EQUILIBRIUM

DEFINITION OF EQUILIBRIUM

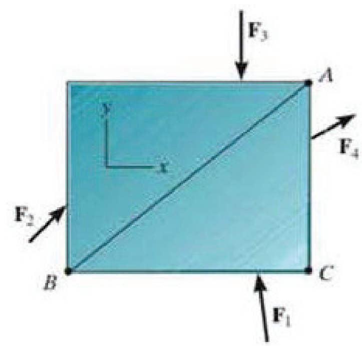
When a system of forces acting on a body has no resultant, the body in which the force system acts is in ***equilibrium***. So, ***equilibrium*** means that both the resultant force and the resultant couple are zero.

Newton's first law of motion states that if the resultant force acting on a particle is zero, the particle will remain at rest or move with constant velocity. The conditions for equilibrium in two dimensions are:

$$\Sigma F_x = 0, \qquad \text{...(3-1a)}$$

$$\Sigma F_y = 0, \qquad \text{...(3-1b)}$$

$$\Sigma M_O = 0 \qquad \text{...(3-1c)}$$



The summations in Eqs. (3.1) must, of course, include *all* the forces that act on the body (both the applied forces and the reactions (the forces provided by supports)).

FREE-BODY DIAGRAM

The ***free-body diagram*** (**FBD**) of a body is a sketch of the body, a portion of a body, or two or more bodies completely isolated or free from all bodies, showing all forces that act on it.

Forces that act on a body can be divided into two general categories:

- *Reactions* are those forces that are exerted on a body by the supports to which it is attached.
- *Applied forces* are those forces acting on a body that are not provided by the supports.

The following is the general procedure for constructing a free-body diagram.

1. A sketch of the body is drawn assuming that all supports (surfaces of contact, supporting cables, etc.) have been removed.
2. All applied forces are drawn and labeled on the sketch. The weight of the body is considered to be applied force acting at the center of gravity. The center of gravity of a homogeneous body coincides with the centroid of its volume.
3. The support reactions are drawn and labeled on the sketch. If the sense of a reaction is unknown, it should be assumed. The solution will determine the correct sense: A positive result indicates that the assumed sense is correct, whereas a negative result means that the correct sense is opposite to the assumed sense.
4. All relevant angles and dimensions are shown on the sketch.

Typical examples of actual supports are shown in the following sequence of photos.

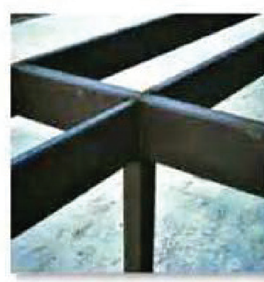
The cable exerts a force on the bracket in the direction of the cable.

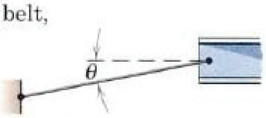
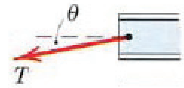
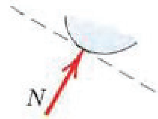
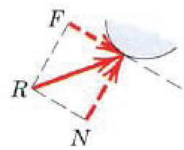
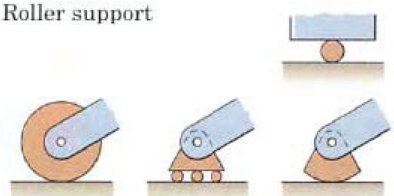
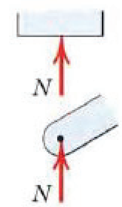
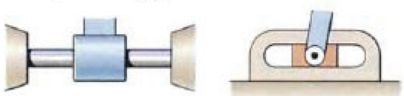
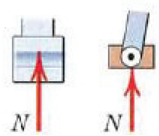
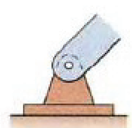
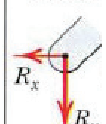
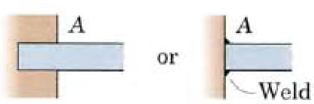
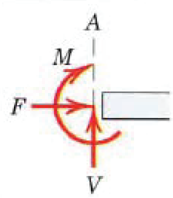
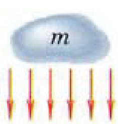
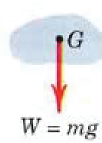
The rocker support for this bridge girder allows horizontal movement so the bridge is free to expand and contract due to a change in temperature.

