Logical Agents

Logical agents combines:

1- A knowledge base (KB): a list of facts that are known to the agent.

2- Current percepts

to infer hidden aspects of the current state using *Rules of inference*

- This scheme is useful in non-episodic, partially observable environments

Knowledge Base (KB) – a set of sentences describing the world.

Logic: is a formal language to represent knowledge, such that *conclusions* can be drawn. It associates with:

1- ***Syntax***: rules for constructing valid sentences. E.g., $x + 2 \geq y$ is a valid arithmetic sentence, $\geq x2y +$ is not.

2- ***Semantic***: "meaning" of sentences, or relationship between logical sentences and the real world. Specifically, semantics defines *truth* of sentences. E.g., $x + 2 \geq y$ is true in a world where $x = 5$ and $y = 7$.

Propositional Logic

The simplest, and most abstract logic.

- **A proposition:** is a statement that can be either true or false; it must be one or the other, or it cannot be both.
  - Example: "the sky is blue", "9 is not prime number". What about "5+2", "Is it raining?", "n is prime", "come to the class" is it a proposition?.

- An **atomic sentences** is one whose truth or falsity does not depend on the truth or falsity of any other proposition.

We will abbreviate propositions by using *propositional variables* such as *p, q, r, …*

Syntax

To build *complex* sentences, we use the *connectives* (**not ¬, and ∧, or ∨, imply ⇒, iff ⇔**)

- **Negation:**
  - If **P** is a sentence, **¬P** is a sentence
- **Conjunction:**
  - If **P** and **Q** are sentences, **P ∧ Q** is a sentence
- **Disjunction:**
  - If **P** and **Q** are sentences, **P ∨ Q** is a sentence
- **Implication:**
  - If **P** and **Q** are sentences, **P ⇒ Q** is a sentence
- **Biconditional:**
  - If **P** and **Q** are sentences, **P ⇔ Q** is a sentence

Semantic

- A **model** specifies the true/false status of each proposition symbol in the knowledge base.
  - E.g., **P** is true, **Q** is true, **R** is false
  - With 3 ***symbols***, there are 2^3=8 possible ***models***, and they can be enumerated exhaustively by the **truth table**.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

- **Model of sentence (M(α)):** is the set of all models, where sentence α is *true*.
  Example: α= P ∨ Q, the M(α)= rows 2, 3, 4.

- A sentence is **valid** if it is true in **all** models,
  - e.g., A ∨¬A, A ⇒A, (A ∧ (A ⇒B)) ⇒B.

- A sentence is **satisfiable** if it is true in **some** model
  - e.g., P∨ Q.

- A sentence is **unsatisfiable** if it is true in **no** models
  - e.g., A∧¬A.

- Rules for evaluating *truth* with respect to a model:

NOT	**¬P**	is true	iff	**P**	is false			
AND	**P ∧ Q**	is true	iff	**P**	is true	and	**Q**	is true
OR	**P ∨ Q**	is true	iff	**P**	is true	or	**Q**	is true
IF-THEN	**P ⇒ Q**	is true	iff	**P**	is false	or	**Q**	is true
IF-AND ONLY IF	**P ⇔ Q**	is true	iff	**P ⇒ Q**	is true	and	**Q ⇒ P**	is true

Example: P="it is raining" Q=" I am in home"
¬P : it is not raining.
P ∧ Q : it is raining and I am in home.
P ∨ Q : it is raining or I am in home.
P ⇒ Q : if it is raining then I am in home.
Q ⇒ P : if I am in home then it is raining.
P ⇔ Q: it is raining if and only if I am in home.

Logical equivalence: two sentences α and β are logically equivalent *if they are true in the same set of models.*

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{de Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{de Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Entailment

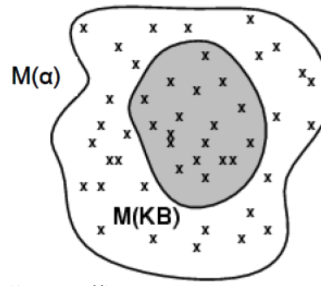
Entailment means that the truth of one sentence follows from the truth of another:

$$\mathbf{KB} \models \alpha$$

Knowledge base KB entails sentence α if and only if α is true in models where KB is true.

- e.g., the KB containing the "Giants won and the Reds lost" entails "The Giants won"

$$\mathbf{KB} \models \alpha \text{ iff } \mathbf{M(KB)} \subseteq \mathbf{M(\alpha)}$$



Inference

An inference procedure can:

– Generate new sentences α entailed by KB (KB ⊨ α)

– Determine whether or not a given sentence is entailed by KB (*i.e.* "prove")

- An inference procedure is ***sound*** if whenever it can derive only true sentences.
- An inference procedure is ***complete*** if can derive every entailed sentence.
- Inferences procedure is desirable to be sound and complete.

Inference Proof Methods

How can we check whether a sentence α is entailed by KB?

1- Inference by enumeration (i.e. using truth tables)

2- Inference rules.

Inference by enumerations

To determine if a sentence α is entailed by KB, then in all rows (models) where KB is true, α should be true.

- Enumeration method is sound and complete but the proofs using this technique grow exponentially in length as the number of symbols increases.

Example1: check (p, p→q ⊨ q).

