Syllabus:

STATICS

1. General Principles.
2. Force System.
3. Equilibrium.
4. Centroids and Centers of Gravity.
5. Moments of Inertia.

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References

1. Higdon, A. and Stiles, W. B. "***Engineering Mechanics***", 3rd Ed., Prentice-Hall India,1974.
2. Meriam, J. L. and Kraige, L. G. "***Engineering Mechanics***", 5th Ed., John Wiley and Sons Inc., 2002.
3. Hibbeler, R. C. and Fan, S. C., "***Engineering Mechanics***", Prentice Hall, 1997.
4. Beer, F. P. and Johnston, E. R., "***Mechanics for Engineers***", 3rd Ed., McGraw Hill, 1984.
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GENERAL PRINCIPLES

MECHANICS

Mechanics is a branch of physical sciences that is concerned with the state of rest or motion of bodies that are subjected to the action of forces. In general, this subject can be divided into three branches:

- Rigid-body mechanics,
- Deformable-body mechanics, and
- Fluid mechanics.

Rigid-body mechanics is divided into two areas:

1. **Statics:** deals with the equilibrium of bodies, that is, those that are at rest or move with a constant velocity.
2. **Dynamics:** is concerned with the accelerated motion of bodies.

Note: We can consider statics as s special case of dynamics, in which the acceleration is zero.

FUNDAMENTAL CONCEPTS

Basic Quantities: The basic quantities which used throughout mechanics are: Length, Time, Mass, and Force. Force is the action of one body to another. A force tends to move a body in the direction of its action.

Note: A force is completely characterized by its magnitude, direction and point of application.

Rigid body: A rigid body can be considered as a combination of a large number of particles in which all the particles remain at a fixed distance from one another, both before and after applying load. In most cases the actual deformations occurring in structures, machines, mechanisms, and the like are relatively small, and the rigid-body assumption is suitable for analysis.

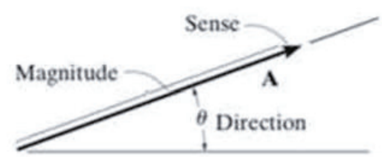
Concentrated Force: A concentrated force represents the effect of a loading which is assumed to act at a point on a body.

SCALARS AND VECTORS

All physical quantities in engineering mechanics are measured using either scalars or vectors.

A scalar is a quantity that has magnitude only. Examples of scalar quantities include length, mass, and time. Because scalars possess only magnitudes, they are real numbers that can be positive, negative, or zero.

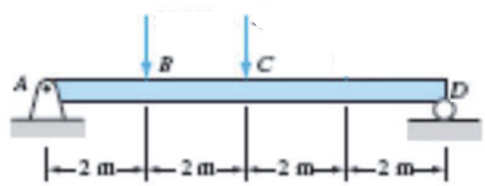
A vector is a quantity that possesses magnitude and direction and obeys the parallelogram law for addition. Examples of vectors encountered in static are force, position, and moment. A vector is shown graphically by an arrow. The length of the arrow represents *the magnitude of the vector*, and angle θ between the vector and a fixed axis defines *the direction of its line of action*. The head or tip of the arrow indicates *the sense of direction of the vector*.



Vector quantities can be further divided into:

1. **Free vector** is one whose action is not confined to or associated with a unique line in space. The wind and moment of a couple are examples of free vectors. Their effect does not depend on their position.

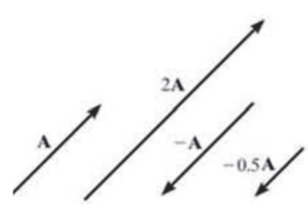
2. **Localized vector** has a definite or specific line of action. Consider the beam shown in figure below. When the load is placed in the position C at the center of beam, the reaction of the supports at A and D on the beam are equal. If the load were moved to position B, the support at A would carry more of the load and the support D carry less. In the other words, the effect of the supports on the beam (the external effect) depend on the position of the load it carries as well as on the slope, sense, and magnitude of that load.



