Engineering Mechanics

- Introduction and basic concepts.
- Force system, units system, parallelogram law, force components,
- Resultant of coplanar forces, components of force in space,
- Moment of a force, moment of coupler, equilibrium, free body diagram, coplanar system,
- Analysis of trusses,
- Friction, nature of friction, theory of friction, coefficient of friction,
- Centroids and center of gravity, centroids of area, centroids determined by integration,
- Moments of inertia, parallel axes theorem, 2nd moment of area by integration, radius of gyration, moment of inertia of composite area.

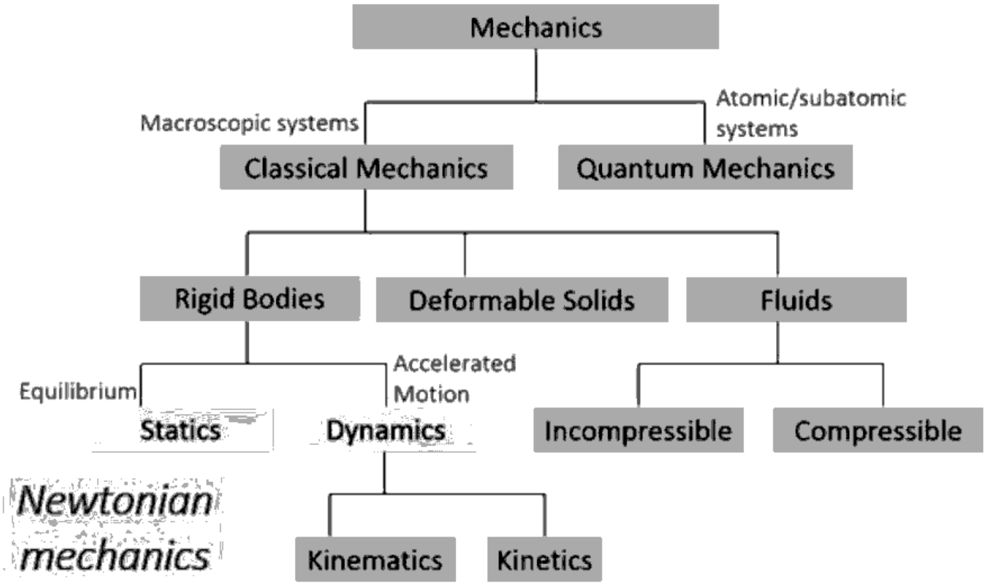
- These lectures were prepared and used by me to conduct lectures for 1st year B. Tech. students as part of Engineering Mechanics course.
- Theories, Figures, Problems, Concepts used in the lectures to fulfill the course requirements are taken from the following references
- I take responsibility for any mistakes in solving the problems. Readers are requested to rectify when using the same.
- I thank the following authors for making their books & lectures available for reference

A. Ali

References:-

- Vector Mechanics for Engineers – Statics & Dynamics, Beer & Johnston; 10 edition.
- Engineering Mechanics Statics Vol. 1, Engineering Mechanics Dynamics Vol. 2, Meriam& Kraige; 6thedition.
- Engineering Mechanics Statics , Engineering Mechanics Dynamics , R. C Hibbeler; 12 edition.
- Engineering Mechanics – Statics, lectures by instructor, R. Ganesh Narayanan.
- Engineering Mechanics – Dynamics, lectures by instructor, Y. Wang.
- Lectures of other instructors in the department.
- Any other references in this field.

Engineering Mechanics



Engineering Mechanics: may be defined as a science which describes and predicts the condition of rest or motion of bodies under the action of forces.

The mechanics of rigid bodies is subdivided into statics and dynamics, the former dealing with bodies at rest, the latter with bodies in motion.

I-Statics:-

Statics is the branch of mechanics which deals with bodies (solids) at rest under the influence of forces.

We consider RIGID BODIES – Non deformable

Vector & Scalar quantities :

Scalar quantities: are the quantities which have only magnitude. Such as:

Time , size , sound , density , light , volume…

Vector quantities: are the quantities which have magnitude and direction.

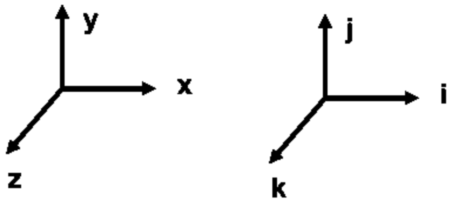
Such as: Force, distance, velocity, displacement, acceleration,….

V = lvl n, where lvl = magnitude, <u>n = unit vector</u>

n = V / lvl

n - dimensionless and in direction of vector 'V'

In our course:

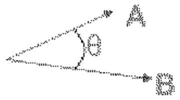


i, j, k – unit vectors

S

Dot product of vectors: A.B = AB cos θ; A.B = B.A (commutative)

A.(B+C) = A.B+A.C (distributive operation)



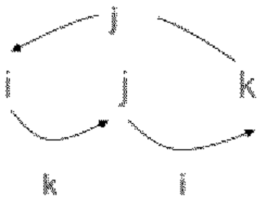
$$A.B = (A_x i+A_y j+A_z k).(B_x i+B_y j+B_z k) = A_xB_x+A_yB_y+A_zB_z$$

i . i = 1

i . j = 0

Cross product of vectors: A x B = C; lCl = lAl lBl Sin θ; AxB = -(BxA)

C x (A+B) = C x A + C x B



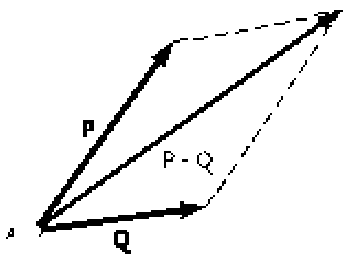
k x j = -i;
i x i = 0

$$\begin{vmatrix} i & j & k \\ A_x & A_Y & A_Z \\ B_X & B_Y & B_Z \end{vmatrix}$$

$$AxB = (A_x i+A_y j+A_z k)x(B_x i+B_y j+B_z k) = (A_yB_z- A_zB_y)i+(\quad)j+(\quad)k$$

ADDITION OF VECTORS:

P + Q = Q + P



<u>**Hint:**</u>

$\mathbf{i} \times \mathbf{i} = 0$	$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$	$\mathbf{k} \times \mathbf{i} = \mathbf{j}$
$\mathbf{i} \times \mathbf{j} = \mathbf{k}$	$\mathbf{j} \times \mathbf{j} = 0$	$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$
$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$	$\mathbf{j} \times \mathbf{k} = \mathbf{i}$	$\mathbf{k} \times \mathbf{k} = 0$

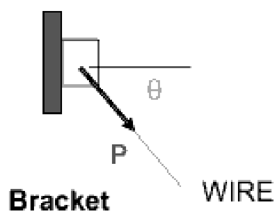
Forces:

Since mechanics is primarily a study of the effects of forces, it is important to have a clear understanding of the concept of a force:

A force

- Action of one body on another which changes or tends to change the motion of the body.
- Required force can move a body in the direction of action,otherwise no effect.

Force system:



Magnitude, direction and point of application is important

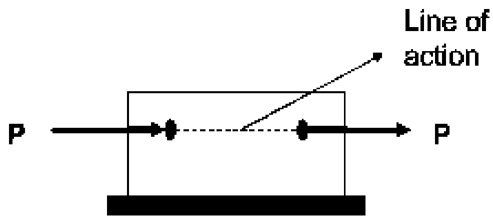
External effect: Forces applied (applied force); Forces exerted by bracket, bolts, foundation….. (reactive force)

Internal effect: Deformation, strain pattern – permanent strain; depends on material properties of bracket, bolts…

Transmissibility principle:

A force may be applied at any point on a line of action without changing the resultant effects of the force applied external to rigid body on which it acts

Magnitude, direction and line of action is important; not point of application



Method of Problem Solution

1- Graphical Method:-

A) A parallelogram

B) The triangle rule

((Magnitudes of Forces can be measured directly))

2- Mathematical Method (Algebraic method)

A) Trigonometric Solution.

B) Alternative Trigonometric Solution

Example: The two forces P and Q act on a bolt A. Determine the resultant.

SOLUTION

Graphical Solution. A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant are measured and found to be

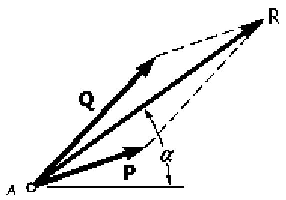
$R = 98$ N   a $= 35°$ R = 98 N a 35° ◄

The triangle rule may also be used. Forces **P** and **Q** are drawn in tip-to-tail fashion. Again the magnitude and direction of the resultant are measured.

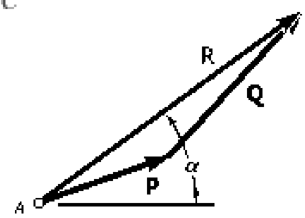
$R = 98$ N   a $= 35°$ R = 98 N a 35° ◄

P =40 N with 20^0

Q = 60 N with 45^0



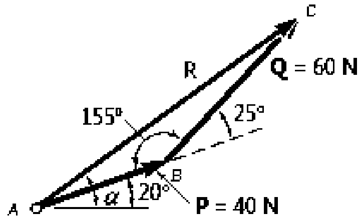
Trigonometric Solution. The triangle rule is again used; two sides and the included angle are known. We apply the law of cosines.



$$R^2 = P^2 + Q^2 - 2PQ \cos B$$
$$R^2 = (40\text{ N})^2 + (60\text{ N})^2 - 2(40\text{ N})(60\text{ N}) \cos 155°$$
$$R = 97.73\text{ N}$$

Now, applying the law of sines, we write

$$\frac{\sin A}{Q} = \frac{\sin B}{R} \qquad \frac{\sin A}{60\text{ N}} = \frac{\sin 155°}{97.73\text{ N}}$$



Solving Eq. (1) for sin A, we have

$$\sin A = \frac{(60\text{ N}) \sin 155°}{97.73\text{ N}}$$

Using a calculator, we first compute the quotient, then its arc sine, and obtain

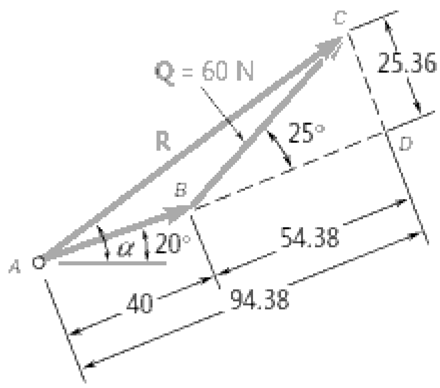
$$A = 15.04° \qquad \alpha = 20° + A = 35.04°$$

We use 3 significant figures to record the answer (cf. Sec. 1.6):

$$\mathbf{R} = 97.7\text{ N} \angle 35.0° \blacktriangleleft$$

Alternative Trigonometric Solution. We construct the right triangle BCD and compute

$$CD = (60\text{ N}) \sin 25° = 25.36\text{ N}$$
$$BD = (60\text{ N}) \cos 25° = 54.38\text{ N}$$



Then, using triangle ACD, we obtain

$$\tan A = \frac{25.36\text{ N}}{94.38\text{ N}} \qquad A = 15.04°$$
$$R = \frac{25.36}{\sin A} \qquad R = 97.73\text{ N}$$

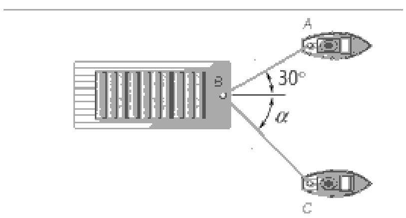
Again, $\alpha = 20° + A = 35.04°$ $\quad \mathbf{R} = 97.7\text{ N} \angle 35.0° \blacktriangleleft$

Ex. (H.W):

A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000-lb force directed along the axis of the barge, determine (*a*) the tension in each of the ropes knowing that $\alpha = 45°$, (*b*) the value of α for which the tension in rope 2 is minimum.

