Order Tracking Analysis

1. Introduction

Mostly, dynamic forces excited in a machine are related to the rotation speed; hence, it is often preferred to analyze vibration signals in terms of orders rather than frequencies. Orders relate the spectral components with the rotational speed and commonly followed by “xRPM” or simply “x” for recognition such as 1x, 3x, 6.5x and so on. Order tracking analysis (OTA) is the process of extracting useful information such as amplitude, frequency and phase for one order or more from the vibration (or acoustic) response to an input rotation speed. It has special importance when the order characteristics vary with time such as during machine speed-up and coast-down. Also, smearing or leakage problem in FFT analysis is effectively reduced when using order-based spectrum due to the fact that harmonic and sub-harmonic components will coincide with the analysis lines. Order tracking is of particular importance in the evaluation of rotor-bearing system characteristics, field balancing and performance evaluation in automobile industry.

The earliest order tracking techniques were based on analog devices. Vibration signal is fed to a tracking bandpass filter whose center frequency is controlled by a tachometer (speed) signal obtained from a tacho-generator. The output of the filter is passed to a peak or RMS detector to obtain the amplitude of the filtered signal. An x-y plotter can be utilized to plot the amplitude against the rotational speed. This method is very time-consuming and can process only limited sweep rates; therefore, it is rarely used at this time. With the vast development of digital equipment and related software during the end of the last century, order tracking techniques have been mainly digitally implemented for more than 20 years. Digital techniques are very accurate and fast such that most of them can be implemented in real time. Digital order tracking techniques are divided into two main categories, waveform reconstruction and non-reconstruction schemes. Both categories and their sub-classes, according to their appearance, will be discussed in details in the following sections.

2. Non-Reconstruction Order Tracking Techniques

As the name refers, the time signal does not need to be reconstructed to obtain the required information about a particular order. Alternatively, these techniques extract the useful information such as amplitude and phase in the frequency or order domain. The obtained parameters represent the averaged values over a certain time interval. This interval can be given in terms of constant time interval, speed interval, percentage speed interval or number of revolutions. The choice of the interval depends on the technique used and test conditions. These techniques are ranging from simple constant-Δf FFT-based to more sophisticated techniques such as computed order tracking (COT) and time-variant discrete Fourier transform (TVDFT).

2.1 FFT-Based Order Tracking

The simplest and commonly applied digital order tracking technique is based on Fourier transform of the time signal. Since FFT is the most computationally efficient algorithm to perform discrete Fourier transform, the signal is divided into a number of blocks with power of two blocksize. FFT is performed for each block and the results are displayed as FFT waterfall or spectrogram plot. The FFT waterfall is a 3D plot in which the FFT spectra are displayed in cascaded form. It has special importance in detecting the resonances as the machine speed sweeps over the entire range. The spectrogram (or spectrograph) is a 2D plot which is similar in some way to the scalogram used in representing CWT coefficients. However, the horizontal axis in the spectrogram represents frequency while the vertical axis represents the time or rotational speed. The amplitudes of the FFT components are color coded in order to be easily detected by visual inspection. Fig. 1 demonstrates the spectrogram obtained from analyzing vibration signal during coast-down test of a large turbogenerator. In addition to the 1x and 2x components, which may be caused by residual unbalance and misalignment, there is large amplitude at 37th harmonic which was excited by a fan of 37 blades in the turbine cooling system. The constant-frequency components, such as structural resonances, appear as vertical lines in this type of spectrograms.

