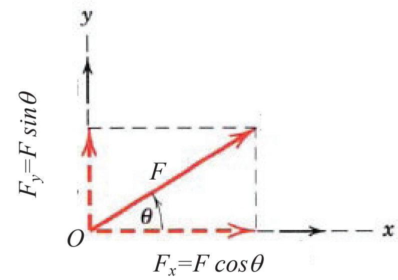
Rectangular Components

For most purposes, rectangular (mutually perpendicular) components of a force are more useful than general oblique components.

The force ***F***, acting at point *O*, has two components:

- Horizontal component $F_x = F\cos\theta$ **…(2-3a)**
- Vertical component $F_y = F\sin\theta$ **…(2-3b)**



where

$$F = \sqrt{F_x^2 + F_y^2} \quad \text{…(2-4a)}$$

$$\tan\theta = \frac{F_y}{F_x} \quad \text{…(2-4a)}$$

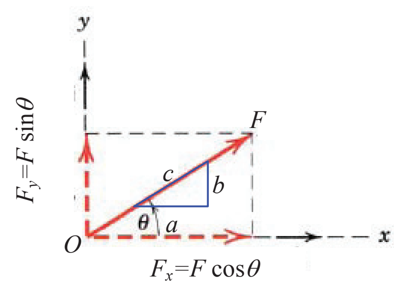
Instead of using the angle *θ*, however, the direction of ***F*** can also be defined using a small "slope" triangle. Since this triangle and the larger triangle are similar, the proportional length of the sides gives

$$\frac{F_x}{F} = \frac{a}{c}$$

or $$F_x = \left(\frac{a}{c}\right)F \quad \text{…(2-5a)}$$

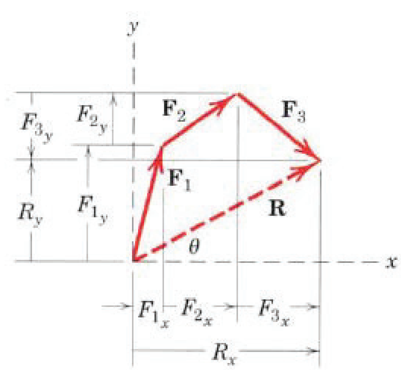
and $$\frac{F_y}{F} = \frac{b}{c}$$

or $$F_y = \left(\frac{b}{c}\right)F \quad \text{…(2-5b)}$$

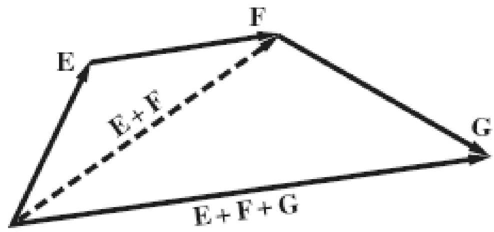


Resultant of Several Coplanar Forces

In determination of the resultant of several forces (more than two forces), using the rectangular component is more convenient than using the parallelogram rule more than once.



(b)



Consider three forces as shown in figure below. So the resultant of these coplanar forces may be determined by the following steps:

1. Resolve each force into x and y components.
2. Add the respective using scalar algebra since they are collinear

$$\overset{+}{\rightarrow} \quad F_{Rx} = \sum F_{ix} = F_{1x} - F_{2x} + F_{3x}$$

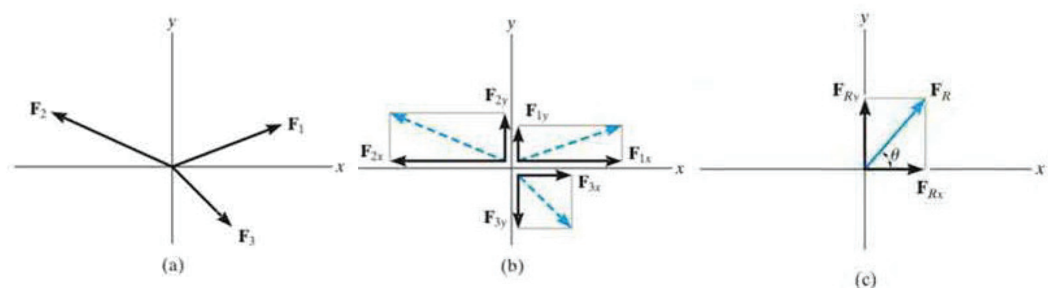
$$+\uparrow \quad F_{Ry} = \sum F_{iy} = F_{1y} + F_{2y} - F_{3y}$$