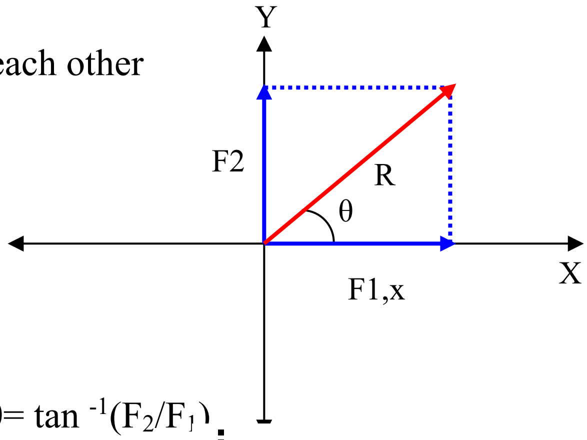
ANS:

a) 3700 Ib , b) 60^0



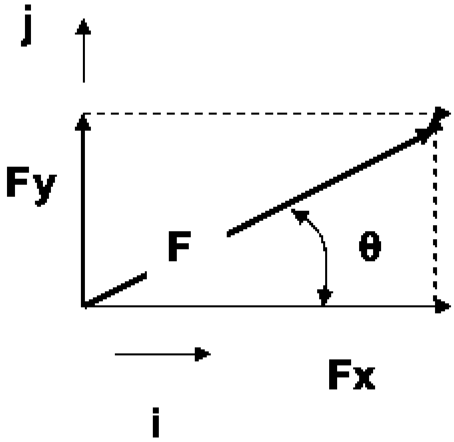
Review:-

A) Two Forces perpendicular each other



$$R = \sqrt{F_1^2 + F_2^2}$$

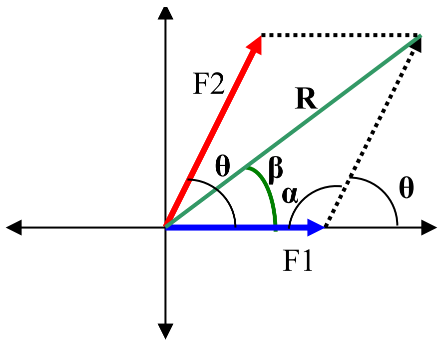
$\tan\theta = \frac{F_2}{F_1}$ ⟶ $\theta = \tan^{-1}(F_2/F_1)$



Fx = fx i ; Fy = fy j; fx, fy are scalar quantities

Therefore, F = fx i + fy j

B) The direction of each forces is know



α=180-θ

To find the Resultant force use the cosine Rule:-

$$R = \sqrt{F_1^2 + F_2^2 - 2F_1F_2 * \cos(\alpha)}$$

To find the direction of R use sine Rule

$$\frac{R}{\sin(\alpha)} = \frac{F_2}{\sin(\beta)}$$

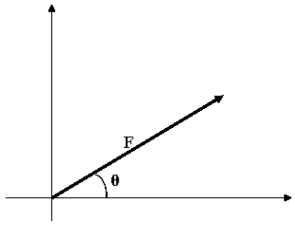
$$\text{Sin}(\beta) = \frac{F_2}{R}\sin(\alpha)$$

Resolve the force in to two components:

Let the force (F) shown in fig. (1) With the direction (**θ**)

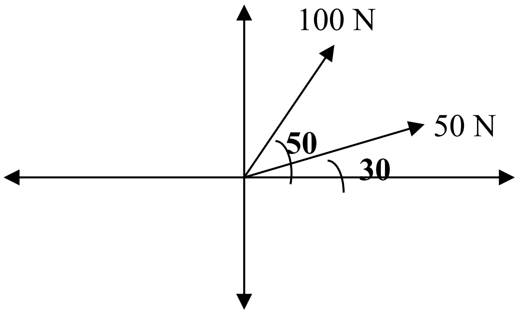
$$F_x = F\cos\theta \qquad F = \sqrt{F_x^2 + F_y^2}$$

$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

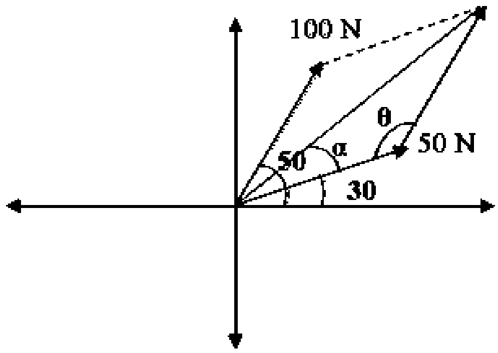


EX :

Find the magnitude and the direction of the Resultant force shown in Figure below



Solution:-



α=50-30=20

θ=180-20

θ=160^0

$$R = \sqrt{50^2 + 100^2 - 2*50*100*\cos(}$$

R=147.9 N

$$\frac{R}{\sin(\theta)} = \frac{F_2}{\sin(\alpha)}$$

$$\sin(\alpha) = \frac{100}{R}\sin(\theta) \longrightarrow$$ **α=13.4 ^{0}**

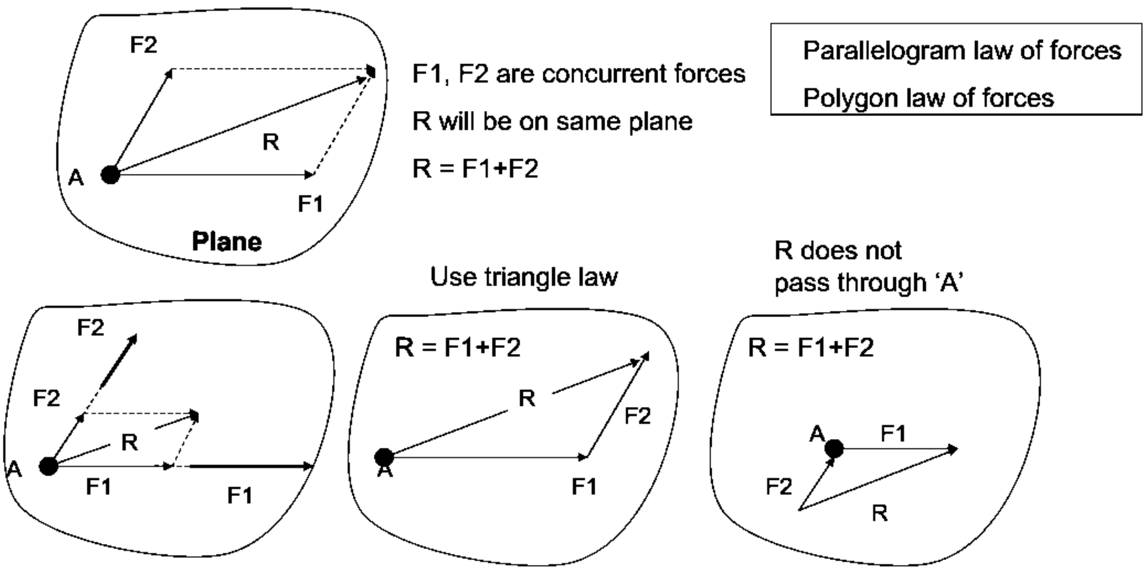
See Resolution force Examples

HW 1: Solve problem 2.1-2.20 page 25 in ref.1

Types of forces system

1- Coplanar forces system:
   - a- concurrent coplanar forces system
   - b- Non-concurrent coplanar forces system

2- Non coplanar forces system:
   - a- concurrent non-coplanar forces system
   - b- Non-concurrent non-coplanar forces system



Concurrent force:- Forces are said to be concurrent at a point if their lines of action intersect at that point

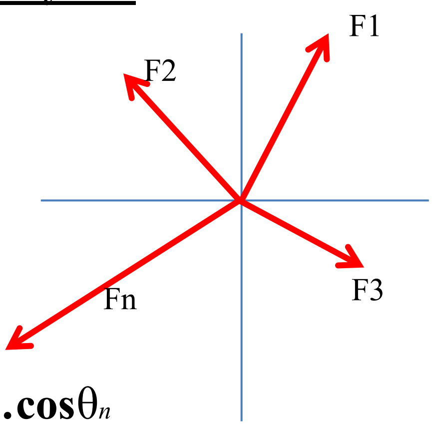
Resultant forces

A simplest force which can replace the original forces system without changing its external effect on a rigid body.

The symbol of resultant force is: →

The unit of resultant force is : Newton (N)

Resultant of concurrent coplanar forces system



We will find out the resultant force for many forces acting on a rigid body by using the following equations:

$$R_x = F_1.\cos\theta_1 \pm F_2.\cos\theta_2 \pm F_3.\cos\theta_3 \pm \ldots \pm F_n.\cos\theta_n$$

$$R_y = F_1.\sin\theta_1 \pm F_2.\sin\theta_2 \pm F_3.\sin\theta_3 \pm \ldots \pm F_n.\sin\theta_n$$

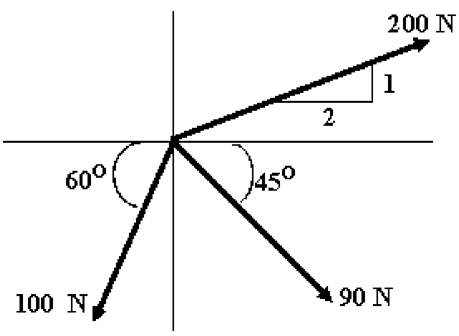
$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

The direction of resultant force may be determined as :

$$\theta = \tan^{-1}\left(\frac{Ry}{Rx}\right)$$

Ex :

Find the resultant force for the concurrent coplanar forces system, shown in figure.



