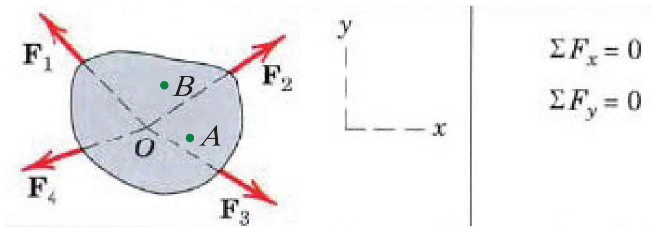
Concurrent Force System (Equilibrium of Particles)

Recall that the resultant of a concurrent force system is a force $\boldsymbol{F_R}$ that passes through the point of concurrency, which we label as point O. The moment equation, $\Sigma M_O = 0$ now is satisfied trivially, so that the number of independent equilibrium equations is reduced from three to two.



i) *Two force equations*

$$\Sigma F_x = 0, \qquad \Sigma F_y = 0, \qquad \textbf{...(3-2)}$$

where x and y are any two non-parallel directions in the xy-plane.

ii) *Two moment equations.*

$$\Sigma M_A = 0, \qquad \Sigma M_B = 0, \qquad \textbf{...(3-3)}$$

where A and B are any two points in the xy-plane (except point O) provided that A, B, and O do not lie on a straight line.

iii) *One force equation and one moment equation.*

$$\Sigma F_x = 0, \qquad \Sigma M_A = 0, \qquad \textbf{...(3-4)}$$

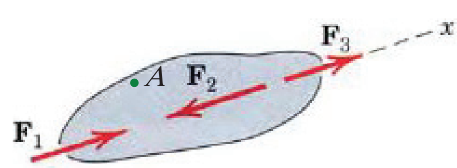
where A is any point in the xy-plane (except point O) and x is any direction that is not perpendicular to the line OA.

***Note:* For a collinear force system** Eq.(3-2) reduces to one equation,

$$\Sigma F_x = 0, \qquad \textbf{...(3-5)}$$

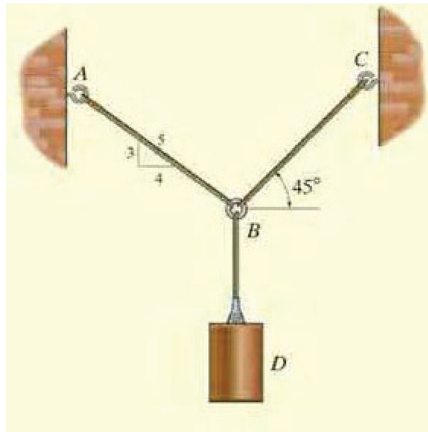
where the x-axis is parallel to the forces. It is also can be use the equation

$$\Sigma M_A = 0, \qquad \textbf{...(3-6)}$$



instead of Eq.(3-5), where A is not on the action line of the forces, of the collinear system. Consequently, only one unknown can be determined when a collinear force system in equilibrium.

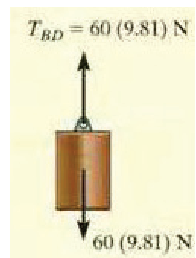
Example 1: Determine the tension in cables BA and BC necessary to support the 60-kg cylinder in Figure (a).



(a)

Solution: Free-Body Diagram:

T_{BD} = the weight of the cylinder = 60(9.82) N



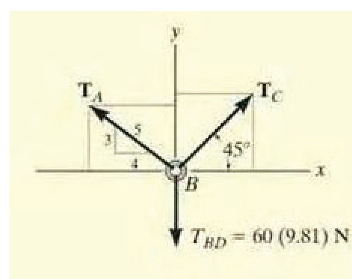
(b)

Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes, we have

$$\xrightarrow{+} \sum F_x = 0; \qquad T_C \cos 45^\circ - \left(\frac{4}{5}\right) T_A = 0 \qquad (1)$$

$$+\uparrow \sum F_y = 0; \qquad T_C \sin 45^\circ + \left(\frac{3}{5}\right) T_A - 60(9.81) = 0 \qquad (2)$$

