FUZZY LOGIC AND REASONING

Fuzzy reasoning system consists of *four* components:
- Knowledge base
- Fuzzification
- Inferencing
- Defuzzification

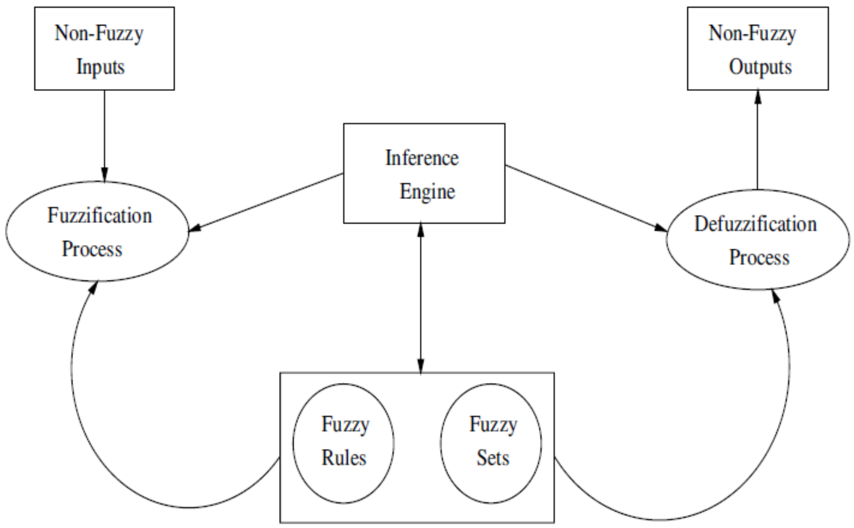


Figure 21.1 Fuzzy Rule-Based Reasoning System

Knowledge base

The knowledge base includes: *fuzzy rules and fuzzy sets.*

Fuzzy Rules

- For fuzzy systems in general, the dynamic behavior of that system is characterized by a set of linguistic fuzzy rules.
- These rules are based on the knowledge and experience of a human expert within that domain. Fuzzy rules are of the general form

if antecedent(s) then consequent(s)

- In general, there are two types of fuzzy rules
  1. *Mamdani fuzzy rules:*

     if A is a and B is b then C is c

     where A and B are fuzzy sets with universe of discourse $X1$, and C is a fuzzy set with universe of discourse $X2$.
  2. *Takagi-Sugeno fuzzy rules:*

     $$\text{if } f_1(A_1 \text{ is } a_1, A_2 \text{ is } a_2, \cdots, A_n \text{ is } a_n) \text{ then } C = f_2(a_1, a_2, \cdots, a_n)$$

     Here, Output sets are linear combinations of the inputs.

- We will focus on *Mamdani* rules in this course.

Fuzzification

- The fuzzification process is concerned with finding a fuzzy representation of non-fuzzy input values.
- This is achieved through application of the membership functions associated with each fuzzy set in the rule input space.
- Assume the fuzzy sets *A* and *B*. The fuzzification process receives the elements $a, b \in X$, and produces the membership degrees $\mu A(a)$, $\mu A(b)$, $\mu B(a)$ and $\mu B(b)$.

Inferencing

The task of the inferencing process is to map the fuzzified inputs to the rule base, and to produce a fuzzified output for each rule.

Consider the rule:

if *A* is *a* and *B* is *b* then *C* is *c*

Fuzzification process:

- *Rule evaluation*:the first step of the fuzzification process is then to calculate the *firing strength*of each rule in the rule base. This is achieved by the *min*-operator, the firing strength is

$$\min\{\mu_A(a), \mu_B(b)\}$$

  For each rule *k*, the firing strength α_k is thus evaluated.
- ***Aggregation***: then the final fuzzy value, β_i, associated with each outcome c_i is computed using the *max*-operator, i.e.