Solution

$R_x = F_1.\cos\theta_1 \pm F_2.\cos\theta_2 \pm F_3.\cos\theta_3$

$$200 * \frac{2}{\sqrt{5}} - 100 * \cos 60 + 90 \cos 45 = +192.4N$$

$$Ry = 200 * \frac{1}{\sqrt{5}} - 100 * \sin 60 - 90 * \sin 45$$

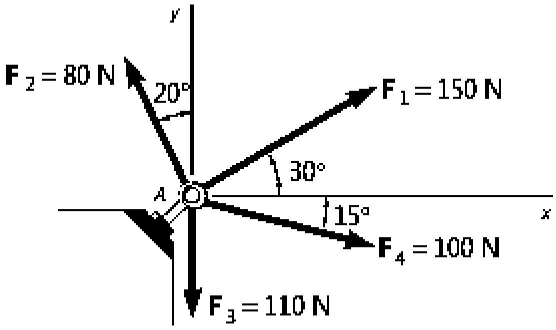
$$= -60.8N$$

$$R = \sqrt{(R_x)^2 + (R_y)^2}$$

$$\sqrt{(192.4)^2 + (60.8)^2} = 202N$$

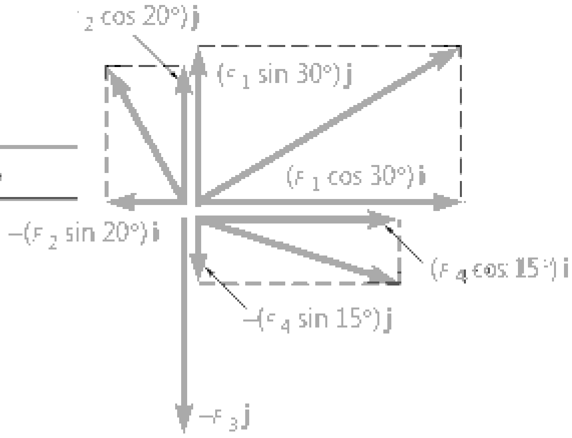
Ex :

Four forces act on bolt A as shown. Determine the resultant of the forces on the bolt.

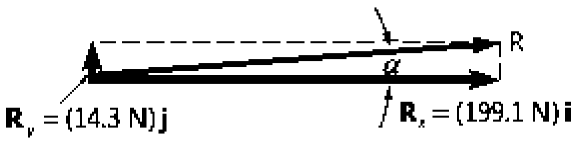


SOLUTION

Force	Magnitude, N	x Component, N	y Component, N
F_1	150	+129.9	+75.0
F_2	80	−27.4	+75.2
F_3	110	0	−110.0
F_4	100	+96.6	−25.9
		$R_x = +199.1$	$R_y = +14.3$



Thus, the resultant R of the four forces is



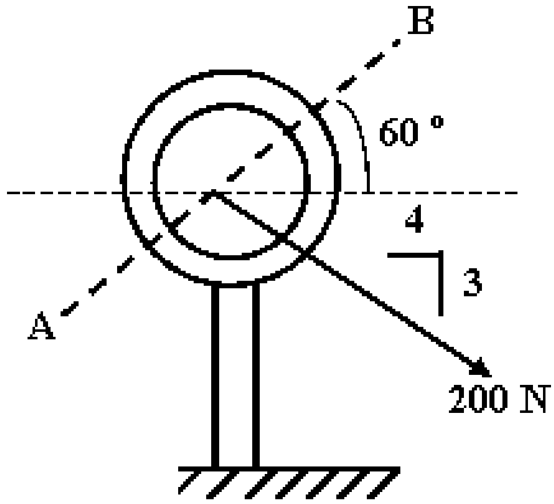
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (199.1\ \text{N})\mathbf{i} + (14.3\ \text{N})\mathbf{j}$$

$$\tan \alpha = \frac{R_y}{R_x} = \frac{14.3\ \text{N}}{199.1\ \text{N}} \qquad \alpha = 4.1°$$

$$R = \frac{14.3\ \text{N}}{\sin \alpha} = 199.6\ \text{N} \qquad \mathbf{R} = 199.6\ \text{N} \measuredangle 4.1°$$

Ex :-

The **200 N** force is a resultant of two forces, one of the forces "**P**" has the direction along the tine **AB** and the other force "**Q**" is on the horizontal direction, determine them.



Solution:

$$R_x = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$R * \cos\theta_R = F_1.\cos\theta_1 \mp F_2.\cos\theta_2$$

$$200 * \tfrac{4}{5} = P * \cos 60 + Q\cos(0)$$

$$160 = 0.5P + Q$$

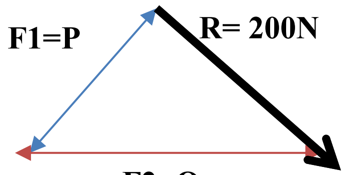
$$R_y = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$R * \sin\theta_R = F_1.\sin\theta_1 \mp F_2.\sin\theta_2$$

$$-200 * \tfrac{3}{5} = P * \sin 60 - Q\sin(0)$$

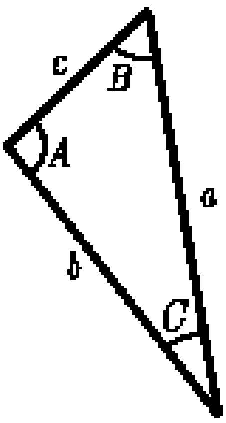
$$P = 138.5N \swarrow$$

$$\therefore Q = 229.25N$$



HW 2: Solve problem 2.21-2.42 page 33 in ref.1

HIN



Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

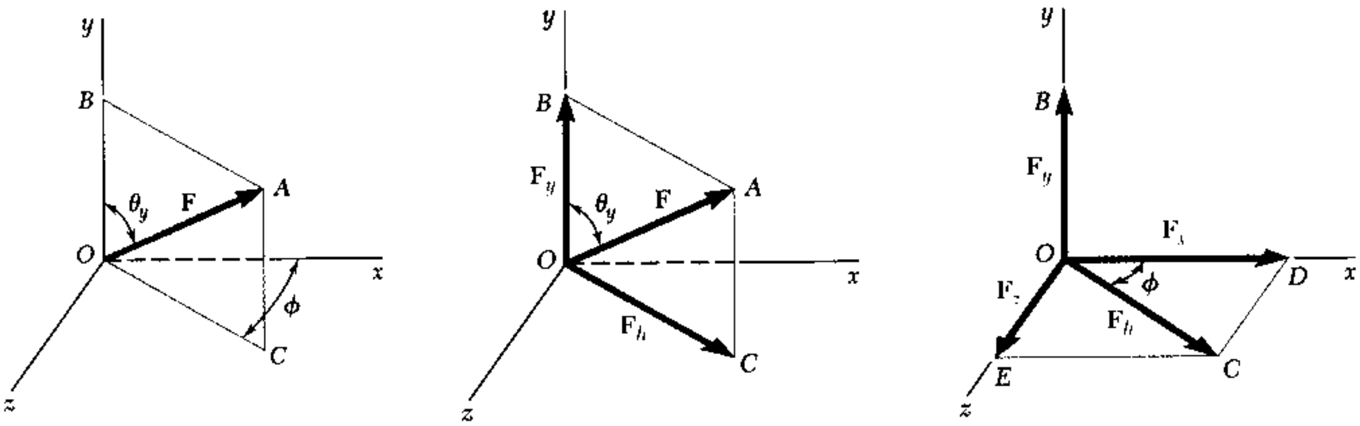
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

RECTANGULAR COMPONENTS
OF A FORCE IN SPACEF

The problems considered in the first part be formulated and solved in a single plane. In this section and in this section, we will discuss problems involving the three dimensions of space.

Consider a force F acting at the origin O of the system of rectangular coordinates x, y, z. To define the direction of F, we draw the vertical plane OBAC containing F.



- Vector **F** is contained in the plane *OBAC*
- Resolve **F** into horizontal and vertical components.

  $F_y = F\cos\theta_y$

  $F_h = F\sin\theta_y$
- Resolve F_h into rectangular components

  $F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$

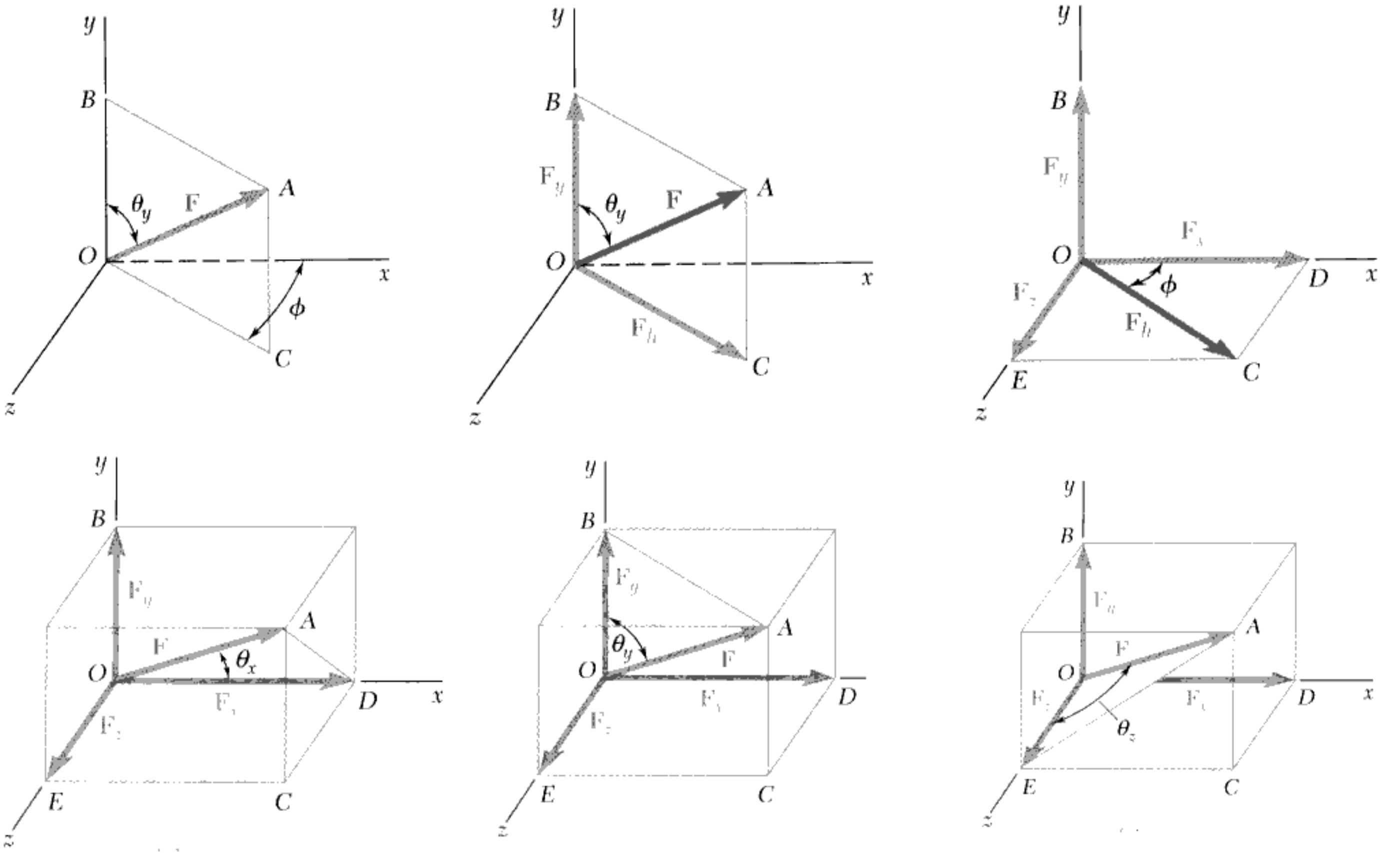
  $F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$

