Moment of Force

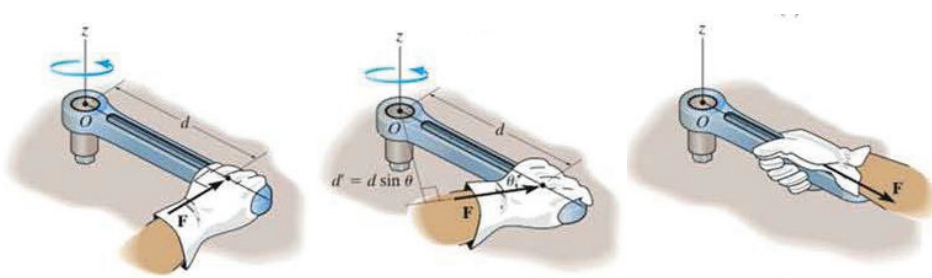
When a force is applied to a body it will produce a tendency for the body to rotate about a point that is not on the line of action of the force. This tendency to rotate is sometimes called a *torque***,** but most often it is called *the moment of a force* or simply *moment.*

The magnitude of the moment is:

$$M = F\,d \qquad \textbf{...(2-8)}$$

Where d is the *moment arm or perpendicular distance* from the axis at point O to the line of action of the force. Units of moment magnitude consist of force times distance, i.e., N.m or lb.ft.

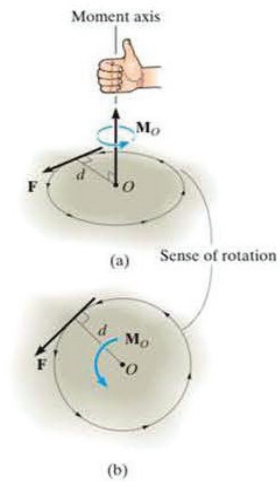
Note 1: If the force $\boldsymbol{F}$ is applied at an angle $\theta \neq 90^o$, Figure b, then it will be difficult to turn the bolt since the moment arm $\boldsymbol{d' = d \sin\theta}$ will be smaller than d.
Note 2: If $\boldsymbol{F}$ is applied along the wrench, Figure c, its moment arm will be zero since the line of action of $\boldsymbol{F}$ will intersect point O (the z-axis). As a result, the moment of $\boldsymbol{F}$ about O is also zero and no turning can occur



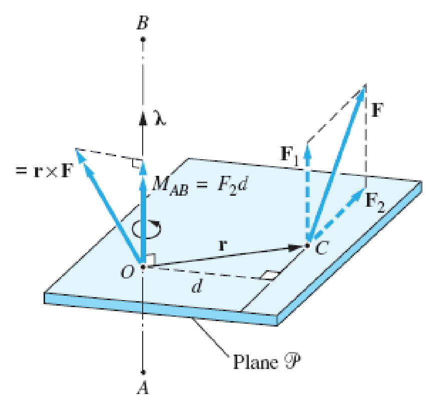
(a) (b) (c)

The moment $\boldsymbol{M_o}$ about O, or about an axis passing through O and perpendicular to the plane, is a *vector quantity* since it has a specified magnitude and direction.

Direction: The direction of $\boldsymbol{M_o}$ is defined by its *moment axis,* which is perpendicular to the plane that contain the force $\boldsymbol{F}$ and its moment arm $\boldsymbol{d}$. The right-hand rule is used to establish the sense of direction of $\boldsymbol{M_o}$. According to this rule, the nature curl of the fingers of the right-hand, as they are drawn towards the palm, represent the tendency for rotation caused by the moment. As the action is performed, the thumb of the right-hand will give the direction sense of $\boldsymbol{M_o}$. Notice that the moment vector is represented in three-directionally by a curl around an arrow as in Figure b. Since in this case the moment will tend to cause a counterclockwise rotation, the moment vector is actually directed out of page.



If the force does not lie in a plane perpendicular to the moment axis, it may be resolved into two components, one being parallel to the moment axis and the other lying in a plane perpendicular to the axis. The component of parallel to the reference axis has no tendency to rotate the body about the axis and has no moment with respect to this axis. The moment of the other component is thus the moment of the force with respect to the line or axis.