VECTOR OPERATIONS

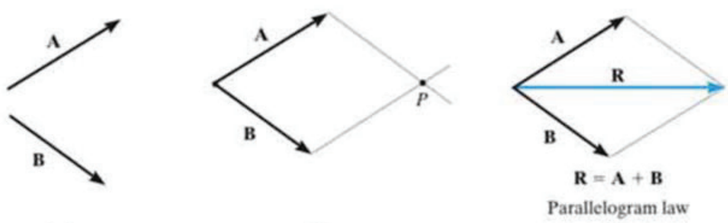
Multiplication and Division of a Vector by a Scalar:

If a vector is multiplied by a positive scalar, its magnitude is increased by that amount. When multiplied by a negative scalar it will also change the directional sense of the vector.



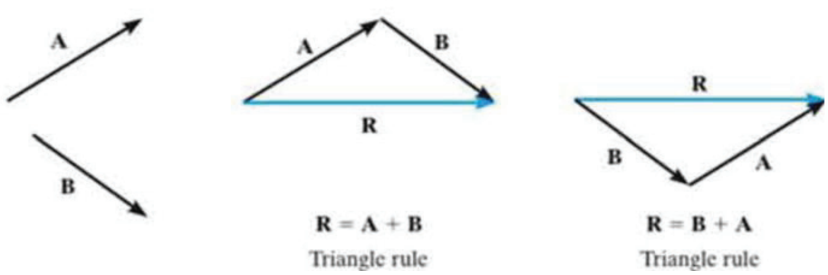
Vector Addition: All vector quantities obey the parallelogram law of addition. To illustrate the two "*component*" vectors **A** and **B** in the figure below are added to form a "*resultant*" vector **R** = **A** + **B** using the following procedure:

- First join the tails of the components at a point so that it makes them concurrent
- From the head of **B**, draw a line parallel to **A**. Draw another line from the head of ***A*** that is parallel to **B**. These tow lines intersect at point P to form the adjacent sides of a parallelogram.
- The diagonal of this parallelogram that extends to P forms **R**, which then represents the resultant vector **R** = **A** + **B**.

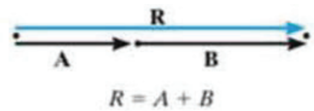


We can also add **B** to **A** using the triangle rule, which is a special case of the parallelogram law, whereby vector **B** is added to vector **A** in a "head-to tail" fashion, i.e., by connecting the head of **A** to the tail of **B**. The resultant **R** extends from the tail of **A** to the head of **B**. In a similar manner, **R** can also obtained by adding **A** to **B**. By comparison, it is seen that vector addition is commutative; in other words, the vectors can be added in either, i.e.

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad \ldots(1\text{-}6)$$



As a special case, if the two vectors **A** and **B** are collinear, i.e., both have the same line of action, the parallelogram law reduce to an *algebraic* or *scalar addition* $R = A + B$.

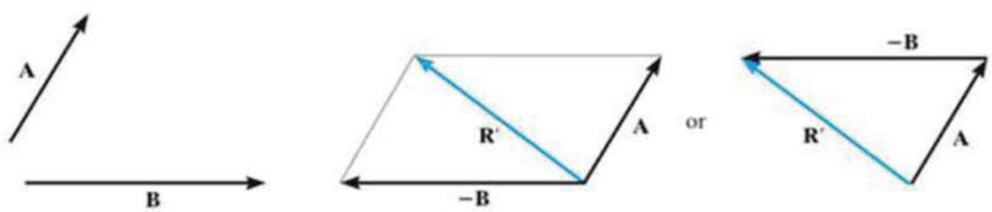


Addition of collinear vectors

Vector Subtraction: The resultant of the difference between two vectors **A** and **B** of the same type may be expressed as

$$\mathbf{R'} = \mathbf{A} - \mathbf{B} = \mathbf{A} + (\mathbf{-B}) \qquad \text{...(1-7)}$$

Subtraction is therefore defined as special case of addition, so the rules of vectors addition also apply to vector subtraction.