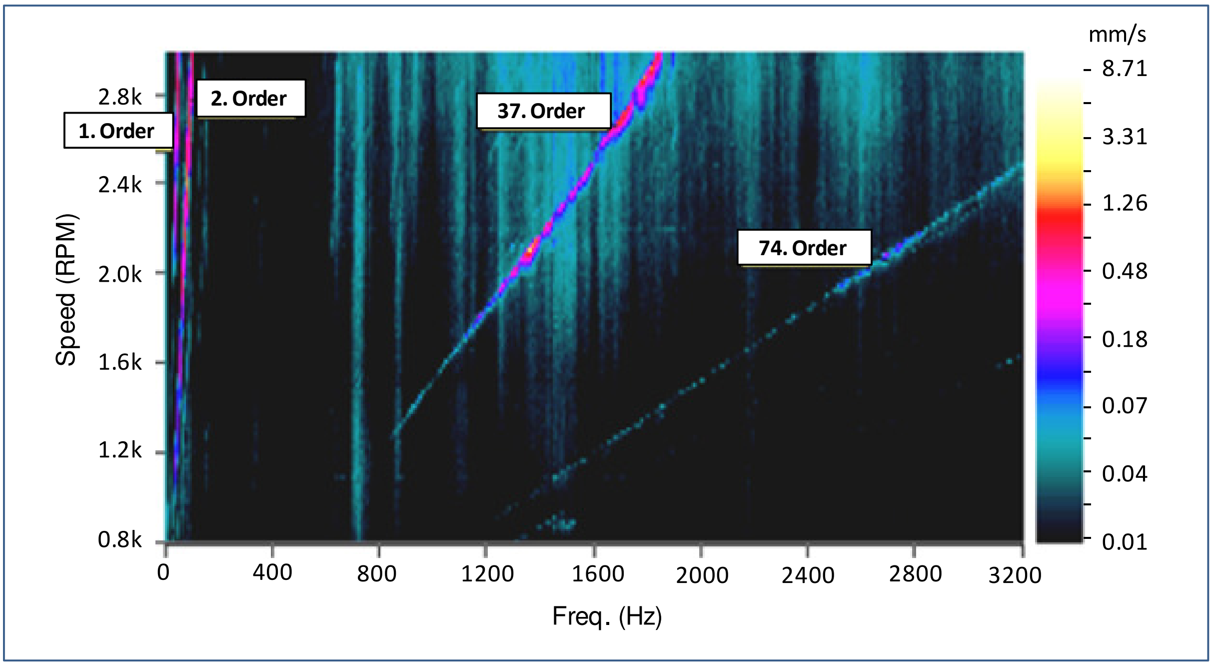

Figure-1 Spectrogram of Coast-down Test for a Large Turbogenerator

Despite the fact that this technique is fast and easy to apply, it suffers from a number of limitations. The blocksize (time interval) is not related to the RPM of the machine and this can be problematic in two ways. At low shaft speed, the interval is too short to cover the low or sub-harmonic orders which results in power leakage (smearing) between closely-spaced orders. On the other hand, at high shaft speed, the interval becomes too long to capture rapid variations and spikes in the signal. Moreover, since the analysis interval covers a certain sweep in the speed, the power of the high-frequency components, such as gearmesh frequency and its sidebands, spreads over several FFT lines resulting in smearing problem. For the above reasons, the order-based analysis becomes highly desirable.

2.2 Constant Angle Order Tracking

From the above discussion, it is clear that analysis time interval should be adequately large for low machine speeds and reasonably small for high speeds. To achieve this goal, the constant angle of rotation can be used instead of constant time interval. Given that the blocksize remain constant, this implies that the signal must be sampled at constant $\Delta\theta$ rather than constant Δt. The straightforward method to accomplish this scenario is to use a shaft encoder with suitable number of pulses per revolution (PPR) to drive the ADC sampling clock and the anti-aliasing tracking filter. However, the shaft encoder requires special considerations which make it difficult to apply in many cases. Furthermore, a sophisticated and high cost measurement circuit is required to interact with the encoder pulses. To overcome these problems, an alternative method, known as Computed Order Tracking (COT), was developed and patented by Potter at the Hewlett Packard. In fact, constant angle order tracking is also known as COT due to popularity of this algorithm.

COT as proposed by Potter is based on resampling the constant Δt time signal to obtain a new set of values with constant $\Delta\theta$. The resampled data is now in the angle domain, as opposed to the original time domain. The resampling process is computationally demanding and it includes two steps, oversampling and then interpolation to obtain the required equi-angle samples. This approach requires an accurate tachometer signal to obtain reliable data about the machine speed. The intervals of resampling are calculated by integrating the speed function. By resampling, any variable-frequency order component is converted into regular sinusoid signal. When DFT or FFT is used to process the resampled signal, the spectral (analysis) lines will represent constant-order components since the transform is based on angular domain rather than time domain. The order resolution Δo can be found in similar fashion to the frequency resolution of the normal Fourier transform, i.e. it is the reciprocal of the total angle of rotation of N-samples Θ:

$$\begin{aligned} \Delta o &= \frac{1}{\Theta} \\ \Theta &= N\,\Delta\theta \end{aligned} \tag{1}$$

There is an equivalent Shannon's sampling theorem in the order domain which can be stated in the following equations:

$$\begin{aligned} o_s &= \frac{1}{\Delta\theta} \\ o_{Nyq} &= \frac{o_s}{2} \end{aligned} \tag{2}$$

Where o_s is the angular sampling rate and o_{Nyq} is the angular Nyquist rate or the maximum order that can be processed. It is worth mentioning that when N is carefully chosen to satisfy integer number of revolutions, the smearing problem is further minimized even when there is no smoothing window function. However, when there is one non-harmonic component or more, the smoothing window is required to reduce smearing problem. For FFT algorithm which needs a power of 2 blocksize, the block can be zero-padded to provide the extra samples required beyond the integer number of revolutions. When the order-based FFT is used to analyze the same vibration signal discussed in previous section, the resulting spectrogram will have constant-order lines as can be shown in Fig. 2. The order components are more clear and easier to indentify than that in the previous spectrogram.