Eq.(1) can be written as $T_A = 0.8839 T_C$. Substituting this into Eq.(2) yields



(c)

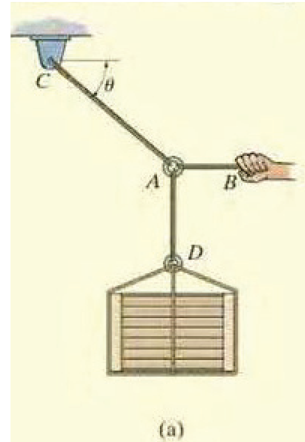
$$T_C \sin 45^\circ + \left(\frac{3}{5}\right)(0.8839 T_C) - 60(9.81) = 0$$

So that $\qquad T_C = 475.66 \text{ N} = 466 \text{ N}$

Substituting this result into either Eq.(1) or Eq.(2), we get

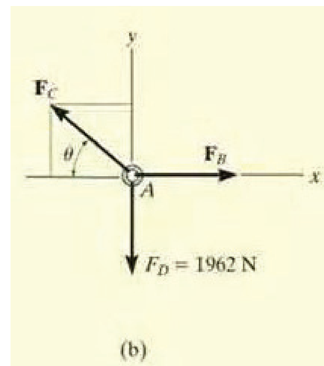
$$T_A = 420 \text{ N}$$

Example 2: The 200-kg crate in Figure (a) is suspended using ropes *AB* and *AC*. Each rope can withstand a maximum force of 10 kN before it breaks. If *AB* always remains horizontal, determine the smallest θ to which the crate can be suspended before one of the ropes breaks.



Solution: Free-Body Diagram: F_D is equal to the weight of the crate, i.e.,

$$F_D = 200(9.81)\text{N} = 1962 \text{ N} = 1.962 \text{ kN} < 10 \text{ kN}$$



Equations of Equilibrium:

$$\xrightarrow{+} \sum F_x = 0; \qquad -F_C\cos\theta + F_B = 0; \qquad F_C = \frac{F_B}{\cos\theta} \qquad (1)$$

$$+\uparrow \sum F_y = 0; \qquad F_C\sin\theta - 1962 = 0 \qquad (2)$$

From Eq.(1), F_C is always greater than F_B since $\cos\theta \leq 1$. Therefore, rope *AC* will reach the maximum tensile force of 10 kN *before* rope *AB*. Substituting F_C =10 kN into Eq.(2), we get

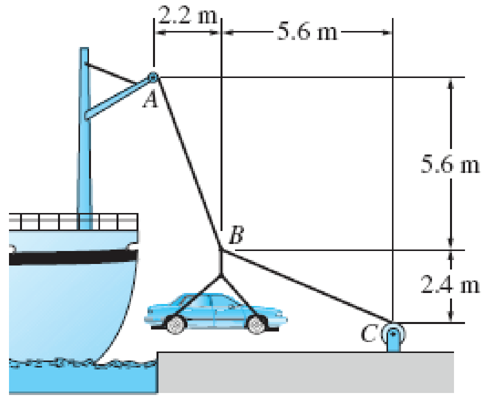
$$[10\times10^3]\sin\theta - 1962 = 0$$

$$\theta = \sin^{-1}(0.1962) = 11.31^\circ = 11.3^\circ$$

The force developed in rope *AB* can be obtained by substituting the value of θ and F_C into Eq.(1).

$$10\times10^3 = \frac{F_B}{\cos 11.31^o} \Longrightarrow F_B = 9.81 \text{ kN} < 10 \text{ kN} \qquad \text{o.k.}$$

Example 4: The 1200-kg car is being lowered slowly onto the dock the hoist *A* and winch *C*. Determine the forces in cables *BA* and *BC* for the position shown.



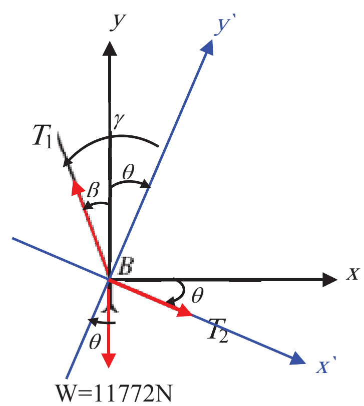
Solution: Draw FBD of the problem.