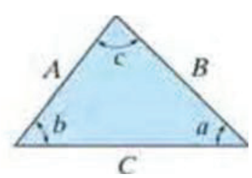
Parallelogram law

Triangle construction

Vector subtraction

Cosine Law and Sine law:

The **cosine law** and **sine law** are applicable to compute angles and sides of a triangle

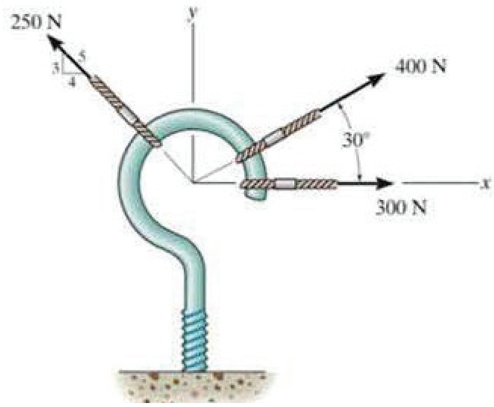


- **Law of cosines**: $C = \sqrt{A^2 + B^2 - 2A.B\cos c}$ **...(1-8)**
- **Law of sines:** $\dfrac{A}{\sin a} = \dfrac{B}{\sin b} = \dfrac{C}{\sin c}$ **...(1-9)**

FORCE SYSTEM

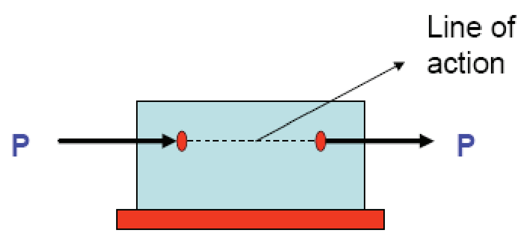
FORCES

A force may be defined as the action of one body on another body which changes or tends to change the motion of the body acted on. Because of the inertia possessed by all material bodies, they react or oppose any force which acts on them (Newton's third law).



Note1: Forces may be considered as localized vectors and they can not be defined unless all the following characteristics mentioned:

- Magnitude,
- Direction (sense and slope),
- Location of any point on its line of action.



Note2: The third characteristic shows that if two forces have the same direction, they will produce the same external effect on a rigid body. This fact leads to the *principle of transmissibility* which states that the external effect of a force on a rigid body is independent of the point of application of the force along its line of action.

System of Forces

When several forces act in a given situation, they are called *system of forces* or *force system.* Force systems can be classified according to the arrangement of the lines of action of the forces of the system as follows:

- **Collinear:** All forces of the system have a common line of action.
- **Concurrent, Coplanar:** The action lines of all the forces of the system are in the same plane and intersect at a common point.
- **Parallel, Coplanar:** The action lines of all the forces of the system are parallel and lie in the same plane.
- **Nonconcurrent, Nonparallel, Coplanar:** The action lines of all the forces of the system are in the same plane, but they are not all parallel and they do not intersect at a common point.
- **Concurrent, Noncoplanar:** The action lines of all the forces of the system are intersect at a common point, but they are not all in one plane.
- **Parallel, Noncoplanar:** The action lines of all the forces of the system are parallel and but they are not all in the same plane.
- **Nonconcurent, Nonparallel, Noncoplanar:** The action lines of all the forces of the system do not all intersect at a common point, they are not parallel, and they do not lie in the same plane.

Resultant

The resultant of a force system is the simplest force system which can replace the original system without changing its external effect on a rigid body. The resultant of a force system can be:

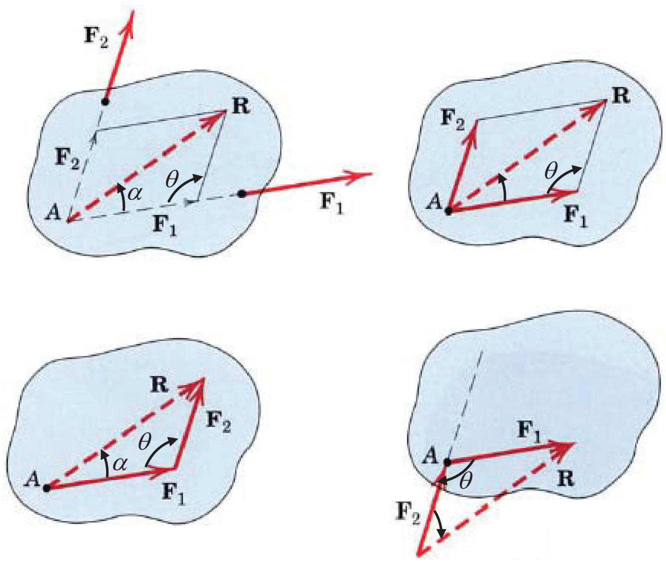
- a single force,
- a pair of parallel forces having the same magnitudes but opposite sense (called a ***couple***), or
- a force and a couple.

