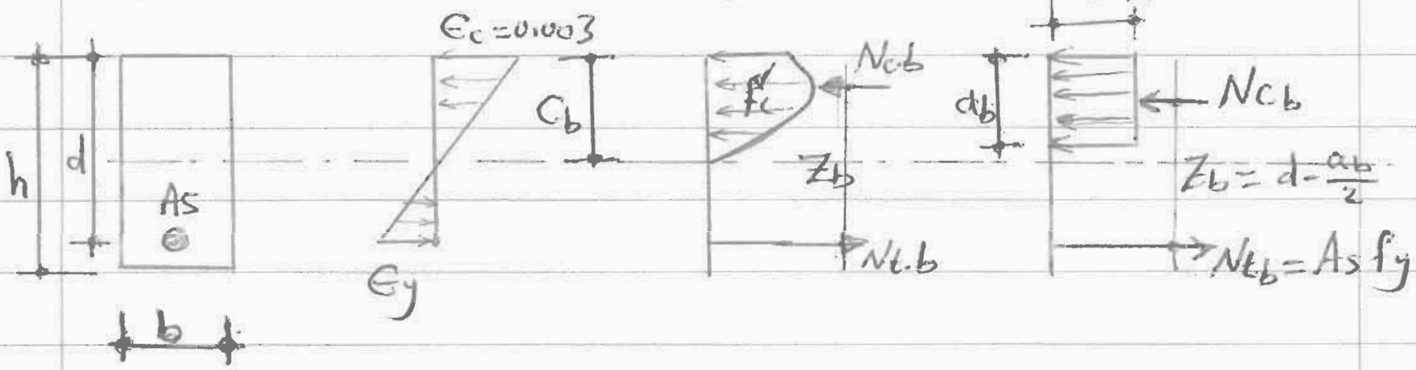
3- Use Safety factors

$\phi = 0.90$ for Bending Moment
$\phi = 0.85$ for Shear
} Beams

$\phi = 0.70$ for Columns

Percentage of steel re-bars (Steel ratio)

* Balanced Steel (ρ_b), represented that amount of reinforcement necessary to make the beam fail by cracking of the concrete at the same load which causes the steel to yield.



From triangles

$$\frac{0.003 + \epsilon_y}{d} = \frac{0.003}{C_b} \; ,$$

$$\epsilon_y = \frac{f_y}{E_s} \; ; \; E_s = 2 \times 10^5 \text{ MPa}$$

$$\therefore \frac{0.003 + \frac{f_y}{2\times10^5}}{d} = \frac{0.003}{C_b}$$

$$\boxed{C_b = \frac{600}{600 + f_y} \cdot d}$$

$$a_b = \beta_1 \cdot C_b$$

From equilibrium $N_{cb} = N_{tb}$

$$0.85 f'_c \cdot a_b \cdot b = A_{sb} \cdot f_y$$

$$A_{sb} = \rho_b \cdot b \cdot d$$

$$0.85 f'_c \cdot \beta_1 \cdot C_b \cdot b = \rho_b \cdot b \cdot d \cdot f_y$$

$$0.85 f'_c \, \beta_1 \cdot \frac{600}{600 + f_y} \, d \cdot b = \rho_b \cdot b \cdot d \cdot f_y$$

$$\therefore \boxed{\rho_b = 0.85 \, \beta_1 \, \frac{f'_c}{f_y} \cdot \frac{600}{600 + f_y}}$$

* Maximum Steel ratio ($\rho_{max.}$)

Because balanced beam will fail suddenly in compression, the code specified that the steel ratio must not exceed $0.75\ \rho_b$ in order that yielding type failure be ensured;

$$\rho_{max.} = 0.75\ \rho_b$$

* Minimum Steel ratio ($\rho_{min.}$)

In a very lightly reinforced section there is a danger that the tensile force carried by the un-cracked concrete could exceed the tensile capacity of the steel yield strength. This mean that the beam would fail suddenly when the cracking moment is reached. To prevent this situation the ACI 318 code requires a minimum percentage of tension reinforcement.