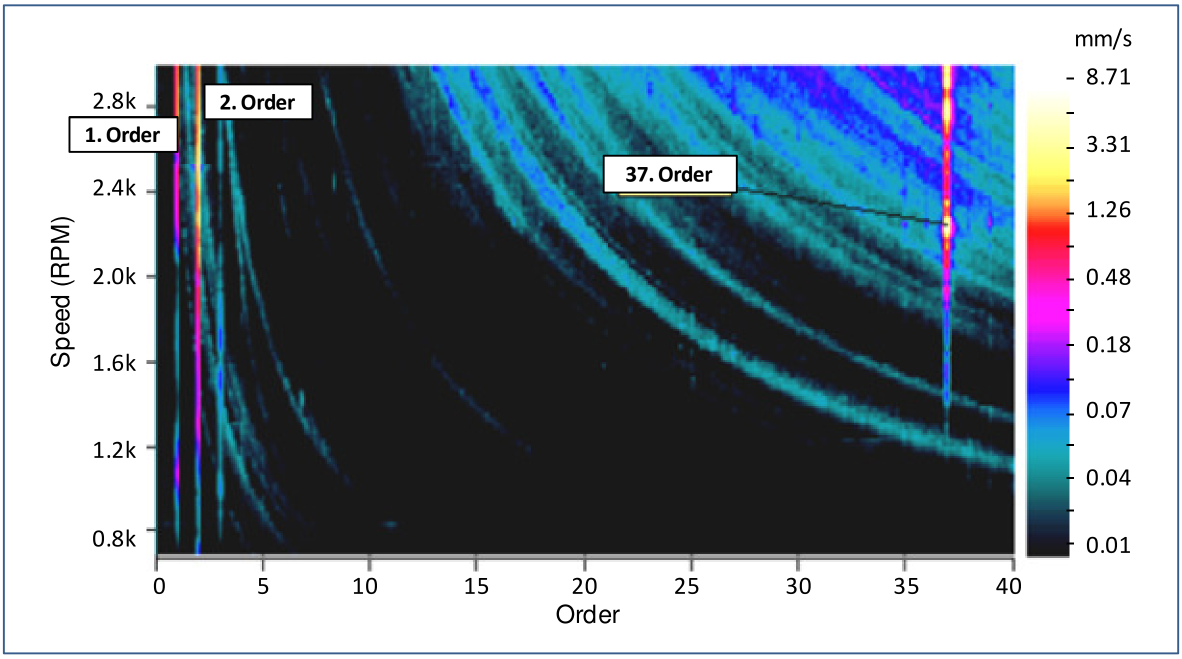

Figure 1 Order Based Spectrogram

2.3 STFT Order Tracking

Short-time Fourier transform (STFT) is a Fourier-related transform used to analyze local sections of a non-stationary signal. It has the ability to characterize the signal amplitude and phase both in time and frequency. In fact, Gabor transform, which forms the basis of Wavelet transform, was inspired by the idea of STFT. The continuous STFT of a signal $x(t)$ is given by:

$$X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)\, w(t-\tau) e^{-i\omega t} dt \qquad (3)$$

Where $w(t)$ is the window function, usually a Hann window or Gaussian bell centered around zero. It is simply the Fourier transform of the signal multiplied by time-shifted window function. The discrete STFT can be expressed as follows:

$$X(m,\omega)=\sum_{n=-\infty}^{\infty}x(n)w(n-m)e^{-i\omega n} \qquad (4)$$

Where m represents shifting factor in terms of time samples. In contrast to FFT-based order tracking, STFT includes multiplying the successive blocks by a window function. Also, the blocks are usually overlapped by a certain amount to avoid artifacts or blocking effect. The overlapping percentage is controlled by the step of shifting factor m. By varying the support length of the window function, different time and frequency resolutions can be obtained. Short window function provide good time resolution but on the expense of frequency resolution. The amplitudes squares (power) of $X(m,\omega)$ are used to plot STFT spectrogram with frequency on the vertical axis and time on the horizontal axis or optionally vice versa.