If the resultant is a force and a couple, the force will not be parallel to the plane containing the couple.

COMPOSITION AND RESOLUTION OF FORCES

The process of replacing a force system by its resultant is called *composition.* The resultant of a pair of concurrent forces can be determined by means of the *parallelogram law*.

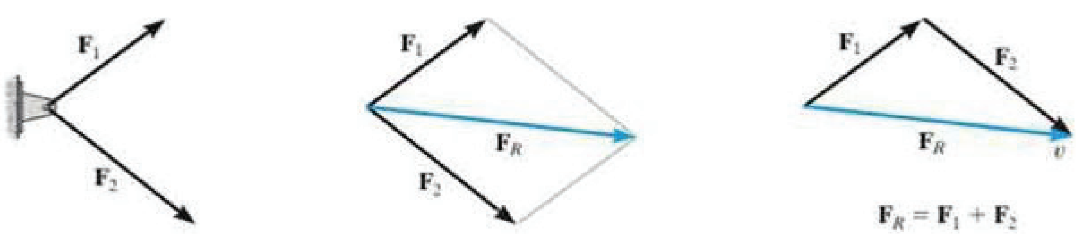
$$R = \sqrt{(F_1)^2 + (F_2)^2 - 2(F_1).(F_2)\cos\theta} \qquad \textbf{...(2-1)}$$



The angle the resultant makes with either force can be determined by the *law of sines*, for example:

$$\frac{F_2}{\sin\alpha} = \frac{R}{\sin\theta} \qquad \textbf{...(2-2)}$$

The process of replacing a force by its components is called *resolution*. A *component of a force* is any one of two or more forces having the given forces as a resultant. So the term "***component***" is used to mean either one of two coplanar concurrent forces or any one of three noncoplanar concurrent forces having the given force as a resultant

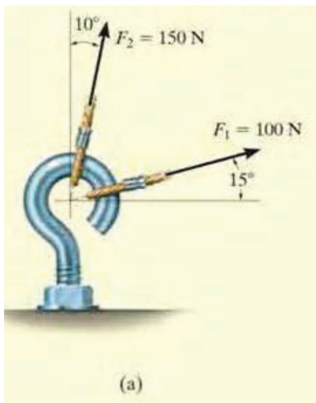


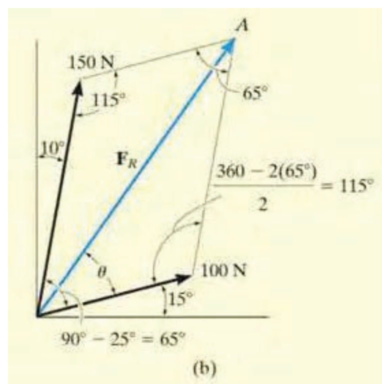
RESULTANT OF THE FORCE SYSTEM

a) Resultant of concurrent, coplanar forces

The resultant of concurrent, coplanar forces is force only, while as we will be seeing later, the resultant of noncurrent forces is force or moment or both.

Example 1: The screw in Figure (a) is subjected to two forces, $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$. Determine the magnitude and direction of the resultant force.





Solution: By parallelogram, the resultant F_R is:

$$F_R = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115^o}$$

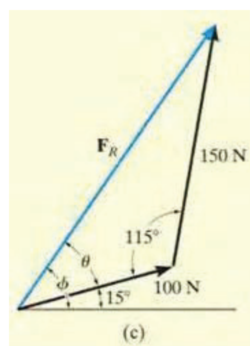
$$= \sqrt{10\,000 + 22\,500 - 30\,000(-0.4226)}$$

$$= 212.6\,N \cong 213 \text{ N}$$

Applying the law of sines to determine θ,

$$\frac{150}{\sin\theta} = \frac{212.6}{\sin 115^o} \quad \Longrightarrow \quad \sin\theta = \frac{150}{212.6}(\sin 115^o)$$