Weight of the car $W = mg = 1200 \times 9.81 = 11772$ N

$$\theta = \tan^{-1}\left(\frac{2.4}{5.6}\right) = 23.2^o, \qquad \text{and} \qquad \beta = \tan^{-1}\left(\frac{2.2}{5.6}\right) = 21.45^o$$

Note: To find T_1 and T_2 we can use the Cartesian components along x and y directions, but this yields two simultaneous equations. So to avoid this we can rotate the coordinate system such that on the directions (x or y) will be along one of the unknown forces (T_1 or T_2).

Thus we will use the $x`y`$- coordinate system.



$$\gamma = \theta + \beta = 23.2 + 21.45 = 44.65^o$$

$$+\nearrow \ \Sigma F_{y`} = 0 \Rightarrow \qquad T_1 \cos\gamma - W\cos\theta = 0$$

$$\therefore T_1 = \frac{W\cos\theta}{\cos\gamma} = \frac{11772\cos 23.2}{\cos 44.65} = 15209.25\,N$$

$$+\searrow \ \Sigma F_{x`} = 0 \qquad \Rightarrow \qquad T_2 - T_1 \sin\gamma + W\sin\theta = 0$$

$$\Rightarrow \qquad T_2 = T_1 \sin\gamma - W\sin\theta = 15209.25 \sin 44.65 - 11772 \sin 23.2$$

$$T_2 = 6051.17\,N$$

Example 5: Two smooth pipes, each having a mass of 300 kg, are supported by the forked tines of the tractor in Figure a, below. Draw the free-body diagrams for each pipe and both pipes together, and then determine the reactions on the forked tines.

Solutions: Weight of each pipe $W = mg = 300 * 9.81 = 2943$ N

From the FBD of Pipe *B*

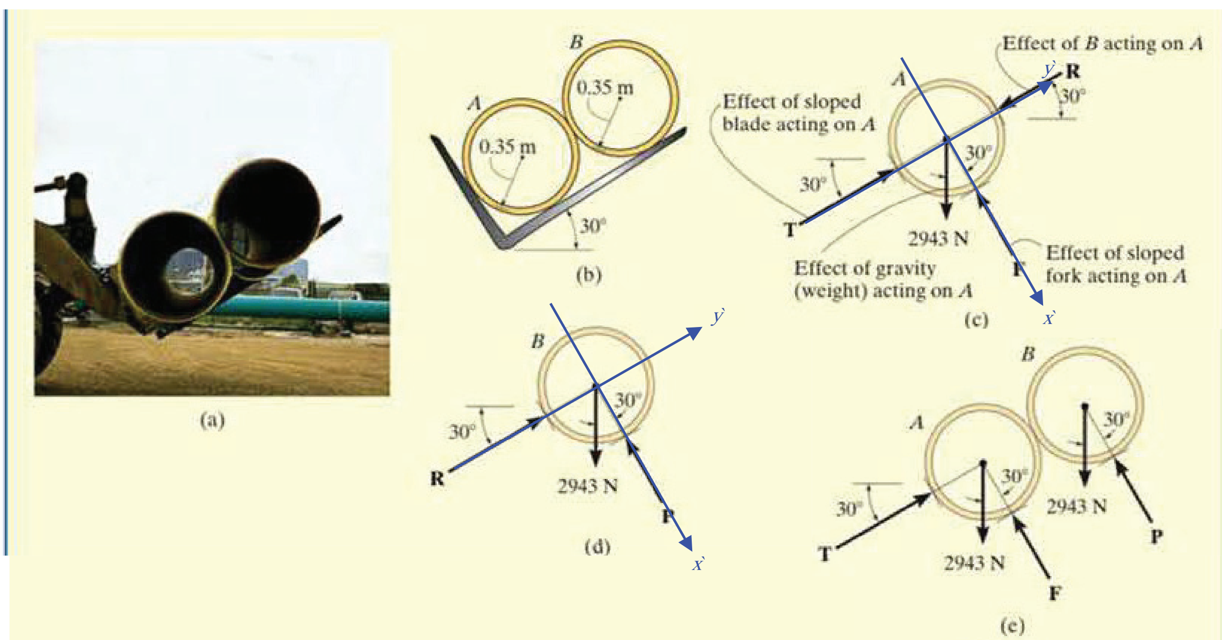
$\Sigma F_{x'} = 0 \quad \Rightarrow \quad W\cos 30 - P = 0 \quad \Rightarrow \quad P = 2943\cos 30 = 2548.7$ N