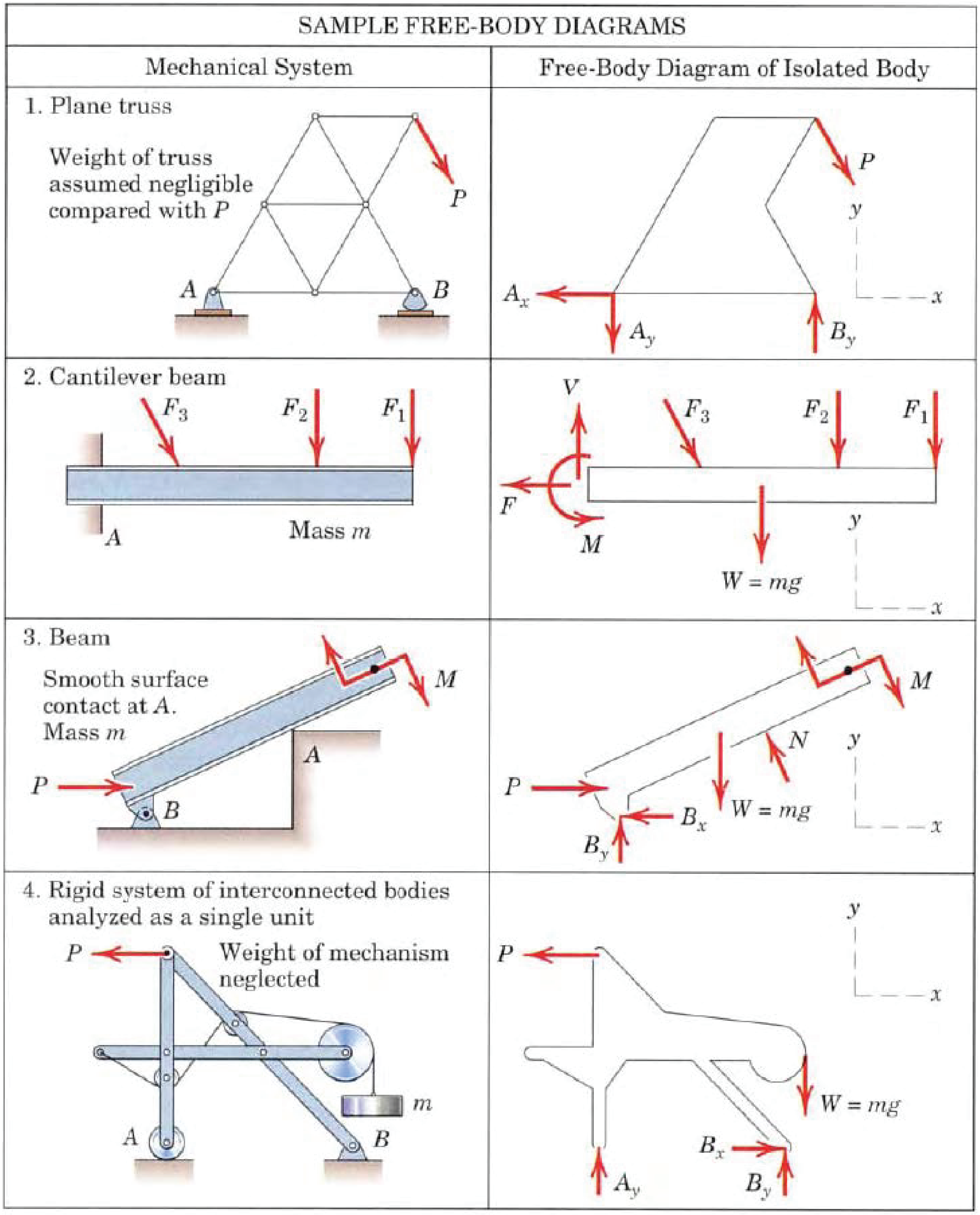
This concrete girder rests on the ledge that is assumed to act as a smooth contacting surface.

This utility building is pin supported at the top of the column.

The floor beams of this building are welded together and thus form fixed connections.



MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS	
Type of Contact and Force Origin	**Action on Body to Be Isolated**
1. Flexible cable, belt, chain, or rope. Weight of cable negligible 	 Force exerted by a flexible cable is always a tension away
2. Smooth surfaces 	 Contact force is compressive and is normal to the surface.
3. Rough surfaces 	 Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.
4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	Pin free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the axis; usually shown as two components R_x and R_y.
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.


SAMPLE FREE-BODY DIAGRAMS
Mechanical System
Free-Body Diagram of Isolated Body
1. Plane truss
Weight of truss assumed negligible compared with P
P
A
B
P
y
x
A_x
A_y
B_y
2. Cantilever beam
F_3
F_2
F_1
A
Mass m
V
F_3
F_2
F_1
F
M
W = mg
y
x
3. Beam
Smooth surface contact at A.
Mass m
M
A
P
B
M
N
y
P
B_x
W = mg
x
B_y
4. Rigid system of interconnected bodies analyzed as a single unit
P
Weight of mechanism neglected
m
A
B
y
P
x
W = mg
B_x
A_y
B_y

GENERAL CASE

It often is convenient to use a set of three independent equations different from those in Eqs. (3.1). The alternative equations are described next.

1. **Two force equations and one moment equation:** The *x*- and *y*-directions in Eqs. (3.1) do not have to be mutually perpendicular—as long as they are *not parallel*. Hence, the equilibrium equations can be restated as

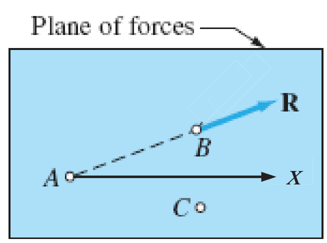
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_O = 0 \qquad \ldots(3.2)$$

where x and y are any two non-parallel directions and O is an arbitrary point.

2. **Two moment equations and one force equation:** It is possible to replace one of the force equations in Eqs. (3.2) by a moment equation, obtaining

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \ldots(3.3)$$

Here, A and B are any two distinct points, and x is any direction that is *not perpendicular* to the line AB. Note that if $\Sigma M_A = 0$ and $\Sigma M_B = 0$ are satisfied, the resultant only can be a force R that lies along the line AB, as shown in Figure. The equation $\Sigma F_x = 0$ (x not perpendicular to AB) then can be satisfied only if $R = 0$.



3. **Three moment equations:** We also can replace both force equations in Eqs. (3.2) by two moment equations. The result is

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0 \qquad \ldots(3.4)$$

where A, B, and C are any three distinct, *non-collinear* points, as indicated in Figure above. Again the equations $\Sigma M_A = 0$ and $\Sigma M_B = 0$ are satisfied only if the resultant is a force R that lies along the line AB. The third equation $\Sigma M_C = 0$ (C not on the line AB) then guarantees that $R = 0$.