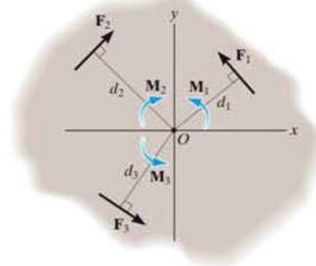


$$M_{AB} = F_2 d \qquad \textbf{…(2-9)}$$

Resultant moment: For two-dimensional problems, where all the forces lie in the *x*-*y* plane the resultant $(\boldsymbol{M_R})_o$ about point *O* (the *z*-axis) can be determined by *finding the algebraic sum* of the moments caused by all forces in the system. As a convention, we will generally consider *positive moment* as *counterclockwise* since they are directed along the positive *z*-axis (out of page). *Clockwise moment* will be *negative*. Doing this, the directional sense of each moment can be represented by a *plus* or *minus* sign.

Using this sign convention, the resultant moment in figure below is therefore:

↺+ $(M_R)_O = \sum Fd;\ \ (M_R)_O = F_1\, d_1 - F_2\, d_2 + F_3\, d_3$



If the numerical result of this sum is positive scalar, $(M_R)_O$ will be counterclockwise moment (out of page); if the result is negative, $(M_R)_O$ will be clockwise moment (into the page).

Principle Moments of Forces:

When determining the moment of a force about a point, it is often convenient to use the *principle of moments*, also known as *Varignon's theorem* which indicates that:

The moment of a force about a point is equal to the sum of the moments of its components about that point.

Example 15: For each case illustrated in Figures below, determine the moment of the force about point *O*.

Solution:

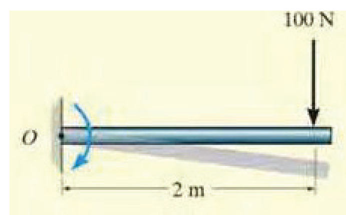
Figure (a) $M_O = -(100)(2) = -200 = 200$ N.m ↻

Figure (b) $M_O = -(50)(0.75) = -37.5 = 37.5$ N.m ↻

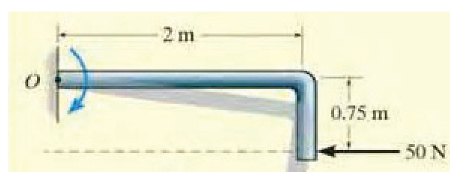
Figure (c) $M_O = -(40)(4 + 2\cos 30^o) = -229 = 229$ lb.ft ↻

Figure (d) $M_O = (60)(1 \sin 45^o) = 42.4$ lb.ft ↺

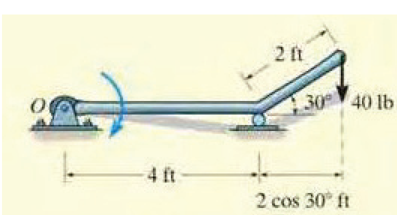
Figure (e) $M_O = (7)(4-1) = 21.0$ kN.m ↺



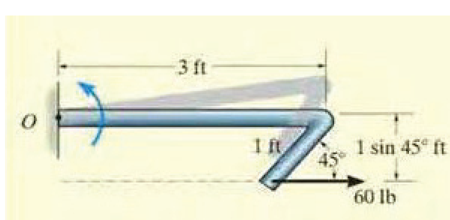
(a)



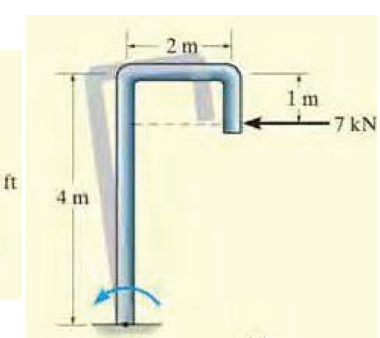
(b)



(c)

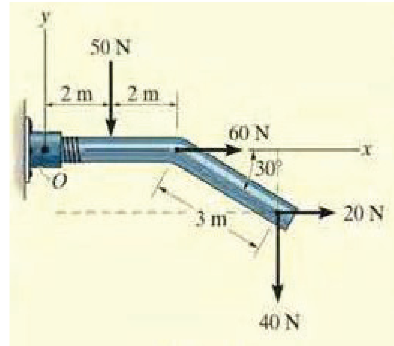


(d)



(e)

Example 16: Determine the resultant moment of the four forces acting on the rod shown in Figure about *O*.