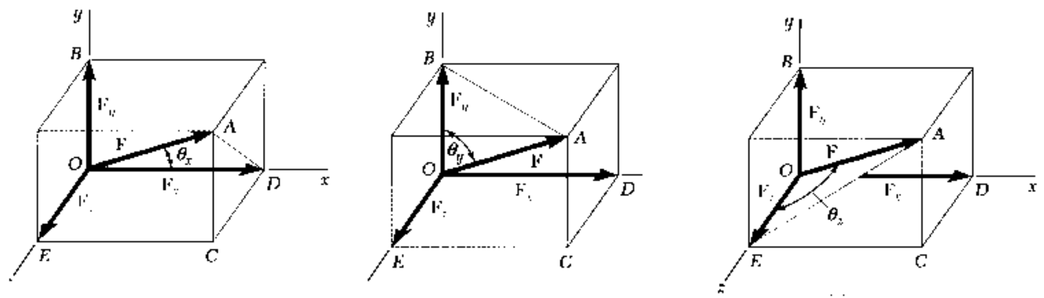
Applying the Pythagorean theorem to the triangles OAB and OCD of Fig. 2.30, we write

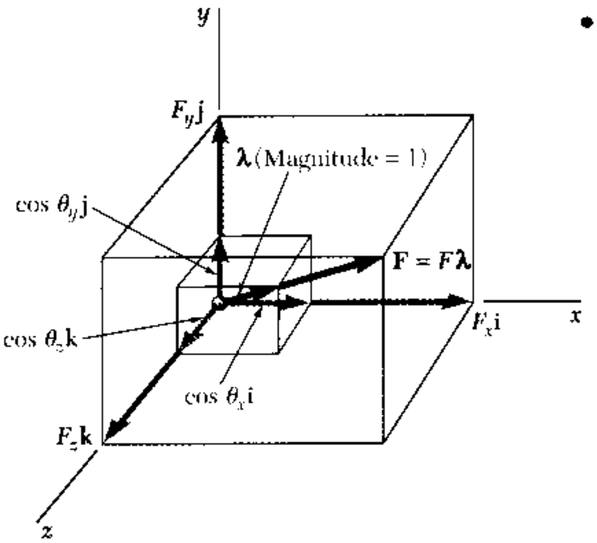
$$F^2 = (OA)^2 = (OB)^2 + (BA)^2 = F_y^2 + F_h^2$$
$$F_h^2 = (OC)^2 = (OD)^2 + (DC)^2 = F_x^2 + F_z^2$$

$$F = (F_x^2 + F_y^2 + F_z^2)^{1/2}$$

SPATIAL COMPONENTS (DIRECTION COSINES)







- With the angles between **F** and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

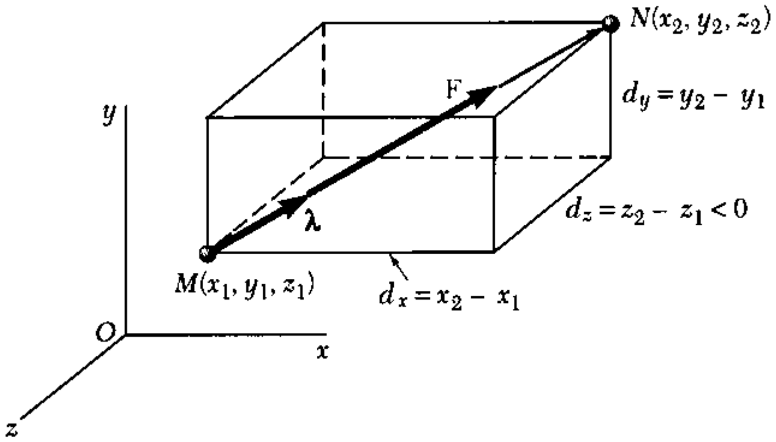
$$= F\vec{\lambda}$$

$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a **unit vector** along the line of action of **F**; $\cos\theta_x$, $\cos\theta_y$, and $\cos\theta_z$ are the **direction cosines**

Rectangular Components in Space

- **Direction** of force **F**
  - Defined by **location** of **two points**
    - *M* (x_1, y_1 and z_1) and *N* (x_2, y_2 and z_2)



$\vec{d}$ = vector joining *M* and *N*

$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$

$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$

$$\vec{F} = F\vec{\lambda}$$

$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$

$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$

ADDITION OF CONCURRENT FORCES IN SPACE

$$\mathbf{R} = \Sigma\mathbf{F}$$

we resolve each force into its rectangular components and write

$$R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = \Sigma(F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k})$$
$$= (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k}$$

from which it follows that

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R_z = \Sigma F_z$$

The magnitude of the resultant and the angles u_x, u_y, u_z that the resultant forms with the coordinate axes are

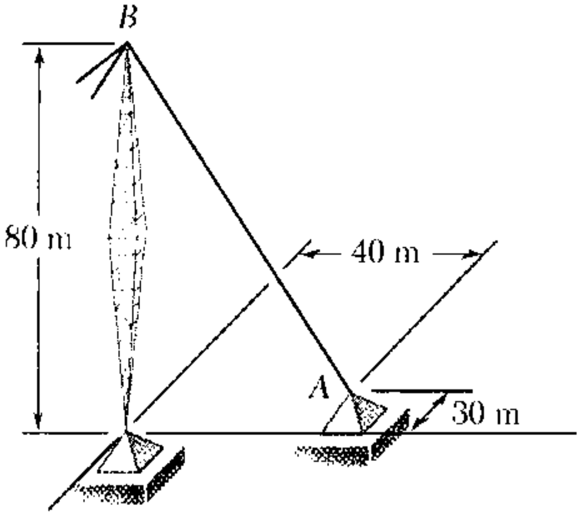
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$\cos u_x = \frac{R_x}{R} \qquad \cos u_y = \frac{R_y}{R} \qquad \cos u_z = \frac{R_z}{R}$$

Example: The tension in the guy wire is 2500 N. Determine:

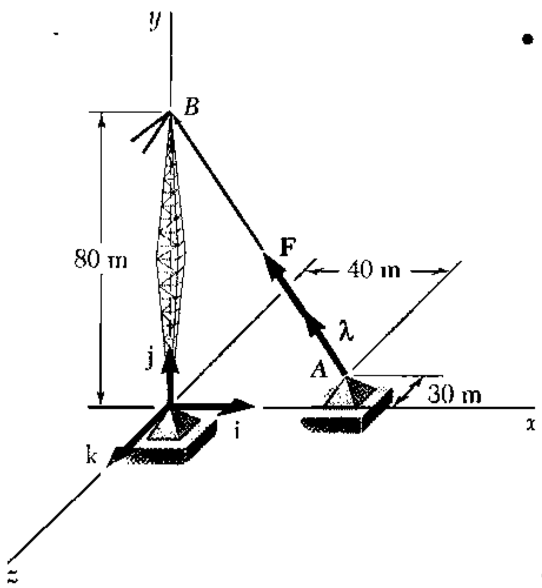
a) components F_x, F_y, F_z of the force acting on the bolt at A,

b) the angles q_x, q_y, q_z defining the direction of the force



SOLUTION:

- Based on the relative locations of the points A and B, determine the unit vector pointing from A towards B.
- Apply the unit vector to determine the components of the force acting on A.
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.



Solution

- Determine the unit vector pointing from A towards B.

$$\overline{AB} = (-40\,\text{m})\vec{i} + (80\,\text{m})\vec{j} + (30\,\text{m})\vec{k}$$

$$AB = \sqrt{(-40\,\text{m})^2 + (80\,\text{m})^2 + (30\,\text{m})^2} = 94.3\,\text{m}$$

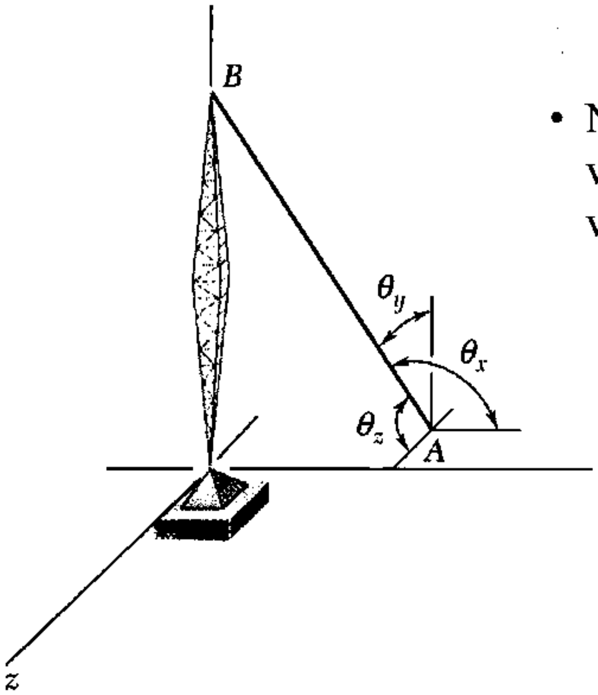
$$\vec{\lambda} = \left(\frac{-40}{94.3}\right)\vec{i} + \left(\frac{80}{94.3}\right)\vec{j} + \left(\frac{30}{94.3}\right)\vec{k} = -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

- Determine the components of the force.

$$\vec{F} = F\vec{\lambda}$$

$$= (2500\,\text{N})\left(-0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}\right)$$

$$= (-1060\,\text{N})\vec{i} + (2120\,\text{N})\vec{j} + (795\,\text{N})\vec{k}$$



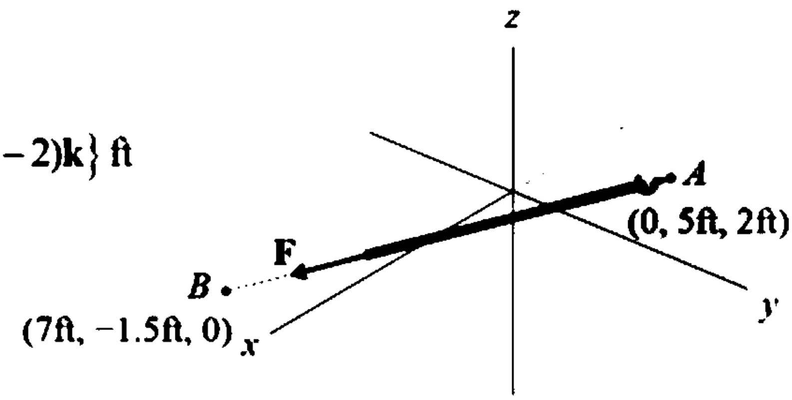
- Noting that the components of the unit vector are the direction cosines for the vector, calculate the corresponding angles.

$$\vec{\lambda} = \cos\theta_x \vec{i} + \cos\theta_y \vec{j} + \cos\theta_z \vec{k}$$
$$= -0.424\vec{i} + 0.848\vec{j} + 0.318\vec{k}$$

$$\theta_x = 115.1^\circ$$
$$\theta_y = 32.0^\circ$$
$$\theta_z = 71.5^\circ$$

HW 2: See Ex. 2.8 and Solve problem 2.71-2.98 page 55 in ref.1

Example: The line of action of force **F** directs from point A to point B. If the magnitude of the force is 120 lb, express the force in Cartesian vector form.