$$\rho_{min.} = \frac{\sqrt{f_c'}}{4 \cdot f_y}$$

and not less than $(1.4/f_y)$

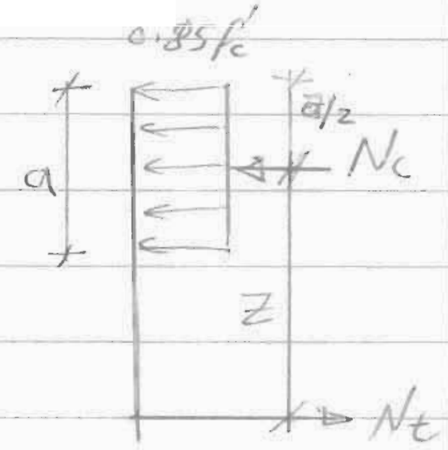
- Moment capacity of a beam

From the equilibrium of a beam section as described in equivalent stress-block distribution (Whitney stress block).

$N_c = 0.85\ f_c' \cdot a \cdot b$

$$N_c = N_t$$

$$0.85 f'_c \cdot a \cdot b = As \cdot f_y$$



(whitney stress block)

$$a = \frac{As \cdot f_y}{0.85 f'_c \cdot b} \quad ;$$

$$Z = d - \frac{a}{2}$$

$$M_n = N_c \cdot Z = N_t \cdot Z$$

$$M_u = \phi M_n \quad ; \phi = 0.9 \; ([illegible])$$

$$M_n = 0.85 f'_c \cdot a \cdot b \,(d - \frac{a}{2})$$

$$M = \cancel{0.85 f'_c} \cdot \frac{As \cdot f_y}{\cancel{0.85 f'_c} \cdot \cancel{b}} \cdot \cancel{b} \cdot (d - \frac{As \cdot f_y}{1.7 f'_c \cdot b})$$

$$As = \rho \cdot b \cdot d$$

$$M = \rho \cdot b \cdot d \cdot f_y \,(d - \frac{\rho \cdot b \cdot d \, f_y}{1.7 \cdot f'_c \cdot b}$$

$$M = \rho \cdot b \cdot d^2 \cdot f_y \,(1 - 0.59 \cdot \rho \cdot \frac{f_y}{f'_c})$$

Concrete protection for reinforcement

For cast in place concrete (nonprestressed) the following minimum concrete cover shall be provided for reinforcement.

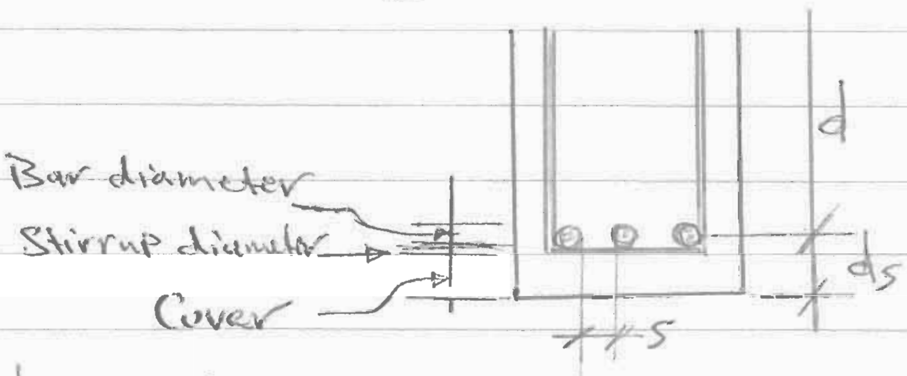
Structural member	Minimum cover (mm)
(a) Concrete cast against and permanently exposed to earth	75
(b) concrete exposed to earth or weather	
No. 19 through No. 57 bars	50
(c) Slabs, walls, joists; No. 36 and smaller	20
Beams and colums	40
Shells, folded plates	20

Steel re-bars Placing :-

- Steel in one layer;

$$d_s = \text{concrete cover} + \text{Stirrups diameter} + \frac{\text{bar. dia.}}{2}$$