Solution: $\circlearrowleft + M_{Ro} = \Sigma Fd;$

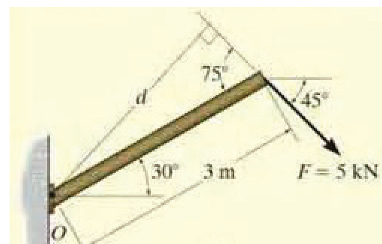
$$M_{Ro} = -50(2) + 60(0) + 20(3\sin 30^o) - 40(4 + 3\cos 30^o)$$

$$= -334 \text{ N.m}$$

$$M_{Ro} = 334 \text{ N.m} \circlearrowright$$

Example 17: Determine the moment of the force in figure about *O*.

Solution I: The moment arm *d* in Figure (a) can be found from trigonometry.



(a)

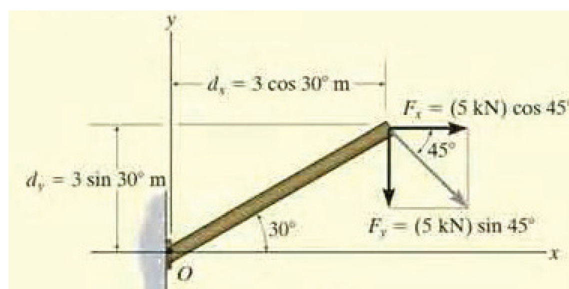
$$d = (3)\sin 75^o = 2.898 \text{ m}$$

Thus,

$$M_O = Fd = -(5)(2.898) = -14.5 \text{ kN.m} = 14.5 \text{ kN.m} \circlearrowright$$

Since the force tends to rotate or orbit clockwise about point *O*, the moment is directed into the page.

Solution II: The *x* and *y* components of the force are indicated in Figure (b).



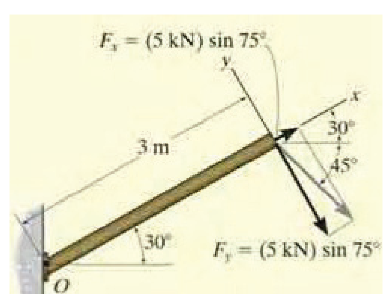
(b)

$$\circlearrowleft + M_O = -F_x\, d_y - F_y\, d_x$$

$$= -(5\cos 45^o)(3\sin 30^o) - (5\sin 45^o)(3\cos 30^o)$$

$$= -14.5 \text{ kN.m} = 14.5 \text{ kN.m} \circlearrowright$$

Solution III: The *x* and *y* axes can be set parallel and perpendicular to the rod's axis as shown in Figure (c). Here $\boldsymbol{F_x}$ produces no moment about point *O* since its line of action passes through this point. Therefore,



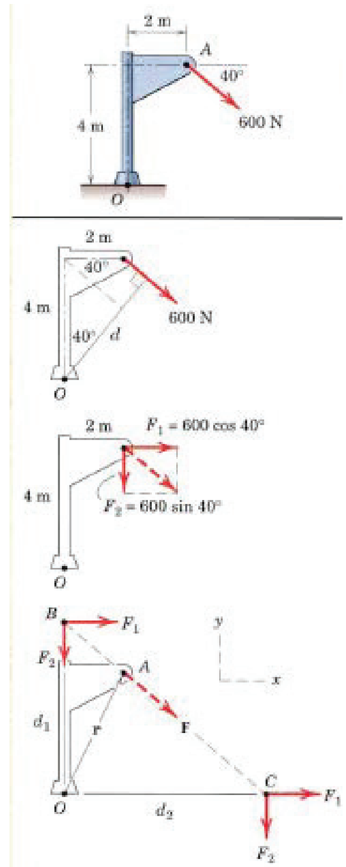
(c)

$$\circlearrowleft + M_O = -F_y\, d_x$$

$$= -(5\sin 75^o)(3)$$

$$= -14.5 \text{ kN.m} = 14.5 \text{ kN.m} \circlearrowright$$

Example 18: Calculate the magnitude of the moment about the base point O of the 600 N force in four different ways.



Solution I: The moment arm to the 600 N force is

$$d = 4 \cos 40^o + 2 \sin 40^o = 4.35 \text{ m}$$

By $M = Fd$ the moment is

$$\circlearrowleft + M_O = -F\, d = -\, 600(4.35)$$

$$= -2610 \text{ N.m} = 2610 \text{ N.m} \circlearrowright$$

Note 1: The minus sign indicates that the vector is in the negative z-direction.