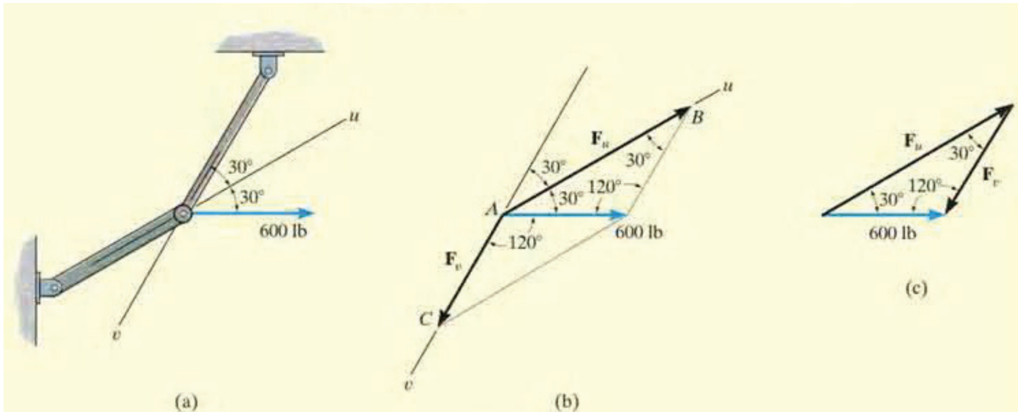
$$\theta = 39.8^{\circ}$$



Thus, the direction ϕ of $\boldsymbol{F_R}$, measured from the horizontal, is

$$\phi = 39.8^{\circ} + 15.0^{\circ} = 54.8^{\circ}$$

Example 2: Resolve the horizontal 600-lb force in Figure (a) into components acting along *u* and *v* axes and determine the magnitude of these components.



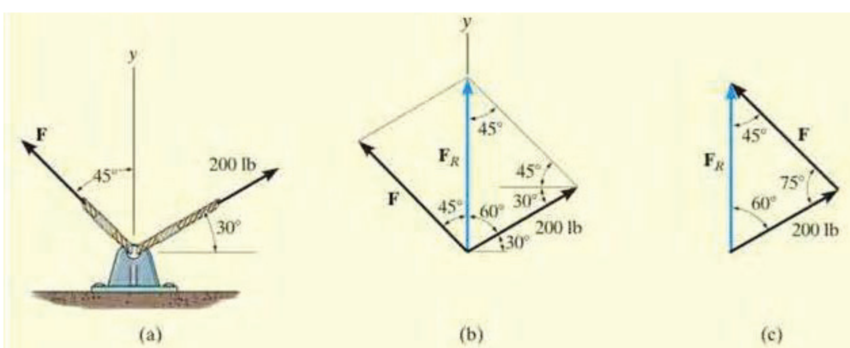
Solution: Applying the law of sines,

$$\frac{F_u}{\sin 120^o} = \frac{600}{\sin 30^o} \quad \Longrightarrow \quad F_u = 1039 \text{ lb}$$

$$\frac{F_v}{\sin 30^o} = \frac{600}{\sin 30^o} \quad \Longrightarrow \quad F_v = 600 \text{ lb}$$

Note: The result for F_u shows that sometimes a component can have a greater magnitude than the resultant.

Example 3: Determine the magnitude of the components force ***F*** in Figure (a) and the magnitude of the resultant force $\boldsymbol{F_R}$ if $\boldsymbol{F_R}$ is directed along the positive *y*-axis.

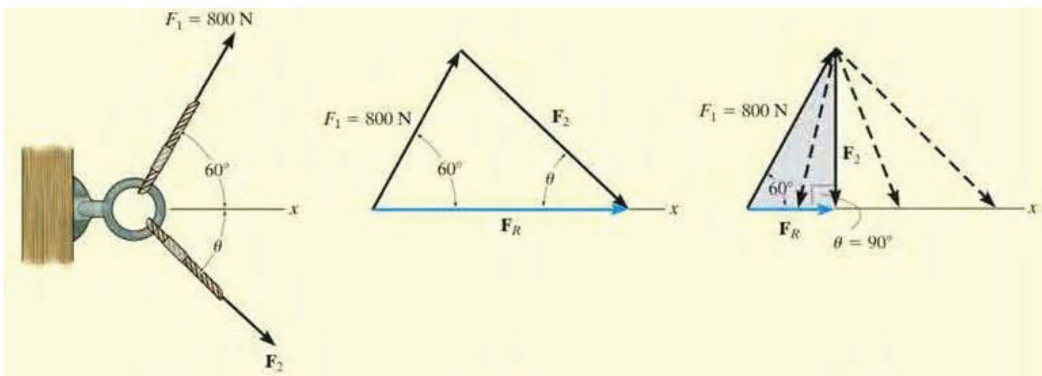


Solution: The magnitude of $\boldsymbol{F_R}$ and ***F*** can be determined by applying the law of sines,

$$\frac{F}{\sin 60^o} = \frac{200}{\sin 45^o} \quad \Longrightarrow \quad F = 245 \text{ lb}$$

$$\frac{F_R}{\sin 75^o} = \frac{200}{\sin 45^o} \quad \Longrightarrow \quad F_R = 273 \text{ lb}$$