Position vector:

$$\mathbf{r}_{AB} = \mathbf{r}_B - \mathbf{r}_A = \{(7-0)\mathbf{i} + (-1.5-5)\mathbf{j} + (0-2)\mathbf{k}\}\ \text{ft}$$
$$= \{7\mathbf{i} - 6.5\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

Unit vector:

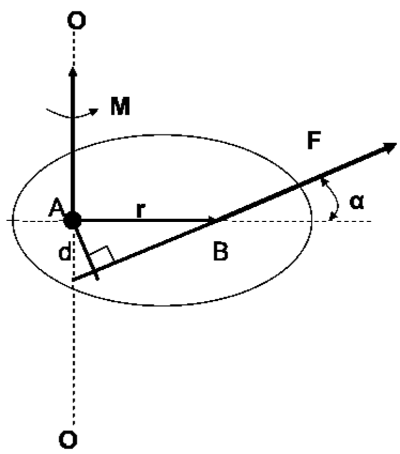
$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\{7\mathbf{i} - 6.5\mathbf{j} - 2\mathbf{k}\}\ \text{ft}}{\sqrt{7^2 + (-6.5)^2 + (-2)^2}\ \text{ft}}$$
$$= 0.717\mathbf{i} - 0.666\mathbf{j} - 0.205\mathbf{k}$$

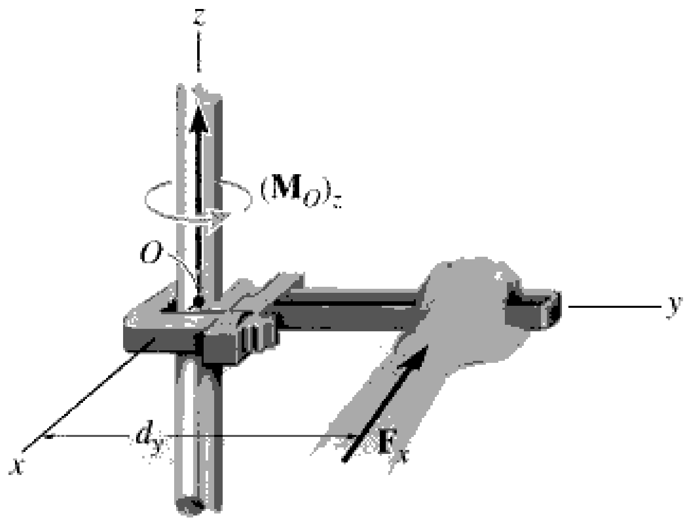
Force vector:

$$\mathbf{F} = F \cdot \mathbf{u}_{AB} = 120\ \text{lb} \cdot \{0.717\mathbf{i} - 0.666\mathbf{j} - 0.205\mathbf{k}\}$$
$$= \{86.1\mathbf{i} - 79.9\mathbf{j} - 24.6\mathbf{k}\}\ \text{lb}$$

Moment of a force

The moment of a force: The tendency of a force to rotate a rigid body about any defined axis (or point or line) is called the Moment of the Force.





Mathematically:

The moment of a force = the applied force X perpendicular distance from the axis of rotation to the LOA of force

M = F * d

M = the moment of a force (**N.m**) or

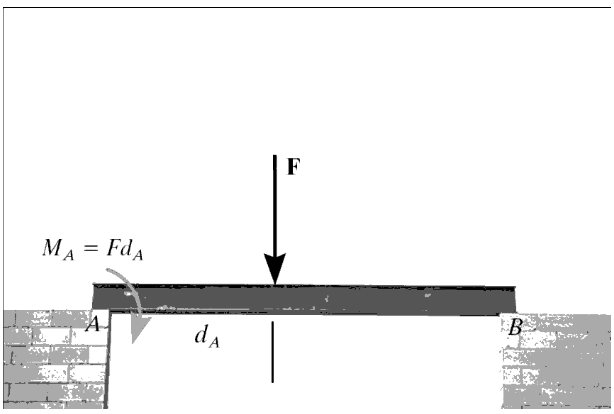
F = applied force (**N**)

d = perpendicular distance between the point of action of the force and moment centre.

Moment is perpendicular to plane about axis O-O.

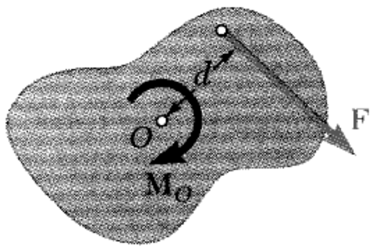
Beams are often used to bridge gaps in walls. We have to know what the effect of the force on the beam will have on the beam supports.

What do you think those impacts are at points A and B?

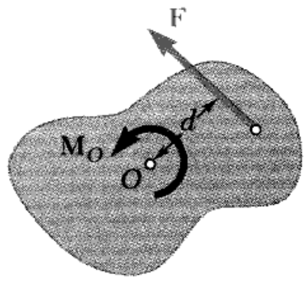


Properties of a Moment

- Moments not only have a magnitude, they also have a sense to them.
- The sense of a moment is clockwise or counter-clockwise depending on which way it will tend to make the object rotate.

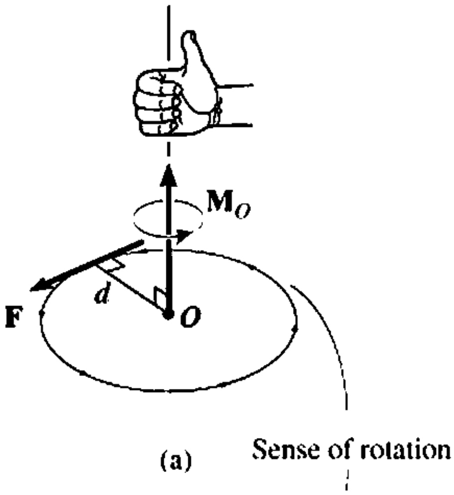


(b) $M_O = -Fd$



(a) $M_O = +Fd$

- The sense of a Moment is defined by the direction it is acting on the Axis and can be found using Right Hand Rule.

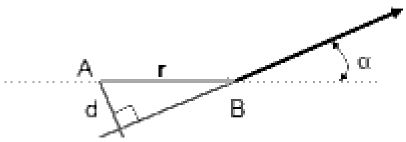


(a)

Cross product:

M = r x F; where 'r' is the position vector which runs from the moment reference point 'A' to any point on the LOA of 'F'

M = Fr sin α; M = Fd



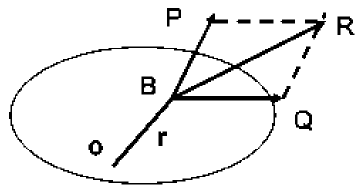
Sin α = d / r

M = r x F = -(F x r): sense is important

Varignon Theorem

The moment of a force about any point is equal to the sum of the moments of the components of the forces about the same point

Concurrent forces – P, Q



$M_o = r \times R = r \times (P+Q) = \underline{r \times P} + \underline{r \times Q}$

Moment of 'P'   Moment of 'Q'

Usefulness:

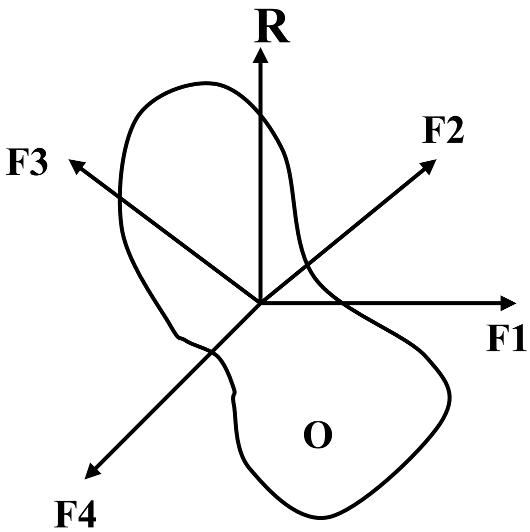
Resultant 'R' – moment arm 'd'

Force 'P' – moment arm 'p'; Force 'Q' – moment arm 'q'

$M_o = Rd = -pP + qQ$

OR

Application of varignons theorem that



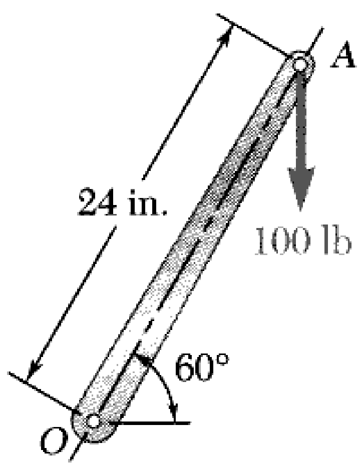
MR= M1+M2+M3+------------Mn

Or

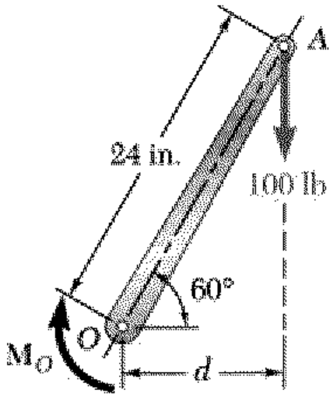
$$MR = \sum_{i=1}^{n} Mi$$

Example

- A 100-lb vertical force is applied to the end of a lever which is attached to a shaft at *O*.
- Determine:
  a) Moment about *O*,
  b) Horizontal force at *A* which creates the same moment,
  c) Smallest force at A which produces the same moment,
  d) Location for a 240-lb vertical force to produce the same moment,
  e) Whether any of the forces from b, c, and d is equivalent to the original force.

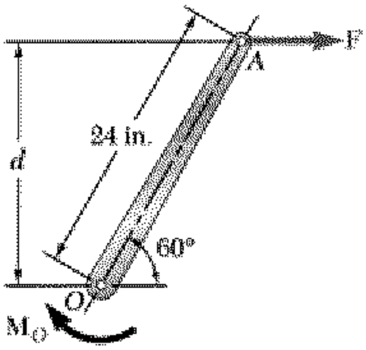


Solution:



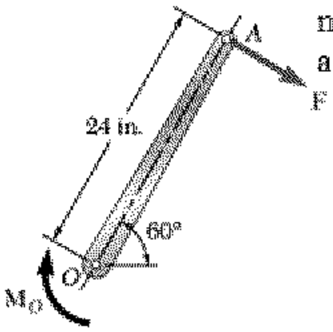
a) Moment about O is equal to the product of the force and the perpendicular distance between the line of action of the force and O. Since the force tends to rotate the lever clockwise, the moment vector is into the plane of the paper.

$$M_O = Fd$$
$$d = (24\text{ in.})\cos 60° = 12\text{ in.}$$
$$M_O = (100\text{ lb})(12\text{ in.})$$
$$\boxed{M_O = 1200\text{ lb}\cdot\text{in}}$$



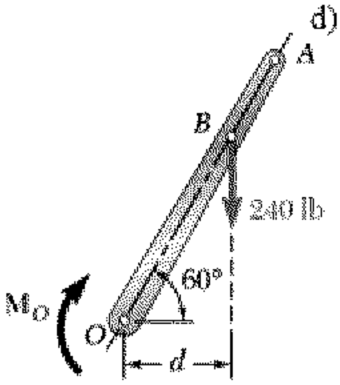
b) Horizontal force at A that produces the same moment,

$$d = (24\text{ in.})\sin 60° = 20.8\text{ in.}$$
$$M_O = Fd$$
$$1200\text{ lb}\cdot\text{in.} = F(20.8\text{ in.})$$
$$F = \frac{1200\text{ lb}\cdot\text{in.}}{20.8\text{ in.}}$$
$$\boxed{F = 57.7\text{ lb}}$$