Solution II: Replace the force by its rectangular components at A,

$$F_{1x} = 600 \cos 40^o = 460 \text{ N}, \qquad F_{1y} = 600 \sin 40^o = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

$$\circlearrowleft + M_O = -F_{1x}\, d_y - F_{2y}\, d_x$$

$$= -460(4) - 386(2)$$

$$= -2610 \text{ N.m} = 2610 \text{ N.m} \circlearrowright$$

Solution III: By the *principle of transmissibility*, move the 600-N force along its line of action to point B, which eliminates the moment component F_{2y}. The arm of F_{1x} becomes

$$d_{1y} = 4 + 2 \tan 40^o = 5.68 \text{ m}$$

and the moment is

$$\circlearrowleft + M_O = -F_{1x}\, d_{1y}$$

$$= -460(5.68)$$

$$= -2610 \text{ N.m} = 2610 \text{ N.m} \circlearrowright$$

Solution IV: Moving the force to point C eliminates the moment component F_{1x}. The moment arm of F_{2x} becomes

$$d_{2x} = 2 + 4 \cot 40^{\circ} = 6.77 \text{ m}$$

and the moment is

$$\circlearrowleft + M_O = -F_{2y}\, d_{2x}$$

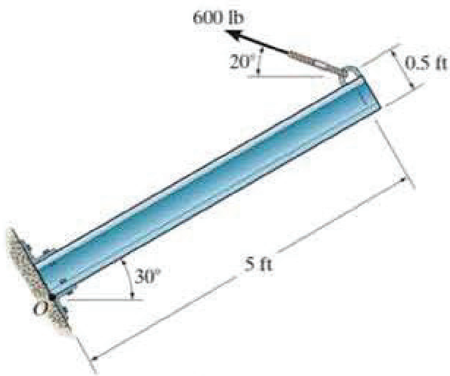
$$= -386(6.77)$$

$$= -2610 \text{ N.m} = 2610 \text{ N.m} \circlearrowright$$

***Note* 2:** The fact that the points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.

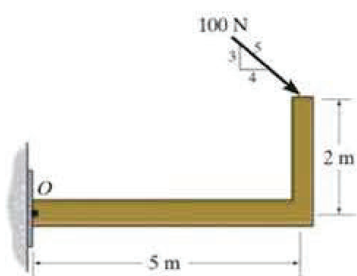
FUNDAMENTAL PROBLEMS

F4–1. Determine the moment of the force about point O.



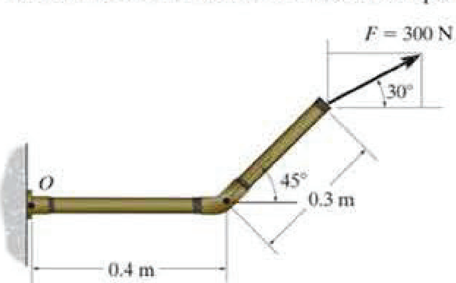
F4–1

F4–2. Determine the moment of the force about point O.



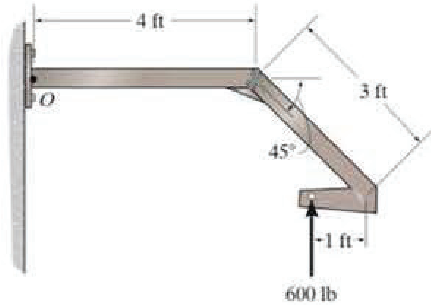
F4–2

F4–3. Determine the moment of the force about point O.



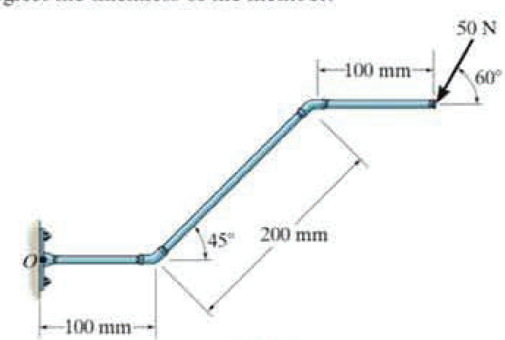
F4–3

F4–4. Determine the moment of the force about point O.