$\Sigma F_{y'} = 0 \quad \Rightarrow \quad R - W\sin 30 = 0 \quad \Rightarrow \quad R = 2943\sin 30 = 1471.5$ N

From the FBD of Pipe *A*

$\Sigma F_{x'} = 0 \quad \Rightarrow \quad F - W\cos 30 = 0 \quad \Rightarrow \quad F = 2943\cos 30 = 2548.7$ N

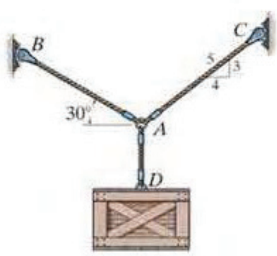
$\Sigma F_{y'} = 0 \quad \Rightarrow \quad T - W\sin 30 - R = 0 \quad \Rightarrow \quad T = 2943\sin 30 + 1471.5 = 2943$ N



FUNDAMENTAL PROBLEMS

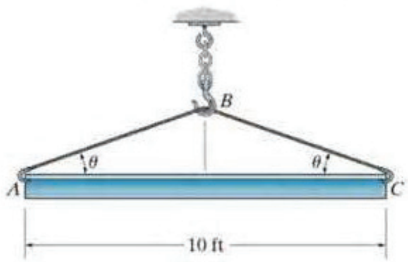
All problem solutions must include an FBD.

F3–1. The crate has a weight of 550 lb. Determine the force in each supporting cable.



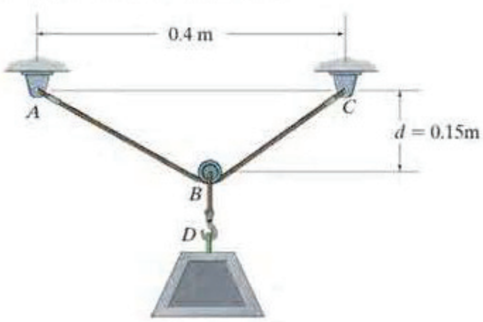
F3–1

F3–2. The beam has a weight of 700 lb. Determine the shortest cable ABC that can be used to lift it if the maximum force the cable can sustain is 1500 lb.



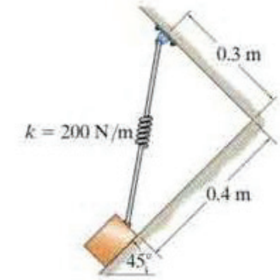
F3–2

F3–3. If the 5-kg block is suspended from the pulley B and the sag of the cord is $d = 0.15$ m, determine the force in cord ABC. Neglect the size of the pulley.



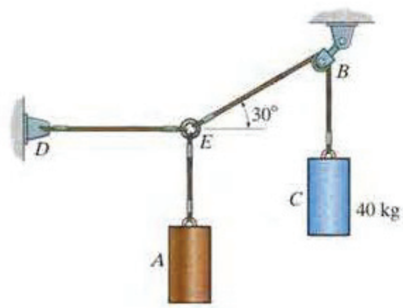
F3–3

F3–4. The block has a mass of 5 kg and rests on the smooth plane. Determine the unstretched length of the spring.



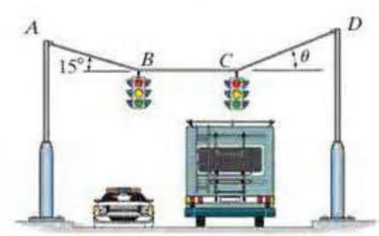
F3–4

F3–5. If the mass of cylinder C is 40 kg, determine the mass of cylinder A in order to hold the assembly in the position shown.