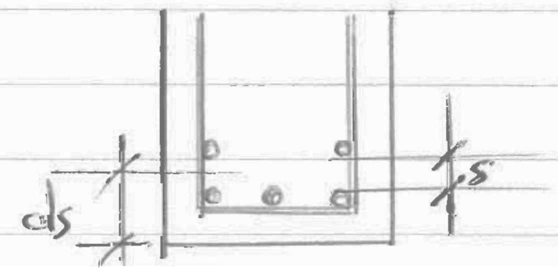
$$= 40 + 10\text{ mm} + \frac{d_b}{2}$$



- Steel in two layers;

$$d_s = \text{concrete cover} + \text{Stirrup dia.} + \text{bar dia.} + \frac{s}{2}$$

$$= 40 + 10 + d_b + \frac{s}{2}$$



Clear spacing (S) :-

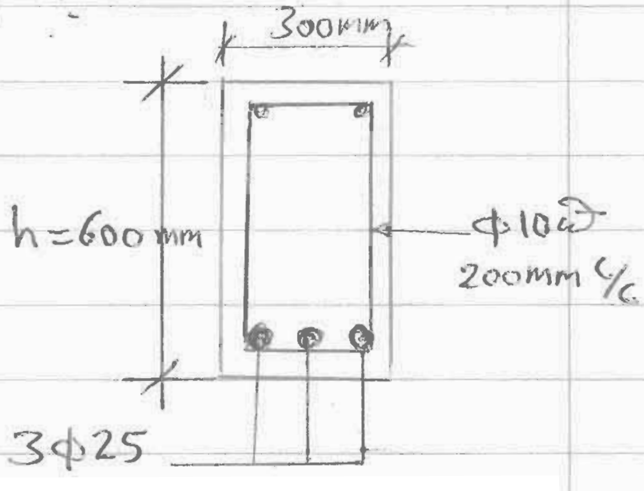
Clear distance between parallel bars in a layer shall be not less than d_b nor 25 mm.

Normal maximum size of coarse aggregate shall be not larger than ;

(a) 1/3 the depth of slabs, nor

(b) 3/4 the minimum clear spacing between individual reinforcing bars.

Example : A rectangular beam has a width of 300 mm & overall depth (h) of 600 mm, As = 3 ϕ 25, $f'_c = 25$ MPa, $f_y = 420$ MPa, calculate the design moment capacity ?



Sol.

$$A_s = 3\phi 25 = 3 * 491$$
$$= 1473 \ mm^2$$

$$\rho = \frac{A_s}{b \cdot d} \ ;$$

$$d = h - d_s = 600 - (40 + 10 + \frac{25}{2})$$
$$= 537.5 \ mm$$

$$\therefore \rho = \frac{1473}{300 * 537.5} = 0.0091$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} = \frac{\sqrt{25}}{4 * 420} = \frac{1.25}{420} = 0.003$$

$$\text{or } \rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.0033 \ \checkmark$$

$$\rho_{max} = 0.75\, \rho_b = 0.75 * 0.85 \cdot \beta_1 \cdot \frac{f'_c}{f_y} \cdot \frac{600}{600+f_y}$$

$$= 0.75 \left(0.85 * 0.85 * \frac{25}{420} * \frac{600}{600+420}\right)$$

$$= 0.019 > \rho \quad \underline{\underline{o.K}} \quad \text{Tension Failure (T.F.)}$$

$$M_n = \rho \cdot b \cdot d^2 \cdot f_y \left(1 - 0.59\, \rho \, \frac{f_y}{f'_c}\right)$$

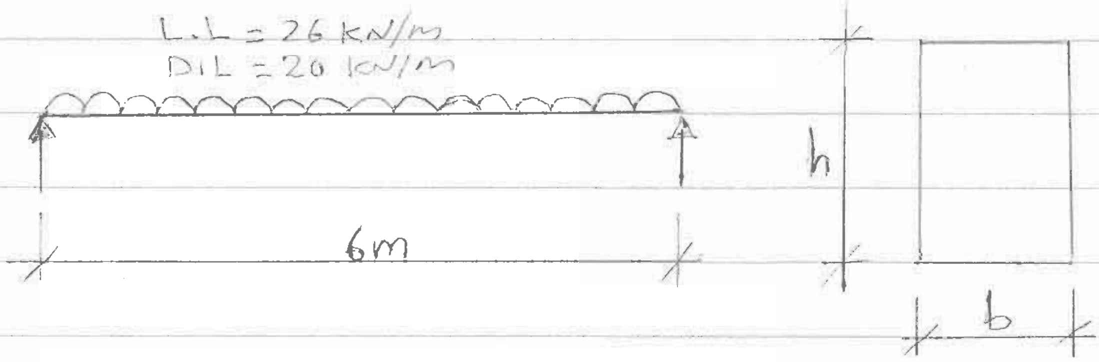
$$= 0.0091 * 300 * (537.5)^2 * 420 \left(1 - 0.59 * 0.0091 \, \frac{420}{25}\right) \times 10^{-6}$$

$$= 301.45 \text{ kN.m}$$

$$M_u = \phi M_n = 0.9 * 301.45 = 271.30 \text{ kN.m}$$

Example:

Design a reinforced concrete beam with a rectangular cross section to carry a service live load of 26 kN/m and total super imposed dead load of 20 kN/m over a 6 m simple span. Using $f'_c = 21$ MPa, $f_y = 420$ MPa.