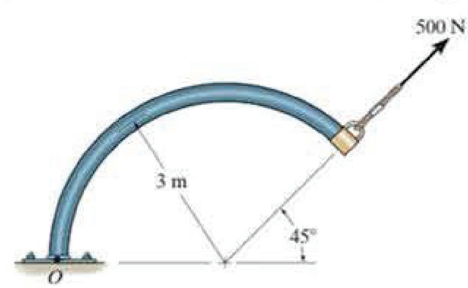
F4–4

F4–5. Determine the moment of the force about point O. Neglect the thickness of the member.



F4–5

F4–6. Determine the moment of the force about point O.



F4–6

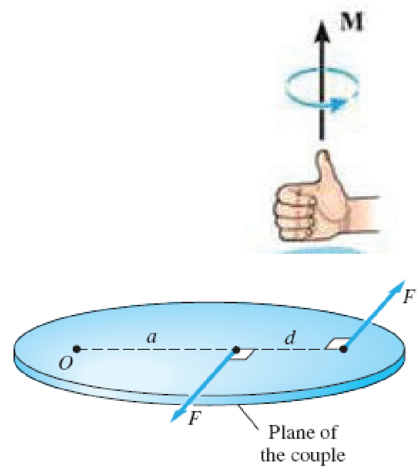
Couples

A couple consists of two parallel, noncollinear forces that are equal in magnitude and opposite in direction

A couple is a purely rotational effect, it has a moment but no resultant force (resultant equals zero thus it has no tendency to translate the body in any direction). A couple possesses two important characteristics:

a) A couple has no resultant force ($\Sigma F = 0$), and

b) The moment of a couple is the same about any point in the plane of the couple. So it may be considered as *a free vector quantity* (not localized vector and can be moved to any parallel position) having both magnitude and direction (aspect of plane and sense of rotation).



The magnitude of the couple is:

$M_O = F(a+d) - Fa$

$\therefore M = Fd$ **…(2-11)**

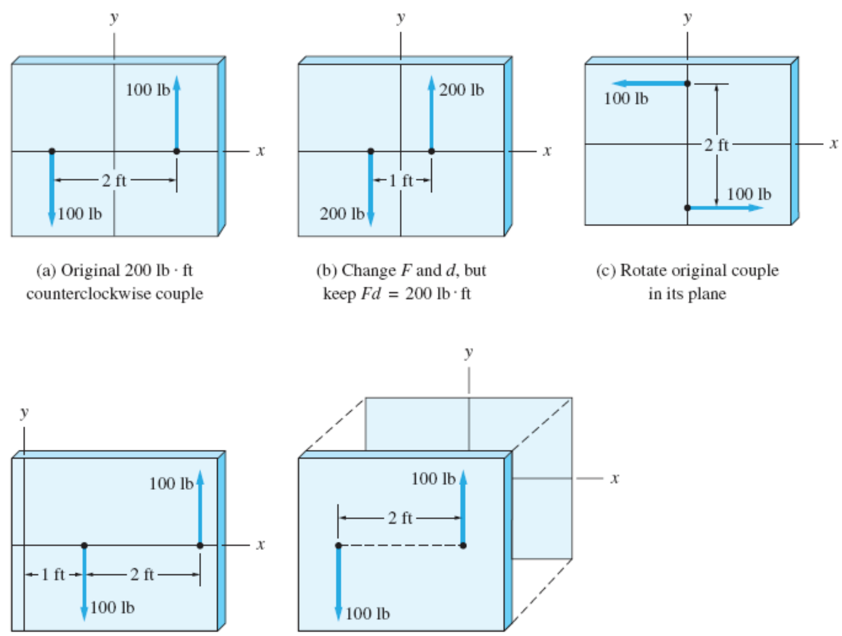
where F is the magnitude of one forces, and d is the perpendicular distance or moment arm between the forces.

The *direction* and the sense of the couple moment are determined by the right-hand rule, where the thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces. In all cases, M will act perpendicular to the plane containing these forces.

Equivalent couples: If two couples produce a moment with the *same magnitude and direction*, then these two couples are ***equivalent***.