Answer 1: Model checking method. Look at the truth table to see that if all of the premises (p, p→q) get a truth value of T then so do the conclusions (q).

p	*Q*	*P→q*
F	F	T
F	T	T
T	F	F
<u>T</u>	***<u>T</u>***	<u>T</u>

Answer 2: we can use deduction theorem*: KB→ a is valid*

p	*q*	*P→q*	*P ∧(p→q)*	*P ∧(p→q)→q*
F	F	F	F	T
F	T	F	F	T
T	F	F	F	T
T	T	T	T	T

Example2: check (¬p, p ∨ q ⊨ q) by deduction theorem.
Answer:

P	q	P ∨ q	¬p	(p ∨ q)∧ ¬p	(p ∨ q)∧ ¬p→q
F	F	F	T	F	T
F	T	T	T	T	T
T	F	T	F	F	T
T	T	T	F	F	T

H.W: use "model-checking" method to prove the above example.

Example3: Suppose "weather problem":
p= "it is hot"
q= "it is humid"
r="it is raining"
Let KB = {(p ^ q) → r, (q → p), q}, which mean
(1) "If it is hot and humid, then it is raining,"
(2) "If it is humid, then it is hot," and
(3) "It is humid."

Now let's ask the query "Is it raining?" That is, KB ⊨ r?

Answer: Using the deduction theorem approach to answering this query:

P	Q	R	(P ^ Q) => R	Q => P	Q	KB	R	KB => R
T	T	T	T	T	T	T	T	T
T	T	F	F	T	T	F	F	T
T	F	T	T	T	F	F	T	T
T	F	F	T	T	F	F	F	T
F	T	T	T	F	T	F	T	T
F	T	F	T	F	T	F	F	T
F	F	T	T	T	F	F	T	T
F	F	F	T	T	F	F	F	T

Inference Rules
It is faster way to implement the inference process **by** using a **proof procedure** that uses **sound rules of inference** to deduce (i.e., derive) new sentences that are true in all cases where the premises are true.

Given the sentences in KB, construct a **proof** that a given *conclusion* sentence can be derived from KB by applying a sequence of sound inferences using either sentences in KB or sentences derived earlier in the proof, until the conclusion sentence is derived.

This step-by-step, local proof process also relies on the **monotonicity** property. That is, adding a new sentence to KB does not affect what can be entailed from the original KB and does not invalidate old sentences.

Some Useful Rules

Name	**Premise(s)**	**consequence**
Modus Ponens	$p, p \rightarrow q$	q
Modus Tollens	$p \rightarrow q, \neg q$	$\neg p$
Syllogism	$p \rightarrow q, q \rightarrow r$	$q \rightarrow r$
And Introduction	p, q	$p \wedge q$
And Elimination	$p \wedge q$	q
Or Introduction	p	$p \vee q$
Double Negation	$\neg\neg p$	p
Unit Resolution	$p \vee q, \neg p$	q
Resolution	$p \vee q, \neg q \vee r$	$p \vee r$

H.W: verify all rules by truth-table.

Examples

1- Prove the conclusion "is it raining" for the "weather problem" given above by inference rules.

Answer:

1.	Q	Premise
2.	$Q \rightarrow P$	Premise
3.	P	Modus Ponens(1,2)
4.	$(P \wedge Q) \rightarrow R$	Premise
5.	$P \wedge Q$	And Introduction(1,3)
6.	R	Modus Ponens(4,5)

2- Given the premises: p, $p \rightarrow q$, $s \vee r$, $r \rightarrow \neg q$. Prove that: $s \vee t$.

Answer:

Step	Reason
1) p	Premise
2) $p \rightarrow q$	Premise
3) q	Modus Ponens (using 1, 2)
4) $r \rightarrow \neg q$	Premise
5) $q \rightarrow \neg r$	Contrapositive statement of 4
6) $\neg r$	Modus Ponenes (using 3, 5)
7) $s \vee r$	Premise

8) s	Unit Resolution (using 6, 7)
9) s∨ t	Or Introduction (using 8)

3- Prove the conclusion r, from the following premises.

q = "My friends are free."

p = "I get my Christmas bonus."

r = "I will take a road trip with my friends."

s = "My friends find a job after Christmas."

Premises:

$(p \wedge q) \rightarrow r$

$\neg s \rightarrow q$

p

$\neg s$

Conclusion: r

Answer:

Steps	Reasons
1) $\neg s \rightarrow q$	Premise
2) $\neg s$	Premise
3) q	Modus Ponens (using 1, 2)
4) p	Premise
5) $p \wedge q$	Conjunction
6) $(p \wedge q) \rightarrow r$	Premise
7) r	Modus Ponens (using 5, 6)

H.w) Prove that "he should Read Ch8" follows logically from these premises:
1. he is CS student → ¬he is Illiterate
2. he is Illiterate ∨ he can Read
3. he can Read ∧ he taking AI → he should Read Ch8

4. he is CS student
5. he taking AI

Disadvantages of propositional logic

Propositional Logic is not a very expressive language because:

- Hard to identify "individuals." E.g., Mary, 3. we need a unique symbol for each individual
- Can't directly talk about properties of individuals or relations between individuals. E.g., tall(Bill)
- Generalizations can't easily be represented. E.g., all triangles have 3 sides.