$$\beta_i = \max_{\forall k}\{\alpha_{k_i}\}$$

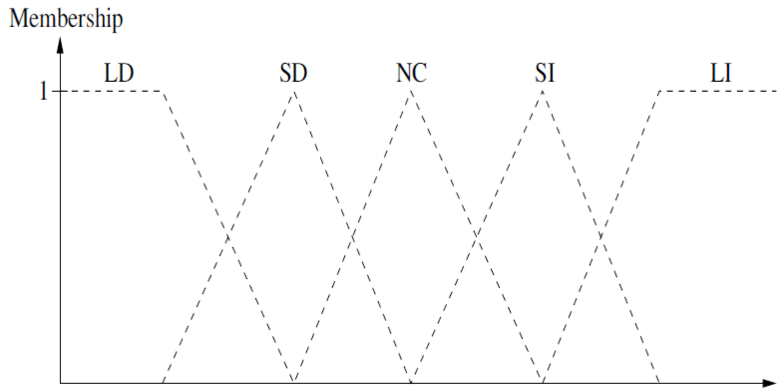
where α_{ki} is the firing strength of rule *k* which has outcome c_i.

Defuzzification

The task of the defuzzification process is to convert the output of the fuzzy rules into a scalar, or non-fuzzy value.

Center of Gravity method:

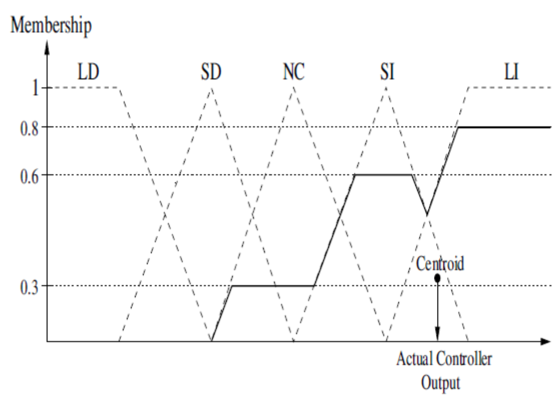
- It is the most popular method for perform the defuzzification process.
- For this approach, each membership function is clipped at the corresponding rule firing strengths.
- The centroid of the composite area is calculated and the horizontal coordinate is used as the output of the controller.
- For example, suppose the following fuzzy sets of output variable *C*.



(a) Output Membership Functions

- Assume three rules with the following *C* membership values: *μLI*= 0.8, *μSI*= 0.6 and *μNC*= 0.3.
- The output of the defuzzification process is calculated as:

$$output = \frac{\sum_{i=1}^{n_x} x_i \mu_C(x_i)}{\sum_{i=1}^{n_x} \mu_C(x_i)}$$

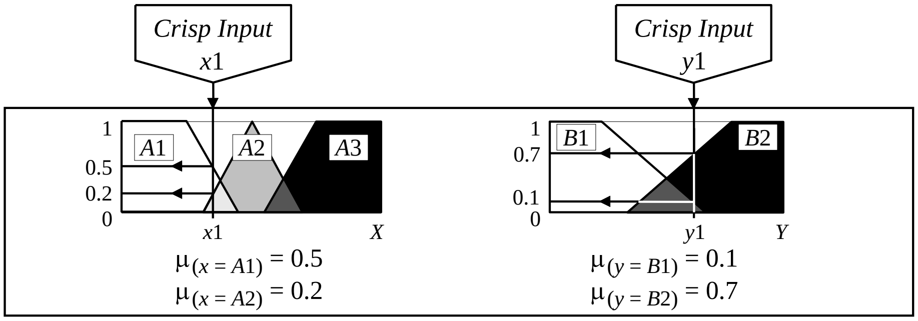


Example:We examine a simple two-input one-output problem that includes three rules:

Rule: 1	Rule: 1
IF *x* is *A*3	IF *project_funding* is *adequate*
OR *y* is *B*1	OR *project_staffing* is *small*
THEN *z* is *C*1	THEN *risk* is *low*
Rule: 2	Rule: 2
IF *x* is *A*2	IF *project_funding* is *marginal*
AND *y* is *B*2	AND *project_staffing* is *large*
THEN *z* is *C*2	THEN *risk* is *normal*
Rule: 3	Rule: 3
IF *x* is *A*1	IF *project_funding* is *inadequate*
THEN *z* is *C*3	THEN *risk* is *high*

Step 1: Fuzzification

The first step is to take the crisp inputs, *x*1 and *y*1 (*project funding* and *project staffing*), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Step 2: Rule Evaluation

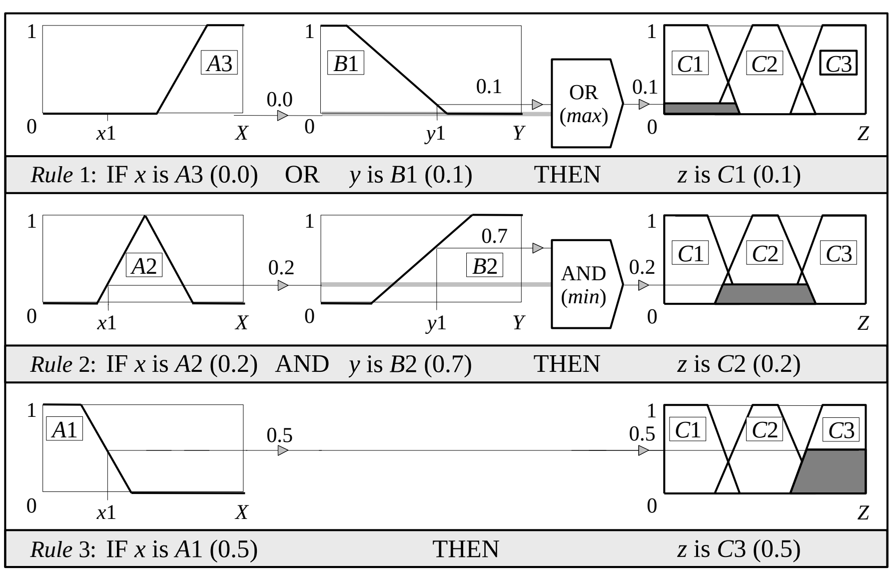
The second step is to take the fuzzified inputs, μ(*x*=*A*1) = 0.5, μ(*x*=*A*2) = 0.2, μ(*y*=*B*1) = 0.1 and μ(*y*=*B*2) = 0.7, and apply them to the antecedents of the fuzzy rules. If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation. This number (the truth value) is then applied to the consequent membership function.

To evaluate the disjunction of the rule antecedents, we use the *OR fuzzy operation*. Typically, fuzzy expert systems make use of the classical fuzzy operation **union**:

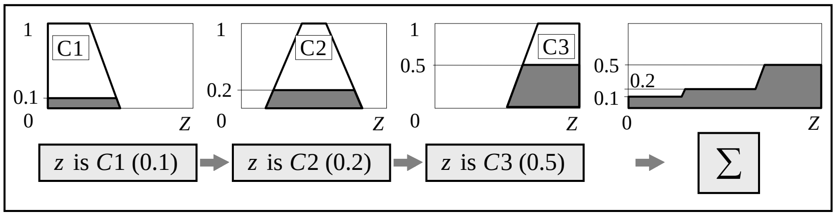
$$\mu A \cup B(x) = max\,[\mu A(x), \mu B(x)]$$

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the *AND fuzzy operation intersection:*

$$\mu A \cap B(x) = min\,[\mu A(x), \mu B(x)]$$



Aggregation of the rule outputs



Centre of gravity (COG):

$$COG = \frac{(0+10+20)\times 0.1+(30+40+50+60)\times 0.2+(70+80+90+100)\times 0.5}{0.1+0.1+0.1+0.2+0.2+0.2+0.2+0.5+0.5+0.5+0.5} = 67.4$$