Figure below illustrates the four operations that may be performed on a couple without changing its moment; all couples shown in the figure are equivalent. The operations are

1. Changing the magnitude F of each force and the perpendicular distance d while keeping the product Fd constant,
2. Rotating the couple in its plane,
3. Moving the couple to a parallel position in its plane
4. Moving the couple to a parallel plane



(a) Original 200 lb · ft counterclockwise couple

(b) Change F and d, but keep Fd = 200 lb · ft

(c) Rotate original couple in its plane

(d) Move original couple to a parallel position in its plane

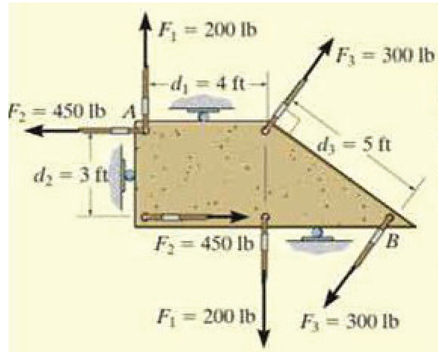
(e) Move original couple to a parallel plane

Resultant of Couples

Because couples are vectors, they may be added by the usual rules of vector addition. Being free vectors, the requirement that the couples to be added must have a common point of application does not apply.

The resolution of couples is no different than the resolution of moments of forces.

Example 27: Determine the resultant couple moment of three couples acting on the plate in Figure below.



Solution: As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft.

$$\curvearrowleft + M_R = \Sigma M;$$

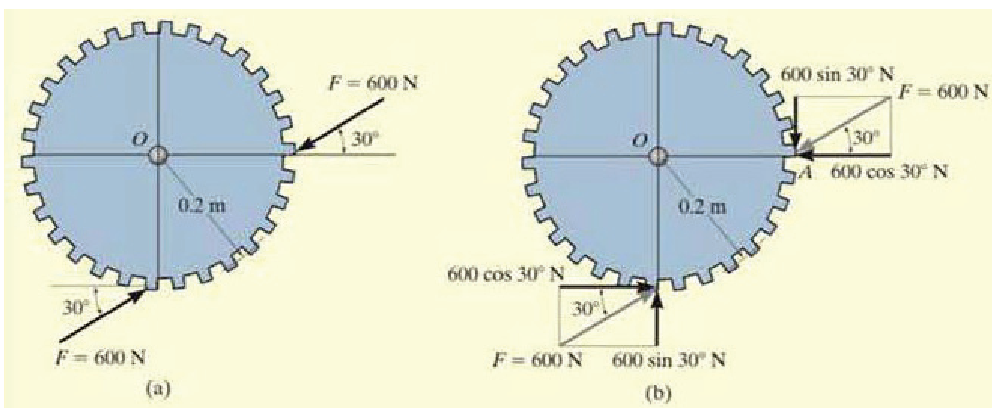
$$M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

$$= (-200)(4) + (450)(3) - (300)(5)$$

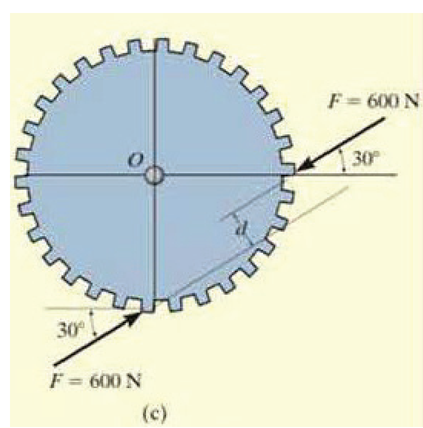
$$= -950 \text{ lb.ft} = 950 \text{ lb.ft} \curvearrowright$$

The negative sign indicates that $\boldsymbol{M_R}$ has a clockwise rotational sense.

Example 28: Determine the magnitude and direction of the couple moment acting on the gear in Figure (a)



Solution: The easiest solution requires resolving each force into its components as shown in Figure (b).



$\curvearrowleft + M_R = \Sigma M_O;$ $\quad M = (600 \cos 30^o)(0.2) - (600 \sin 30^o)(0.2)$

$$= 43.9 \text{ N.m} \curvearrowright$$

$\curvearrowleft + M_R = \Sigma M_A;$ $\quad M = (600 \cos 30^o)(0.2) - (600 \sin 30^o)(0.2)$