3. The resultant force is then computed by using Pythagorean Theorem,

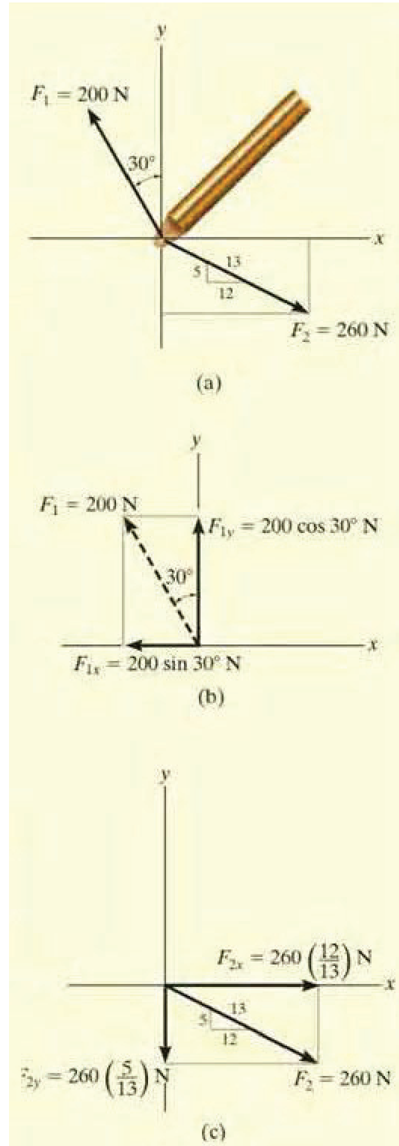
$$R = \sqrt{F_{Rx}^2 + F_{Ry}^2}$$

and the angle θ, which specifies the direction of resultant, is determined from trigonometry:

$$\tan\theta = \frac{F_{Ry}}{F_{Rx}}$$



Example 5: Determine the *x* and *y* components of $\boldsymbol{F_1}$ and $\boldsymbol{F_1}$ acting on the boom shown in Figure (a).



Solution: $\boldsymbol{F_1}$ is resolved into *x* and *y* components,

$$\boldsymbol{F_{1x}} = -200 \sin 30^o = -100 \text{ N} = 100 \text{ N} \leftarrow$$

$$\boldsymbol{F_{1y}} = 200 \cos 30^o = 173 \text{ N} = 173 \text{ N} \uparrow$$

The force $\boldsymbol{F_2}$ is resolved into its *x* and *y* components,

$$\frac{F_{2x}}{260} = \frac{12}{13} \Longrightarrow F_{2x} = 260\left(\frac{12}{13}\right) = 240\, N \longrightarrow$$

Similarly

$$F_{2y} = 260\left(\frac{5}{13}\right) = 100\, N \downarrow$$

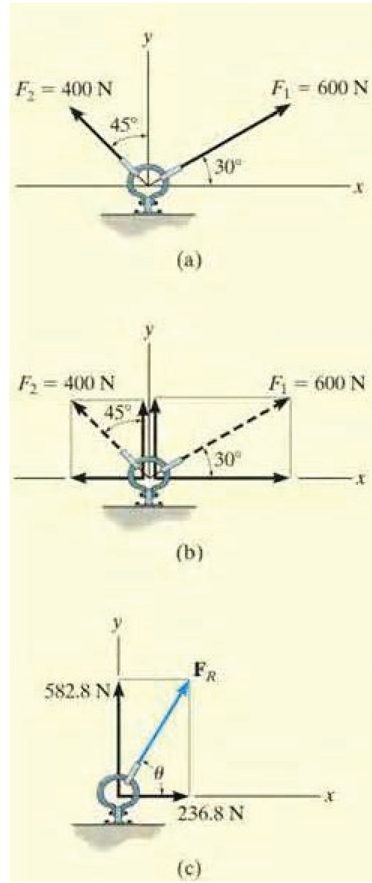
Hence;

$$\boldsymbol{F_{2x}} = 240 \text{ N} = 240 \text{ N} \longrightarrow$$

$$\boldsymbol{F_{2y}} = -100 = 100 \text{ N} \downarrow$$

Example 6: The link in Figure (a) is subjected to two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$. Determine the magnitude and direction of the resultant force.

Solution:



$\xrightarrow{+}$ $F_{Rx}=\sum F_x$; $F_{Rx}= 600\cos 30^o - 400\sin 45^o$

$=236.8 \rightarrow$

$+\uparrow$ $F_{Ry}=\sum F_y$; $F_{Ry}= 600\sin 30^o + 400\cos 45^o$

$=582.8 \uparrow$

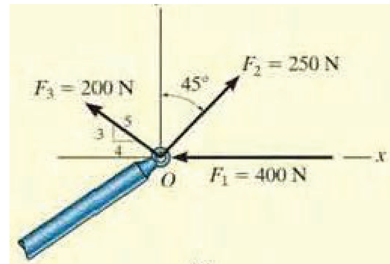
The resultant force has a *magnitude* of,

$$F_R = \sqrt{236.8^2 + 582.2^2}$$
$$= 629\,N$$

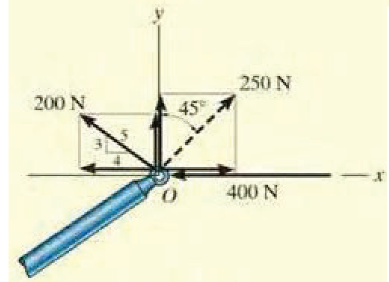
and its direction,

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{582.8}{236.8}\right) = 67.9^o$$

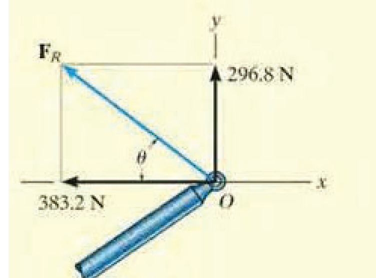
Example 7: The end of the boom O in Figure (a) is subjected to three ***concurrent and coplanar forces*.** Determine the magnitude and direction of the resultant force.