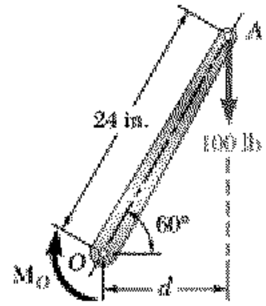
c) The smallest force at A to produce the same moment occurs when the perpendicular distance is a maximum or when F is perpendicular to OA.

$$M_O = Fd$$
$$1200\text{lb}\cdot\text{in.} = F(24\text{ in.})$$
$$F = \frac{1200\text{ lb}\cdot\text{in.}}{24\text{ in.}}$$
$$\boxed{F = 50\text{ lb}}$$

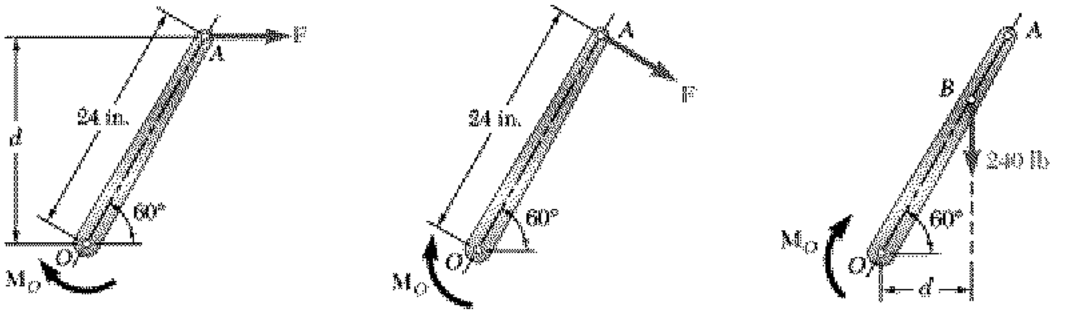


d) To determine the point of application of a 240 lb force to produce the same moment,

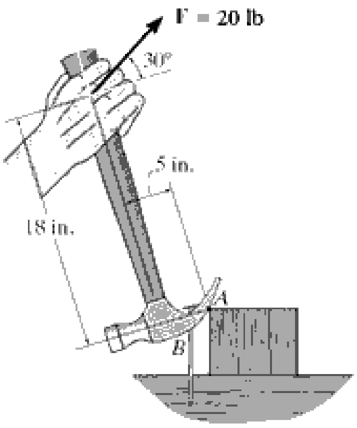
$$M_O = Fd$$
$$1200\text{lb}\cdot\text{in.} = (240\text{ lb})d$$
$$d = \frac{1200\text{lb}\cdot\text{in.}}{240\text{ lb}} = 5\text{ in.}$$
$$OB\cos 60° = 5\text{ in.}$$
$$\boxed{OB = 10\text{ in.}}$$



e) Although each of the forces in parts b), c), and d) produces the same moment as the 100 lb force, none are of the same magnitude and sense, or on the same line of action. None of the forces is equivalent to the 100 lb force.



Example:



Given: A 20 lb force is applied to the hammer.

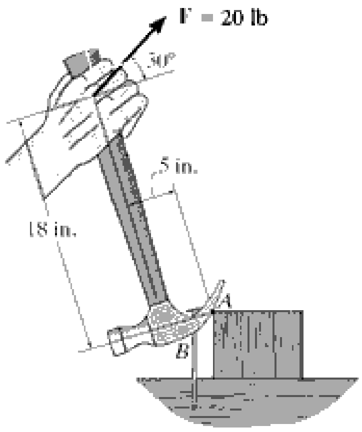
Find: The moment of the force at A.

Plan:

Since this is a 2-D problem:

1) Resolve the 20 lb force along the handle's x and y axes.

2) Determine M_A using a scalar analysis.



Solution:

$+\uparrow F_y = 20 \sin 30° \text{ lb}$

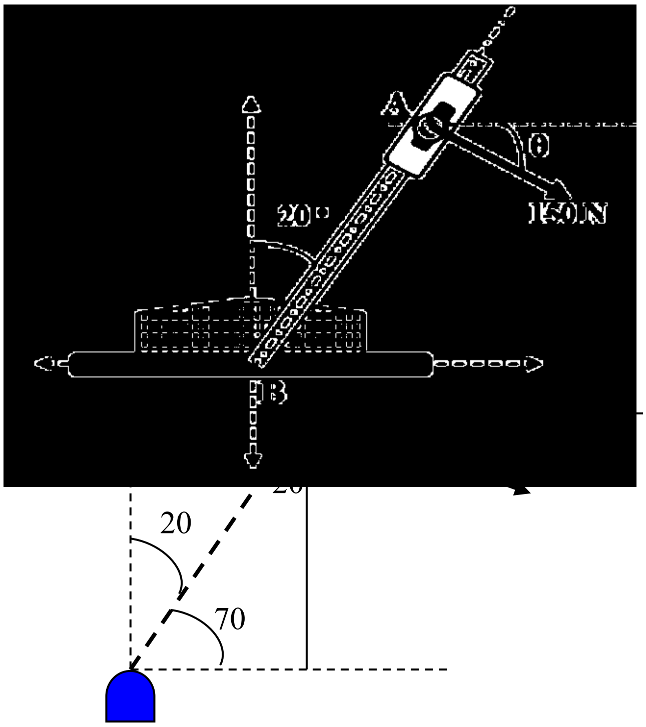
$+\rightarrow F_x = 20 \cos 30° \text{ lb}$

$$+\circlearrowleft M_A = \{-(20 \cos 30°)\text{lb } (18 \text{ in}) - (20 \sin 20°)\text{lb } (5 \text{ in})\}$$

$$= -351.77 \text{ lb·in} = 352 \text{ lb·in (clockwise)}$$

EX:-

Determine the maximum moment about (B)
Which can be caused by the (150N) Force.
In what direction should the force act?
The distance (AB) is 250 mm.



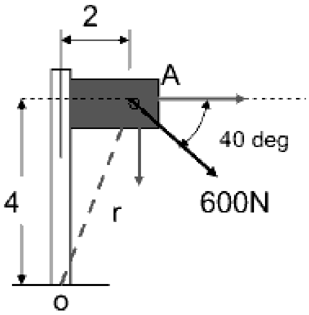
Solution:-

$M_B = 150*0.25 = -37.5 N.m$

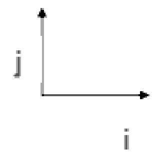
$\theta = 180-(90+70) = 20^o$

Pb:2/5 (Meriam / Kraige):

Calculate the magnitude of the moment about 'O' of the force 600 N



in mm



1) Mo = 600 cos 40 (4) + 600 sin 40 (2)
= 2610 Nm (app.)

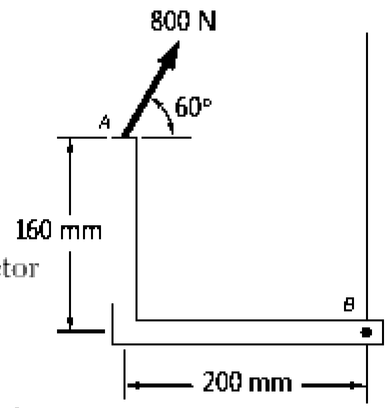
2) Mo = r x F = (2i + 4j) x (600cos40i-600sin40j)

= -771.34-1839 = 2609.85 Nm (CW);

mag = 2610 Nm

EX :

A force of 800 N acts on a bracket as shown. Determine the moment of the force about B.



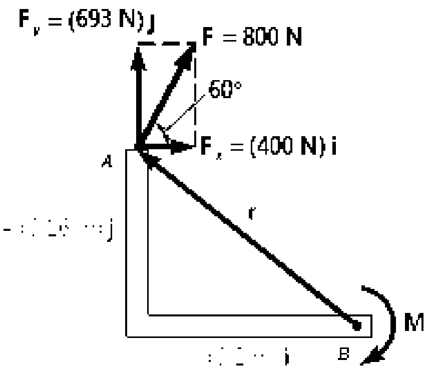
Solution:

The moment $\mathbf{M}_B$ of the force $\mathbf{F}$ about B is obtained by forming the vector product

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}$$

where $\mathbf{r}_{A/B}$ is the vector drawn from B to A. Resolving $\mathbf{r}_{A/B}$ and $\mathbf{F}$ into rectangular components, we have

$$\mathbf{r}_{A/B} = -(0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}$$
$$\mathbf{F} = (800\text{ N})\cos 60°\mathbf{i} + (800\text{ N})\sin 60°\mathbf{j}$$
$$= (400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}$$



Recalling the relations (3.7) for the cross products of unit vectors (Sec. 3.5) we obtain

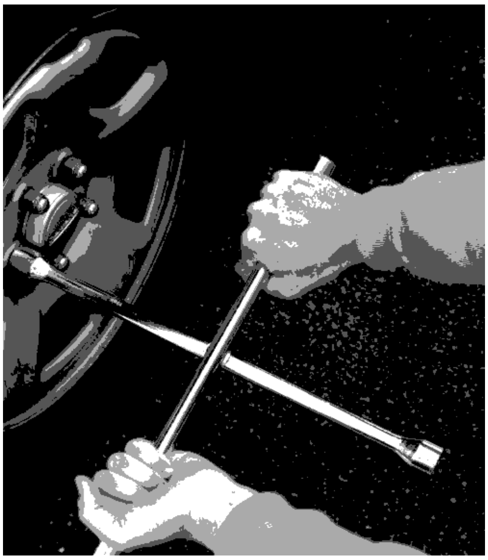
$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F} = [-(0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}] \times [(400\text{ N})\mathbf{i} + (693\text{ N})\mathbf{j}]$$
$$= -(138.6\text{ N}\cdot\text{m})\mathbf{k} - (64.0\text{ N}\cdot\text{m})\mathbf{k}$$
$$= -(202.6\text{ N}\cdot\text{m})\mathbf{k} \qquad \mathbf{M}_B = 203\text{ N}\cdot\text{m} \; \mathbf{i} \; ◄$$

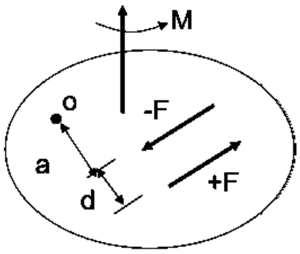
The moment $\mathbf{M}_B$ is a vector perpendicular to the plane of the figure and pointing *into* the paper.

HW : Solve problem 3.1-3.15 page 91 in ref.1

Moment of Couple

Couples:- Moment produced by two equal, opposite and non-collinear forces. It does not produce any translation, only rotation.