2.4 Time-Variant DFT Order Tracking

This approach does not require resampling the data, instead it is based on DFT which has kernel (basis) of variable frequency over the data block; hence "time-variant" prefix comes out. In fact, TVDFT is a special case of chirp z-transform which is defined as the Fourier transform with kernel of variable frequency and damping over time. The kernel frequency for a specific order is excerpted from the tachometer signal. Like the COT method, the TVDFT is able to provide leakage free order analysis but without the need to resample the data in the angle domain. Thus, it is much less computationally demanding than COT, making it more suitable for real-time analysis for rapidly varying speeds and large number of response channels. According to the TVDFT, the individual order/frequency components can be evaluated as follows:

$$\begin{aligned} X_k &= \frac{1}{N}\sum_{n=0}^{N-1}x(n)\,e^{-i\phi_{k,n}} \\ \phi_{k,n} &= \int_0^{n\Delta t} o_k\,\omega(t)\,dt \end{aligned} \qquad (5)$$

Where $\phi_{k,n}$ is the angular displacement for order component k up to the time $n\Delta t$ and $\omega(t)$ is the instantaneous angular speed. Of course, to reduce smearing as much as possible, the blocksize needs to cover an integer number of revolutions. The order resolution is the reciprocal of the number of revolutions covered by the block. As a numeric example, when it is required to obtain 0.1x order resolution, the analysis record length should be extended over 10 shaft revolutions. If this is not the case, then a smoothing time window function, such as Hann or Blackman window, can be used to reduce smearing. In fact, since the data is sampled at constant Δt, it is not guaranteed that the exact integer number of revolution is obtained. Also, if there are non-harmonic components contained in the signal, smearing is more likely to occur even when using integer number of revolutions. Therefore, it is desirable to use a smoothing window function.

To reduce correlation among the closely-spaced orders and/or crossing frequency components, the orthogonality compensation matrix (OCM) can be used. Due to non-orthogonality of TVDFT kernels, evaluation of each order/frequency component requires direct summation according to eq. 5; hence, this method is more suitable to extract a certain number of orders rather than getting order-spectrum to plot the spectrogram. The amplitudes and phase angles of extracted orders can be plotted against the shaft speed to get the so called Bode plot or frequency response function (FRF). The amplitude and/or frequency axes can optionally be in log or linear scale.

In this scheme, the Orthogonality Compensation Matrix (OCM) is used to decorrelate the estimated orders according to the following linear equations:

$$\begin{bmatrix} \tilde{X}_1 \\ \tilde{X}_2 \\ \tilde{X}_3 \\ \vdots \\ \tilde{X}_m \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & \cdots & c_{1m} \\ c_{21} & c_{22} & c_{23} & \cdots & \\ c_{31} & c_{32} & c_{33} & \cdots & \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ c_{m1} & c_{m2} & c_{m3} & \cdots & c_{mm} \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ \vdots \\ X_m \end{bmatrix} \quad (6)$$

Where $\tilde{X}_k$ are the correlated components, X_k are the actual (uncorrelated) components and c_{ij} are the cross-correlation or orthogonality compensation terms. Each term in the OCM represents how much a kernel interacts with another. Since the accuracy of compensation depends on the quality of the correlated components estimation, special care must be taken in calculating these components to achieve better accuracy. In many cases, Hann or Blackman window will help in smoothing out some undesirable blocking effect. The correlated (uncompensated) components can be estimated from the following equation:

$$\tilde{X}_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\,w(n)\,\mathrm{e}^{i\phi_{k,n}} \tag{7}$$

The cross-correlation terms c_{ij} can be calculated by applying the kernel of order i to the conjugate kernel of order j [50]:

$$c_{ij} = \frac{1}{N}\sum_{n=0}^{N-1} \mathrm{e}^{i\phi_{i,n}}\,w(n)\left(\mathrm{e}^{i\phi_{j,n}}\right)^* \tag{8}$$

To solve for the uncorrelated components, the OCM must be inverted and multiplied by the vector of correlated components, or alternatively Gauss elimination can be used to solve the system of linear equations. The second approach is preferable due to ease of implementation, reduced processing time and enhanced stability as compared to the inversion scheme. However, another consideration must be taken in applying the compensation scheme. Unless, all orders with significant energy are included in the compensation scheme, the detected components will suffer from distortion due to contribution of uncompensated order(s). This requires pre-knowledge of all effective orders in the signal prior to applying the compensation scheme. Normal FFT based tracking analysis may be used to estimate the effective orders.