$$= 43.9 \text{ N.m} \curvearrowright$$

e. Parallel Force System

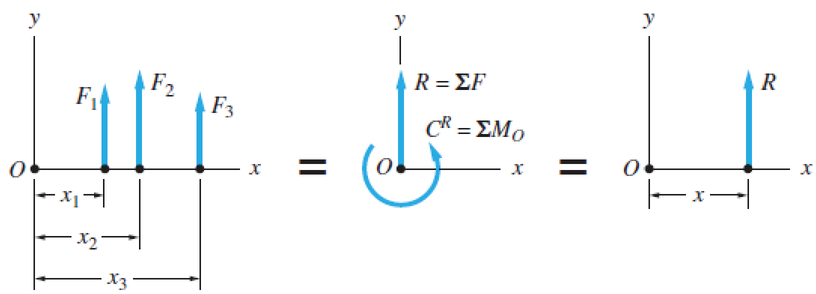
1.Coplanar Force Systems:

The *parallel force system* shown below consists of forces that are parallel to the y-axis. Thus the resultant forces:

$$F_R = \Sigma F_y \qquad \text{...(2-13a)}$$

The position of the resultant force can be determined from:

$$x = \frac{\sum M}{F_R} \qquad \text{...(2-13b)}$$



(a) Parallel, coplanar force system (b) Equivalent force-couple system (c) Resultant is a force (or a couple)

Example 40: Replace the loading system by an equivalent resultant force and specify where the resultant's line of action intersects the beam measured from O.

Solution: The resultant is:

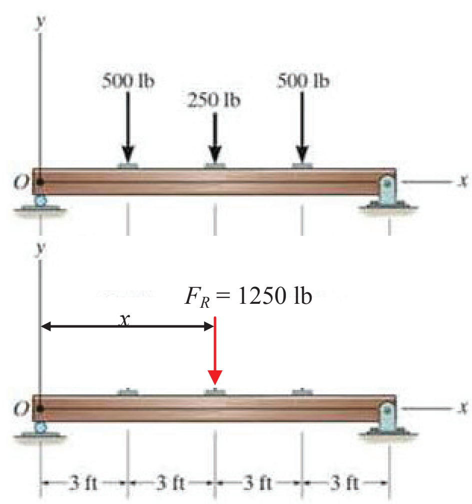
$+\uparrow \quad F_R = \sum F_y$ = -500 – 250 -500 = -1250 lb =1250 lb $\downarrow$

And its position:

$\circlearrowleft + \quad F_R\,(x) = \sum M_O$

-1250*x=-500*3 – 250*6 -500*9

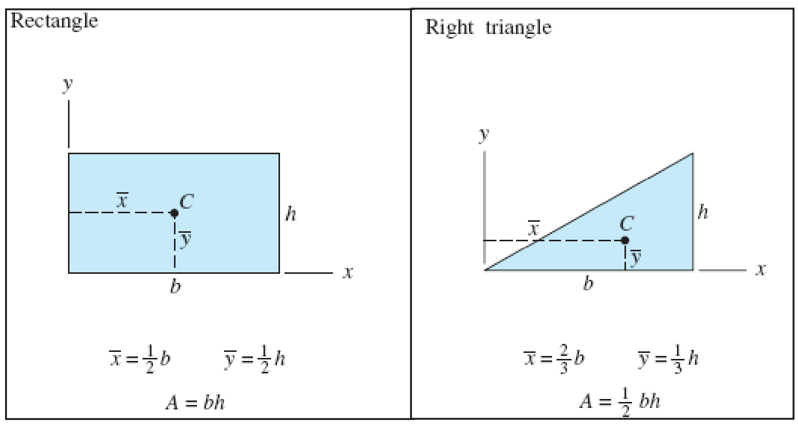
$$\therefore x = \frac{-7500}{-1250} = 6 \text{ ft}$$



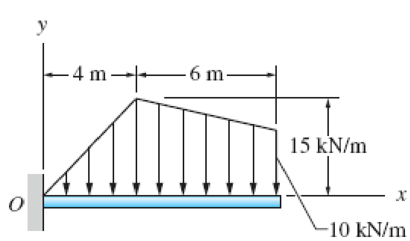
Resultant of Distributed Normal Loads (Line Loads)

- The magnitude of the resultant force is equal to the area under the load diagram.
- The line of action of the resultant force passes through the centroid of the area under the load diagram.

So, if the load surface or the load diagram has a simple shape, such those shown in Figure below, we can use the information in the Figures to find the resultant and its position.