Example 4: It is required that the resultant force acting on the eyebolt in Figure (a) be directed along the positive *x*-axis and that $\boldsymbol{F_2}$ has a *minimum* magnitude. Determine this magnitude, the angle θ, and the corresponding resultant force.



Solution: The magnitude of $\boldsymbol{F_2}$ is a *minimum* or the shortest length when its line of action is *perpendicular* to the line of action of $\boldsymbol{F_R}$, that is, when,

$$\theta = 90$$

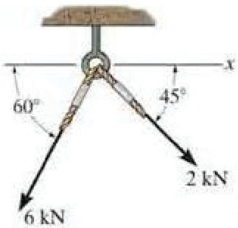
Since the vector addition now forms a right triangle, the two unknown magnitudes can be obtained by trigonometry.

$$F_R = 800 \cos 60^o = 400 \text{ N}$$

$$F_2 = 800 \sin 60^o = 693 \text{ N}$$

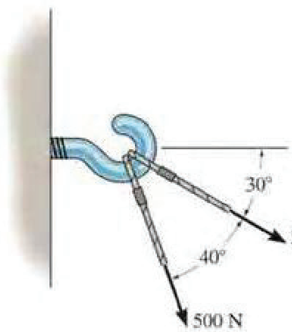
FUNDAMENTAL PROBLEMS*

F2–1. Determine the magnitude of the resultant force acting on the screw eye and its direction measured clockwise from the x axis.



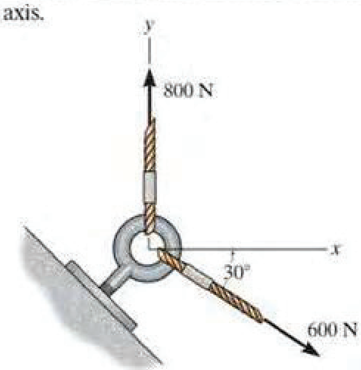
F2–1

F2–2. Two forces act on the hook. Determine the magnitude of the resultant force.



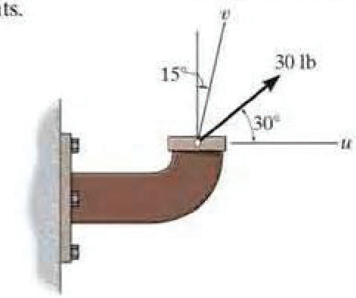
F2–2

F2–3. Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



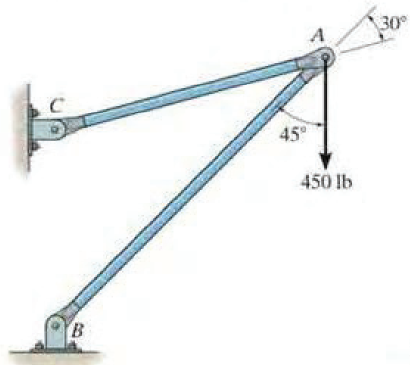
F2–3

F2–4. Resolve the 30-lb force into components along th u and v axes, and determine the magnitude of each of thes components.



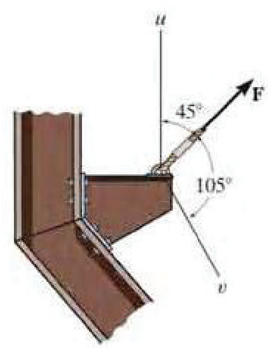
F2–4

F2–5. The force $F = 450$ lb acts on the frame. Resolv this force into components acting along members AB an AC, and determine the magnitude of each component.



F2–5

F2–6. If force **F** is to have a component along the u axis o $F_u = 6$ kN, determine the magnitude of **F** and th magnitude of its component $\mathbf{F}_v$ along the v axis.



F2–6