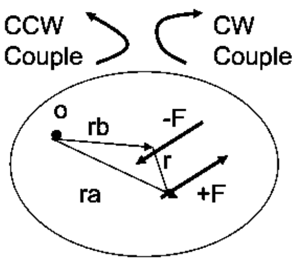
=>-F and F produces rotation

=>Mo = F (a+d) – Fa = Fd;
Perpendicular to plane

⇒Independent of distance from ‘o’, depends on ‘d’ only

⇒ moment is same for all moment centers

Vector algebra method



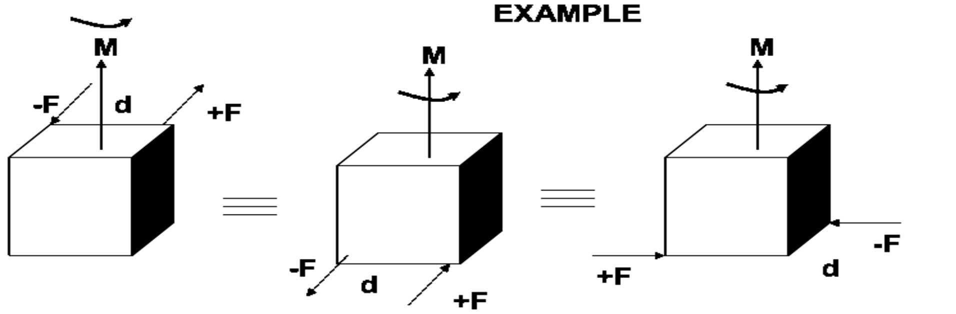
M = ra x F + rb x (-F) = (ra-rb) x F = r x F

Equivalent couples

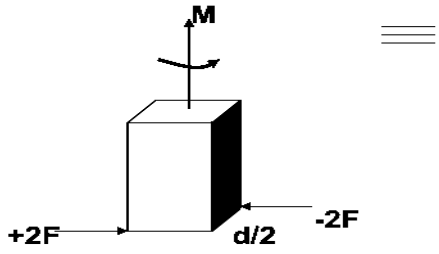
•Changing the F and d values does not change a given couple as long as the product (Fd) remains same

•Changing the plane will not alter couple as long as it is parallel

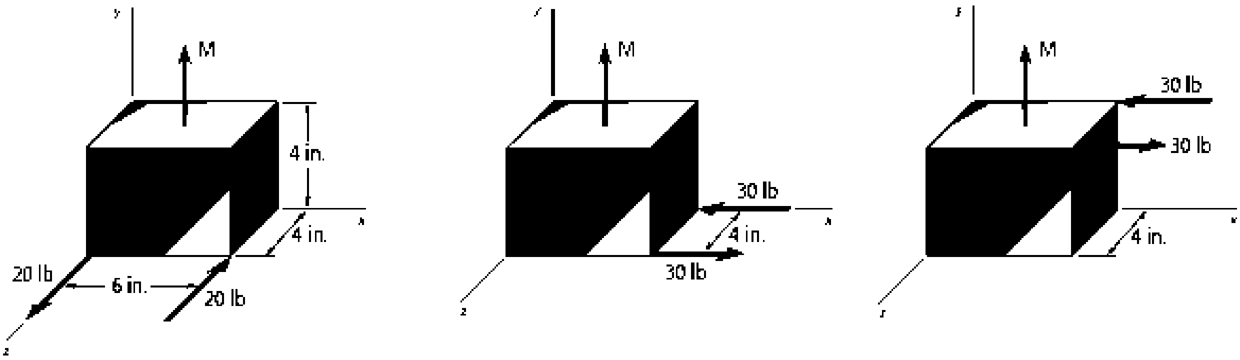
EXAMPLE



All four are equivalent couples



EX:

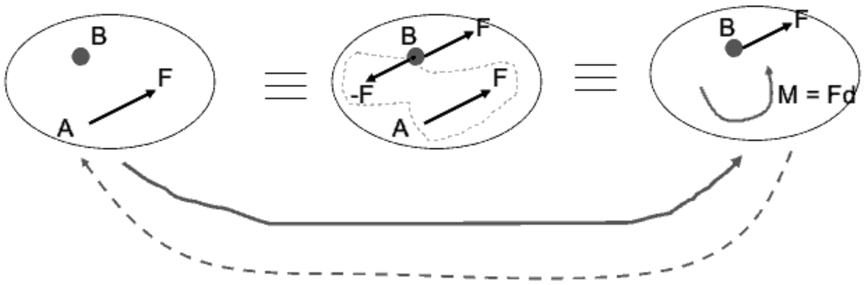


Force-couple system

=>Effect of force is two fold – 1) to push or pull, 2) rotate the body about any axis

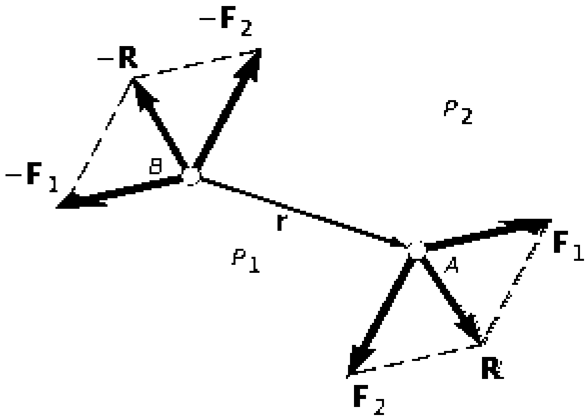
⇒Dual effect can be represented by a force-couple syatem

⇒ a force can be <u>replaced</u> by a force and couple



Addition of couples

we express the moment M of the resulting couple (in the figure)as follows:



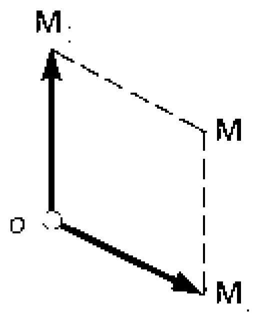
$$\mathbf{M} = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

and, by Varignon's theorem,

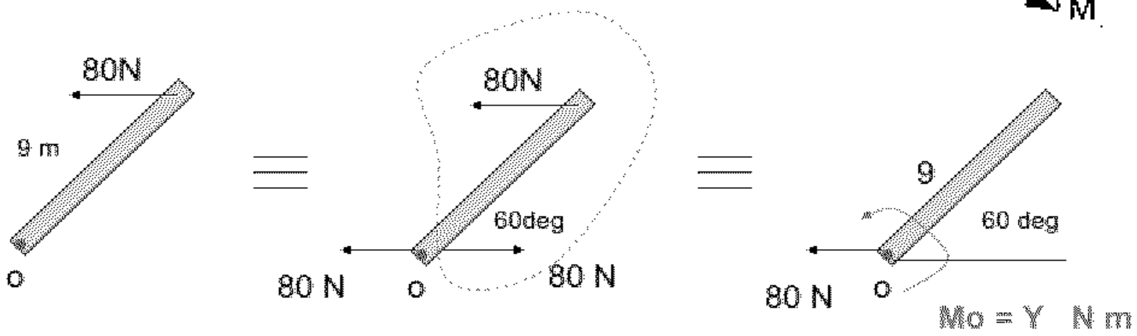
$$\mathbf{M} = \mathbf{r} \times \mathbf{F}_1 + \mathbf{r} \times \mathbf{F}_2$$

But the first term in the expression obtained represents the moment M1 of the couple in P1 , and the second term represents the moment M2 of the couple in P2. We have

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$



EXAMPLE



Mo = 80 (9 sin 60) = 624 N m; CCW

EXAMPLE 4.10

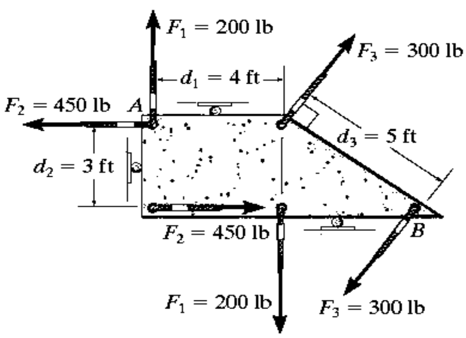


Fig. 4–30

Determine the resultant couple moment of the three couples acting on the plate in Fig. 4–30.

SOLUTION

As shown the perpendicular distances between each pair of couple forces are $d_1 = 4$ ft, $d_2 = 3$ ft, and $d_3 = 5$ ft. Considering counterclockwise couple moments as positive, we have

$$\circlearrowleft +M_R = \Sigma M; \; M_R = -F_1 d_1 + F_2 d_2 - F_3 d_3$$

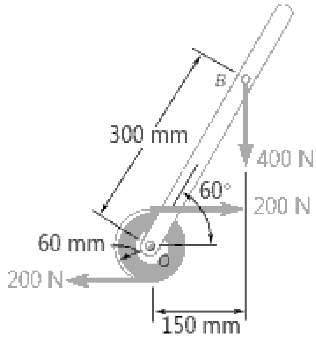
$$= (-200 \text{ lb})(4 \text{ ft}) + (450 \text{ lb})(3 \text{ ft}) - (300 \text{ lb})(5 \text{ ft})$$

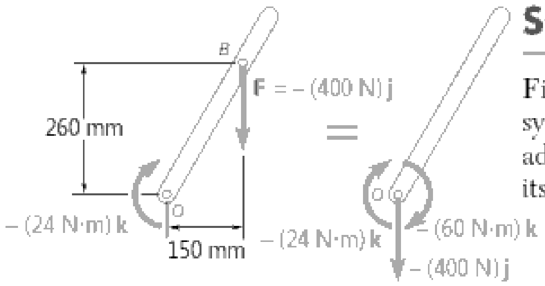
$$= -950 \text{ lb}\cdot\text{ft} = 950 \text{ lb}\cdot\text{ft} \circlearrowright \qquad \textit{Ans.}$$

The negative sign indicates that $\mathbf{M}_R$ has a clockwise rotational sense.

EX:

Replace the couple and force shown by an equivalent single force applied to the lever. Determine the distance from the shaft to the point of application of this equivalent force.