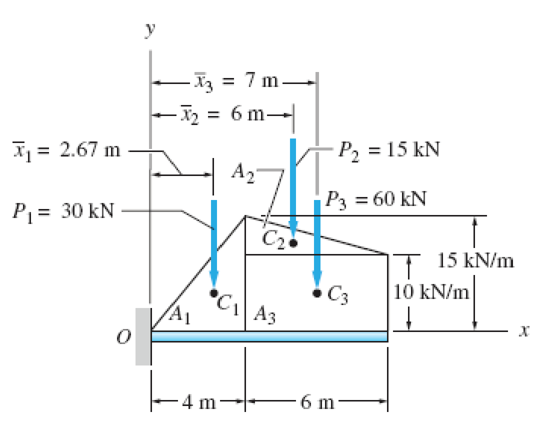


Example 45: Determine the resultant of the line load acting on the beam shown in Fig. (a).



(a)

Solution: The load diagram can be represented as the sum of three line loads corresponding to the two triangles, A_1 and A_2, and the rectangle A_3.



(b)

$$P_1 = \frac{1}{2}(4)(15) = 30\,\text{kN}$$

$$P_2 = \frac{1}{2}(6)(5) = 15\,\text{kN}$$

$$P_3 = 6(10) = 60\,\text{kN}$$

The line of action of each of these forces passes through the centroid of the corresponding load diagram, labeled C_1, C_2, and C_3 in Fig. (b).

$$\bar{x}_1 = \frac{2}{3}(4) = 2.67\,\text{m}$$

$$\bar{x}_2 = 4 + \frac{1}{3}(6) = 6\,\text{m}$$

$$\bar{x}_3 = 4 + \frac{1}{2}(6) = 7\,\text{m}$$

It follows that the magnitude of the resultant of the line load in Fig. (a) is given by

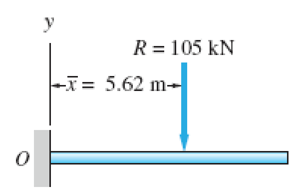
$$\Sigma F_y = R \quad +\!\downarrow \qquad R = P_1 + P_2 + P_3 = 30 + 15 + 60 = 105\text{ kN}$$

To determine $\bar{x}$, the horizontal distance from point O to the line of action of R, we use the moment equation:

$$\Sigma M_O = R\,\bar{x} \quad \circlearrowright + ; \qquad 30(2.67) + 15(6) + 60(7) = 105\,\bar{x}$$

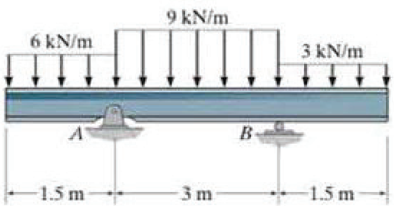
which gives

$$\bar{x} = 5.62\,\text{m}$$



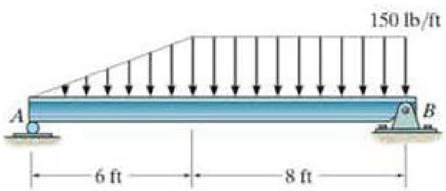
(c)

F4–37. Determine the resultant force and specify where it acts on the beam measured from A.



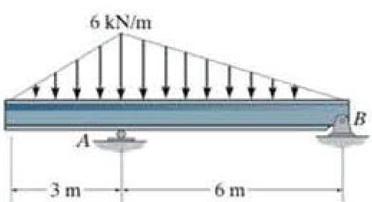
F4–37

F4–38. Determine the resultant force and specify where it acts on the beam measured from A.



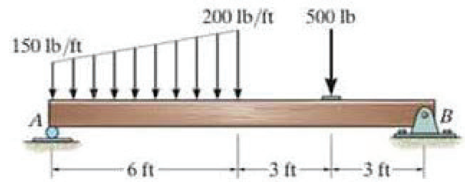
F4–38

F4–39. Determine the resultant force and specify where it acts on the beam measured from A.



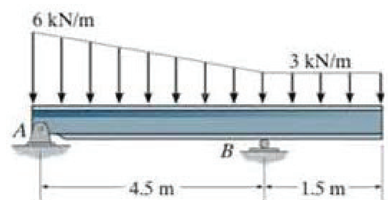
F4–39

F4–40. Determine the resultant force and specify where it acts on the beam measured from A.



F4–40

F4–41. Determine the resultant force and specify where it acts on the beam measured from A.



F4–41