س1/ افرض ان $x / \theta \sim NB(r,\theta)$ وان $g(\theta) \sim Bata(\alpha;\beta)$ والمطلوب جد $f(\theta / x)$ اذا علمت ان

$$p(x / \theta) = \theta^{r}(1-\theta)^{x} \quad x = 0,1 \dots \quad ; 0 < \theta < 1$$

$$g(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}$$

$$f(\theta/x) = \frac{f(x/\theta)g(\theta)}{\int_0^1 f(x/\theta)g(\theta)d\theta}$$

$$f(\theta/x) = \frac{\theta^{r}(1-\theta)^{x}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\alpha)}\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\int_0^1 \theta^{r}(1-\theta)^{x}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1}d\theta}$$

$$f(\theta/x) = \frac{\theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}}{\int_0^1 \theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}d\theta}$$

$$f(\theta/x) = \frac{\theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}}{\frac{\Gamma(r+\alpha)\Gamma(x+\beta)}{\Gamma((r+\alpha)+(x+\beta))}\int_0^1 \frac{\Gamma((r+\alpha)+(x+\beta))}{\Gamma(r+\alpha)\Gamma(x+\beta)}\theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}d\theta}$$

$$f(\theta/x) = \frac{\theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}}{\frac{\Gamma(r+\alpha)\Gamma(x+\beta)}{\Gamma((r+\alpha)+(x+\beta))}}$$

$$f(\theta/x) = \frac{\Gamma((r+\alpha)+(x+\beta))}{\Gamma(r+\alpha)\Gamma(x+\beta)}\theta^{r+\alpha-1}(1-\theta)^{x+\beta-1}$$

$$f(\theta/x) \sim Beta(r+\alpha\,; x+\beta)$$

س2/ لجدول الخسارة الاتي

	θ_1	θ_2
d_1	14	7
d_2	28	0

فاذا علمت ان $p(x/\theta_2)=\frac{1}{2}$ $\quad p(\theta_1)=\frac{1}{2}$ $\quad p(x/\theta_1)=\frac{2}{3}$

والمطلوب قرار بيز.

$$h(\theta_i/x)=\frac{p(x/\theta_i)p(\theta_i)}{\sum_{i=1}^{n}p(x/\theta_i)p(\theta_i)}$$

$$h(\theta_1/x)=\frac{p(x/\theta_1)p(\theta_1)}{\sum_{i=1}^{n}p(x/\theta_i)p(\theta_i)}$$

$$h(\theta_1/x)=\frac{p(x/\theta_1)p(\theta_1)}{p(x/\theta_1)p(\theta_1)+p(x/\theta_2)p(\theta_2)}$$

$$h(\theta_1/x)=\frac{\frac{2}{3}.\frac{1}{2}}{\frac{2}{3}.\frac{1}{2}+\frac{1}{2}.\frac{1}{2}}$$

$$h(\theta_1/x)=\frac{\frac{2}{6}}{\frac{2}{6}+\frac{1}{4}}$$

$$h(\theta_1/x)=\frac{4}{7}$$

$$h(\theta_2/x)=\frac{p(x/\theta_2)p(\theta_2)}{p(x/\theta_1)p(\theta_1)+p(x/\theta_2)p(\theta_2)}$$

$$h(\theta_2/x)=\frac{\frac{1}{2}.\frac{1}{2}}{\frac{2}{3}.\frac{1}{2}+\frac{1}{2}.\frac{1}{2}}$$

$$h(\theta_2/x)=\frac{\frac{1}{4}}{\frac{2}{6}+\frac{1}{4}}$$

$$h(\theta_2/x)=\frac{3}{7}$$

$$E(L/x) = \sum h(\theta_i/x) L(d_i, \theta)$$
$$E(L/x) = h(\theta_1/x) L(d_1, \theta_1) + h(\theta_2/x) L(d_1, \theta_2)$$
$$E(L/x) = \frac{4}{7} \cdot 14 + \frac{3}{7} \cdot 7$$
$$E(L/x) = 11$$
$$E(L/x) = \frac{4}{7} \cdot 28 + \frac{3}{7} \cdot 0$$
$$E(L/x) = 16$$