3. Reconstruction Order Tracking Techniques

These schemes enable to extract the order/frequency components and reconstruct their time histories. Instead of estimating the average amplitude and phase angle over a certain block size, the time history allows the determination of instantaneous values at any time. However, in the presentation of data as spectrogram, the overall time signal can be segmented and the average or peak amplitude of each order/frequency component may be used to represent the amplitude over the segments. While reconstruction order tracking methods are the most accurate techniques, they require extensive computations which limit their use in practical daily analysis procedures. These techniques include the Vold-Kalman and Gabor based order tracking.

3.1 Vold-Kalman Order Tracking

The Kalman filter was first adapted to order tracking by Vold and Leuridan in 1993 where the first generation Vold-Kalman order tracking (VKOT) scheme was proposed. Later, Vold et al. have developed the technique further and proposed the second generation VKOT scheme with improved capabilities such as close-orders and cross-orders separation. The basic idea behind the Vold-Kalman filter is to define local constraints which state that the unknown orders are smooth (having slowly varying amplitude) and that the sum of the orders should approximate the total observed signal. The relationship with the measured data is called the "data equation" while smoothness condition is called the "structural equation". In fact, structural equation works like low-pass filter. The second generation data equation is given by:

$$y(n) = x(n)\, e^{i\phi_n} + \eta(n) \tag{9}$$

Where $y(n)$ is the observed (measured) signal, $x(n)$ is the filtered order, $\eta(n)$ is the error term and ϕ_n is the angular displacement of the required order. The structural equation of the second generation VKOT is given by:

$$\nabla^p x(n) = \varepsilon(n) \tag{10a}$$

Where ∇^p represents the backward difference of order p and $\varepsilon(n)$ is the non-homogeneity term. For example for two-pole low-pass filter action (p=2), the structural equation becomes:

$$x(n) - 2x(n-1) + x(n-2) = \varepsilon(n) \tag{10b}$$

When it is required to separate multiple closely-spaced or cross orders, the data and structural equations are applied for all of the orders simultaneously. The drawback of this approach is that the resulting system of equation is very large as compared to the system obtained from applying data and structural equations to extract one order only.

3.2 Gabor Order Tracking

The STFT discussed earlier is generally not invertible, i.e. it is not possible to reconstruct the time signal from the STFT coefficients. However, when certain conditions are applied, Gabor expansion can be used to recover time signal from the modified STFT. For a given set of discrete time signal $x(n)$, the modified STFT is given by:

$$X(m,k) = \sum_{n=0}^{N-1} x(n) w^*(n - m\Delta M) e^{-i 2\pi kn/N}$$
$$\text{for } 0 \le k \le K-1, 0 \le m \le M-1 \quad (11)$$

Where ΔM is the time-sample step that is a positive integer, K is the total number of frequency bins. The total number of time points $M = N/\Delta M$. The resulting coefficients $X(m,k)$ sometimes known as Gabor coefficients with total size = $M \times K$. Critical sampling occurs when $K = \Delta M$ which gives N Gabor coefficients, while oversampling occurs when $K > \Delta M$. In case of oversampling, the transform in eq. 11 contains redundancy from a mathematical point of view. The original data samples can be reconstructed as follows :

$$x(n) = \sum_{m=0}^{M-1} \sum_{k=0}^{K-1} X(m,k) h(n - m\Delta M) e^{i 2\pi kn/N} \quad \text{for } 0 \le n \le N-1 \quad (12)$$

The above expansion is traditionally known as Gabor expansion. The window function $h(n)$ is called synthesis window as opposed to analysis window $w(n)$. The key issue of implementing Gabor expansion is the choice of both the analysis and synthesis window functions. In fact, the position of these windows can be interchanged; hence, $w(n)$ and $h(n)$ are called "dual functions". Gabor order tracking (GOT) is used to extract a specific order component according to the following steps:

1. After selecting the appropriate analysis window, synthesis windows and the Gabor sampling rate, the Gabor coefficients are evaluated from eq. 11.

2. The resulting Gabor coefficients are multiplied by a mask window to keep only the relevant coefficients while other coefficients are set to zero. The center frequency of the mask window is determined by the tachometer signal and the required order while its bandwidth is carefully selected.

3. Performing Gabor expansion using the masked Gabor coefficients to obtain the filtered order/frequency component.

Step 2 and 3 can be repeated for all the required orders. Despite the computational efficiency of the above scheme as compared to VKOT, it is only suitable to extract well-spaced non-crossing orders.