F3–5

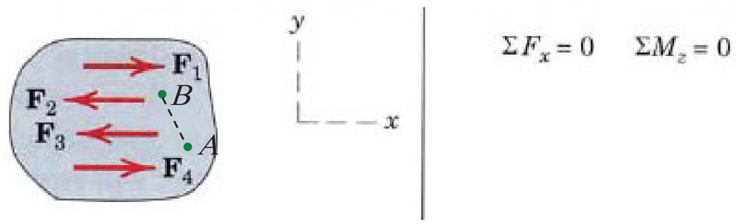
F3–6. Determine the tension in cables AB, BC, and CD, necessary to support the 10-kg and 15-kg traffic lights at B and C, respectively. Also, find the angle θ.



F3–6

Parallel Force System

Assume that all the forces lying in the *xy*-plane are parallel to the *x*-axis. The equation $\Sigma F_y = 0$ is automatically satisfied, and the number of independent equilibrium equations is again reduced from three to two.



i) *One force equation and one moment equation.*

$$\Sigma F_x = 0, \qquad \Sigma M_A = 0, \qquad \textbf{...(3-7)}$$

where x is any direction in the *xy*-plane except the *y*-direction, and A is any point in the *xy*-plane.

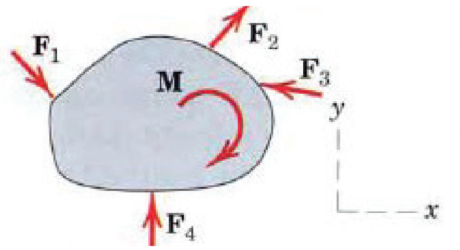
ii) *Two moment equations.*

$$\Sigma M_A = 0, \qquad \Sigma M_B = 0, \qquad \textbf{...(3-8)}$$

where A and B are any two points in the *xy*-plane, provided that the line AB is not parallel to the *x*-axis.

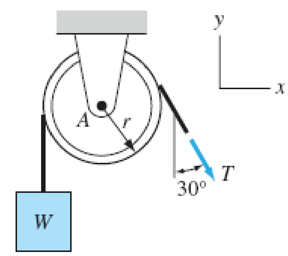
The force system that holds a body in equilibrium is said to be *statically determinate* if the number of independent equilibrium equations equals the number of unknowns that appear on its free-body diagram. Statically determinate problems can therefore be solved by equilibrium analysis alone. If the number of unknowns exceeds the number of independent equilibrium equations, the problem is called *statically indeterminate.* The solution of statically indeterminate problems requires the use of additional principles that are beyond the scope of this text.

General Case:



$$\Sigma F_x = 0 \qquad \Sigma M_z = 0$$
$$\Sigma F_y = 0$$

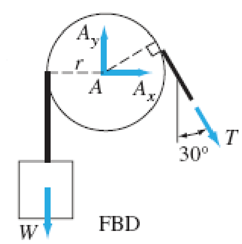
Example 6: The weight W is attached to one end of a rope that passes over a pulley that is free to rotate about the pin at A. The weight is held at rest by the force T applied to the other end of the rope. Using the given FBD, show that $T = W$ and compute the pin reactions at A.



Solution: From the FBD

$$\Sigma M_A = 0 \ \circlearrowleft{+} \qquad Wr - Tr = 0 \implies T = W.$$

***Note*:** This result is significant because it shows that the *tension in a rope does not change when the rope passes over a pulley that is supported by a frictionless pin.*



$$\Sigma F_x = 0 \ \xrightarrow{+} \qquad A_x + T\sin 30^\circ = 0$$

$$\Sigma F_y = 0 \ +\uparrow \qquad A_y - W - T\cos 30^\circ = 0$$

With $T = W$, the last two equations yield

$$A_x = -0.5W \qquad A_y = 1.866W$$

The minus sign indicates that A_x acts to the left; that is, in the direction opposite to what is shown on the FED.