(a)



(b)



(c)

Solution: Each force is resolved into its x and y components, Figure (b). Summing the x components, we have

$$\xrightarrow{+} \quad F_{Rx}=\sum F_x; \qquad F_{Rx}= -400 + 250 \sin 45^o - 200\left(\frac{4}{5}\right)$$

$$= -383.2 = 383.2 \text{ N} \longleftarrow$$

The negative sign indicates that $\boldsymbol{F_{Rx}}$ acts to the left, i.e., in the negative x direction, as noted by small arrow. Summing the y components yields

$$+\uparrow \quad F_{Ry}=\sum F_y; \qquad F_{Ry}= 250 \cos 45^o + 200\left(\frac{3}{5}\right)$$

$$=296.8 \text{ N} \uparrow$$

The resultant force has a *magnitude* of,

$$F_R = \sqrt{(-383.2)^2 + 296.8^2}$$
$$= 485\,N$$

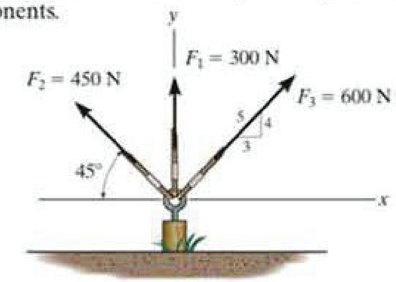
and its direction,

$$\theta = \tan^{-1}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^o$$

Note: Application of this method is more convenient, compared to using two applications of parallelogram law, first to add $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ then adding $\boldsymbol{F_3}$ to this resultant.

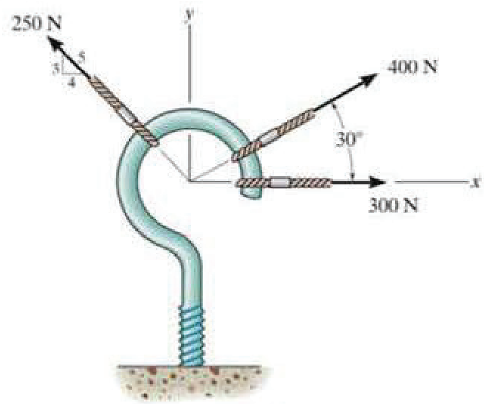
FUNDAMENTAL PROBLEMS

F2–7. Resolve each force acting on the post into its x and y components.



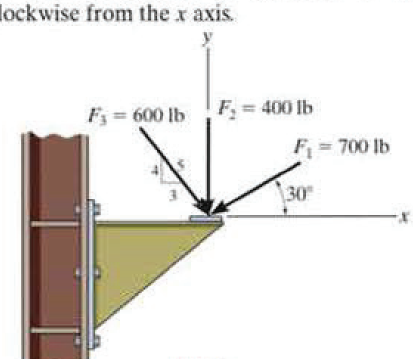
F2–7

F2–8. Determine the magnitude and direction of the resultant force.



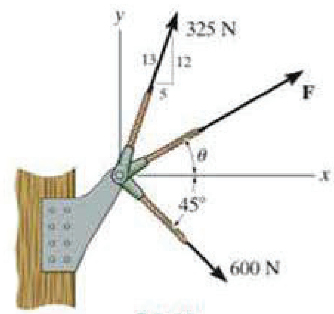
F2–8

F2–9. Determine the magnitude of the resultant force acting on the corbel and its direction θ measured counterclockwise from the x axis.



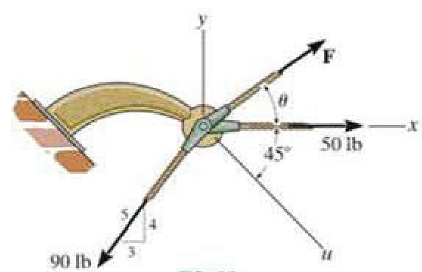
F2–9

F2–10. If the resultant force acting on the bracket is to be 750 N directed along the positive x axis, determine the magnitude of F and its direction θ.



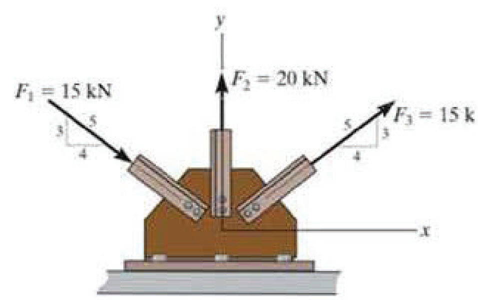
F2–10

F2–11. If the magnitude of the resultant force acting on the bracket is to be 80 lb directed along the u axis, determine the magnitude of F and its direction θ.



F2–11

F2–12. Determine the magnitude of the resultant force and its direction θ measured counterclockwise from the positive x axis.



F2–12