Sol.

$$W_u = 1.4\, D.L. + 1.7\, L.L.$$

$$= 1.4 * 20 + 1.7 * 26$$

$$= 72.2 \text{ kN/m}.$$

$$M_u = \frac{w_u l^2}{8} = \frac{72.2 * (6)^2}{8} = 324.9 \ kN.m$$

$$M_n = \frac{M_u}{\phi} = \frac{324.9}{0.9} = 361 \ kN.m$$

$$\because \ M_n = \rho . b . d^2 f_y \left(1 - 0.59 \rho . \frac{f_y}{f'_c}\right)$$

$$\rho_{max} = 0.75 \rho_b = 0.75 * 0.85 * 0.85 * \frac{21}{420} * \frac{600}{600+420}$$
$$= 0.016$$

$$\rho_{min} = \frac{\sqrt{f'_c}}{4 f_y} \text{ or } \frac{1.4}{f_y} \text{ whichever is greater}$$

$$\therefore \rho_{min} = \frac{1.4}{420} = 0.0033$$

Assume $\rho = 0.01$

$$M_n = \rho b d^2 f_y \left(1 - 0.59 \rho \frac{f_y}{f'_c}\right)$$

$$361 * 10^6 = 0.01 * b . d^2 * 420 \left(1 - 0.59 * 0.01 \frac{420}{21}\right)$$

$$\therefore b.d^2 = 97.45 * 10^6 \ mm^3$$

Take $b = 300$ mm

$\therefore d = 570$ mm

$$h = 570 + 10 + 40 + \frac{25}{2}$$
$$= 632.5$$

Take $h = 650$ mm

Exact $d = h - 40 - 10 - \frac{25}{2} = 587.5$ mm

Substitute in M_n equation

$$361 * 10^6 = \rho * 300 * (587.5)^2 * 420 \left(1 - 0.59 * \rho * \frac{420}{21}\right)$$

Solve the above quadratic equation for (ρ)

$\rho = 0.01$

$$A_{s_{required}} = \rho . b . d = 0.01 * 300 * 587.5$$

$$= 1762.5 \ mm^2$$

If we use $\phi 25$ mm bars $\Rightarrow A_b = 491 \ mm^2$

$$n = \frac{A_s}{A_b} = \frac{1762.5}{491} = 3.5$$

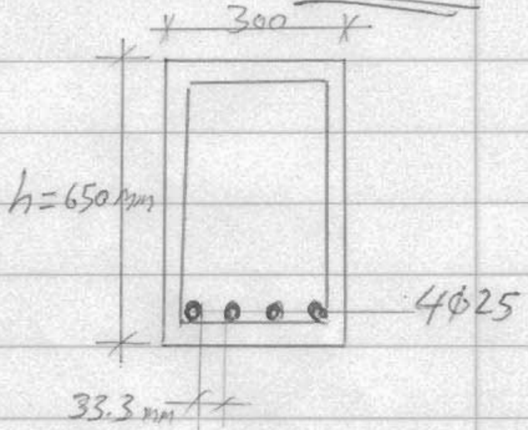
use $4 \phi 25$

$$A_{s_{provided}} = 4 * 491 = 1964 \ mm^2 > A_{s_{required}} = 1762.5 \ mm^2$$

o.K.

$$S = \frac{300 - 2*40 - 2*10 - 4*25}{3}$$

$$= 33.33 \ mm > 25 \ mm$$



* If concrete mix contain
Max. aggregate size = 20 mm
So that

$$S > \frac{4}{3} * 20 = 26.66 \ mm$$

Therefore $S = 33.33$ is o.K.
use $4 \phi 25$ mm bars in one layer

* If Max. aggregate size did not specified

$\therefore$ Max. aggregate size $= \frac{3}{4} S = \frac{3}{4} (33.33)$
$= 25 \ mm$