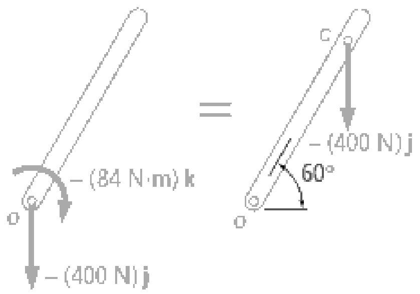




SOLUTION

First the given force and couple are replaced by an equivalent force-couple system at O. We move the force $\mathbf{F} = -(400\text{ N})\mathbf{j}$ to O and at the same time add a couple of moment $\mathbf{M}_O$ equal to the moment about O of the force in its original position.

$$\mathbf{M}_O = \overrightarrow{OB} \times \mathbf{F} = [(0.150\text{ m})\mathbf{i} + (0.260\text{ m})\mathbf{j}] \times (-400\text{ N})\mathbf{j}$$
$$= -(60\text{ N}\cdot\text{m})\mathbf{k}$$

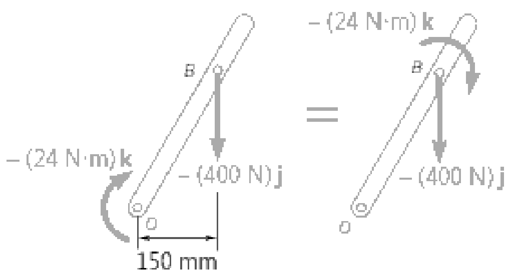


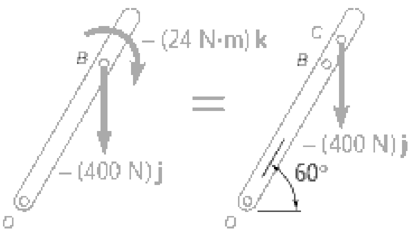
This couple is added to the couple of moment $-(24\text{ N}\cdot\text{m})\mathbf{k}$ formed by the two 200-N forces, and a couple of moment $-(84\text{ N}\cdot\text{m})\mathbf{k}$ is obtained. This last couple can be eliminated by applying $\mathbf{F}$ at a point C chosen in such a way that

$$-(84\text{ N}\cdot\text{m})\mathbf{k} = \overrightarrow{OC} \times \mathbf{F}$$
$$= [(OC)\cos 60°\mathbf{i} + (OC)\sin 60°\mathbf{j}] \times (-400\text{ N})\mathbf{j}$$
$$= -(OC)\cos 60°(400\text{ N})\mathbf{k}$$

We conclude that

$$(OC)\cos 60° = 0.210\text{ m} = 210\text{ mm} \qquad OC = 420\text{ mm} \blacktriangleleft$$





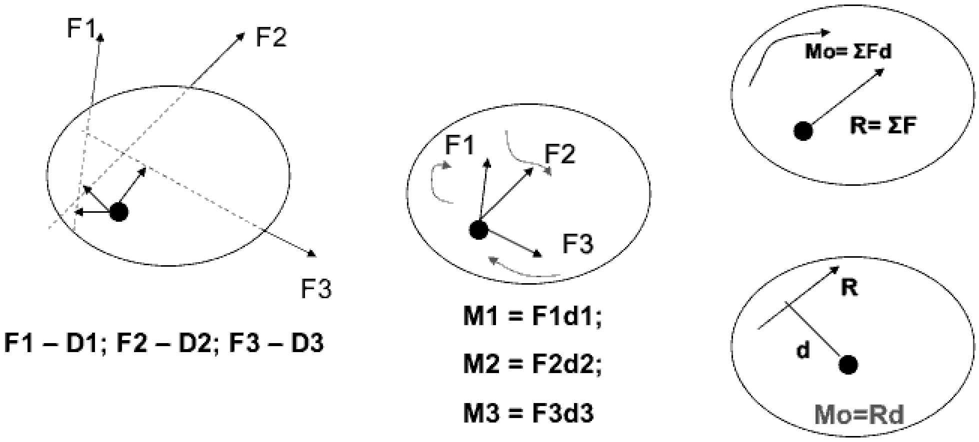
Alternative Solution. Since the effect of a couple does not depend on its location, the couple of moment $-(24\text{ N}\cdot\text{m})\mathbf{k}$ can be moved to B; we thus obtain a force-couple system at B. The couple can now be eliminated by applying $\mathbf{F}$ at a point C chosen in such a way that

$$-(24\text{ N}\cdot\text{m})\mathbf{k} = \overrightarrow{BC} \times \mathbf{F}$$
$$= -(BC)\cos 60°(400\text{ N})\mathbf{k}$$

We conclude that

$$(BC)\cos 60° = 0.060\text{ m} = 60\text{ mm} \qquad BC = 120\text{ mm}$$
$$OC = OB + BC = 300\text{ mm} + 120\text{ mm} \qquad OC = 420\text{ mm} \blacktriangleleft$$

How to obtain resultant force ?



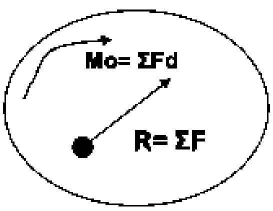
NON-CONCURRENT FORCES

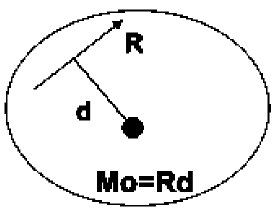
Principle of moments

Summarize the above process: R = ΣF

Mo = ΣM = Σ(Fd)

Mo = Rd





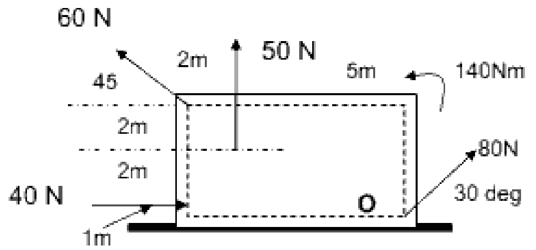
First two equations: reduce the system of forces to a force-couple system at some point 'O'

Third equation: distance 'd' from point 'O' to the line of action 'R'

=> VARIGNON'S THEOREM IS EXTENDED HERE FOR NON-CONCURENT FORCES

EX:

Meriam / kraige; 2/8



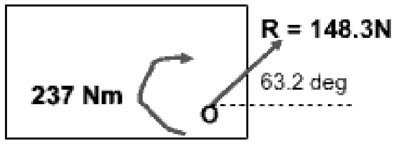
Find the resultant of four forces and one couple which act on the plate

Rx = 40+80cos30-60cos45 = 66.9 N

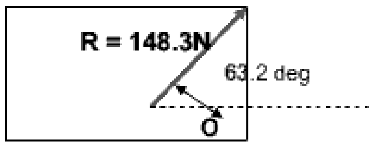
Ry = 50+80sin 30+60cos45 = 132.4 N

R = 148.3 N; $\Theta = \tan^{-1}(132.4/66.9) = 63.2$ deg

Mo = 140-50(5)+60cos45(4)-60sin45(7) = -237 Nm



Final LOA of R: 148.3 d = 237; d = 1.6 m



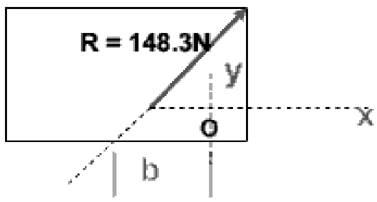
LOA of R with x-axis:

$(Xi + yj) \times (66.9i+132.4j) = -237k$

$(132.4x - 66.9y)k = -237k$

$132.4x - 66.9y = -237$

Y = 0 => x = b = -1.792 m

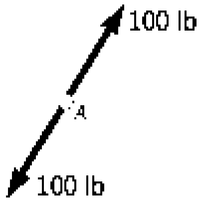


HW : Solve problem 3.70-3.92 page 118 in ref.1

HW : Solve problem 3.147-3.148 page 152 in ref.1

The equilibrium

When the resultant of all the forces acting on a particle is zero, the particle is in equilibrium.



$$\mathbf{R} = \Sigma \mathbf{F} = 0 \qquad .14)$$

Resolving each force **F** into rectangular components, we have

$$\Sigma(F_x\mathbf{i} + F_y\mathbf{j}) = 0 \qquad \text{or} \qquad (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} = 0$$

We conclude that the necessary and sufficient conditions for the equilibrium of a particle are

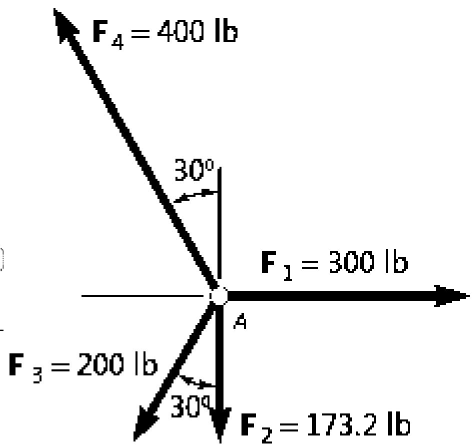
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad (2.15)$$

And

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

See Sample problem 2.9 page 59 Ref. 1

Example:- check the equilibrium conditions For the following system are satisfied ?



Solution:-

$$\Sigma F_x = 300 \text{ lb} - (200 \text{ lb}) \sin 30° - (400 \text{ lb}) \sin 30°$$
$$= 300 \text{ lb} - 100 \text{ lb} - 200 \text{ lb} = 0$$
$$\Sigma F_y = -173.2 \text{ lb} - (200 \text{ lb}) \cos 30° + (400 \text{ lb}) \cos 30$$
$$= -173.2 \text{ lb} - 173.2 \text{ lb} + 346.4 \text{ lb} = 0$$

HW : Solve problem 2.f5-2.102 page 61 in ref.1 (3 D)

Newton's First Law of Motion

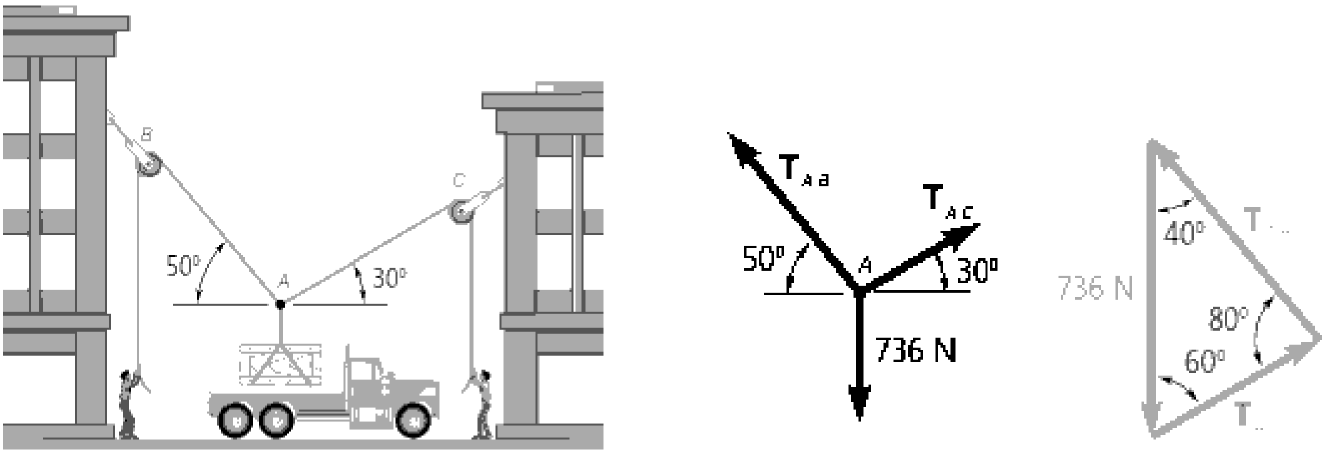
If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

From this law and from the definition of equilibrium given above, it is seen that a particle in equilibrium either is at rest or is moving in a straight line with constant speed. In the following section, various problems concerning the equilibrium of a particle will be considered.

FREE BODY DIAGRAM

Free body diagram: is a sketch showing the body (particle) and all the forces (and reactions) acting on it.

Ex: W=75 kg = 75*9.81=736 N



(a) Space diagram (b) Free-body diagram (c) Force triangle

Since point A is in equilibrium, the three forces acting on it must form a closed triangle when drawn in tip-to-tail fashion.

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{AC}}{\sin 40°} = \frac{736\ \text{N}}{\sin 80°}$$

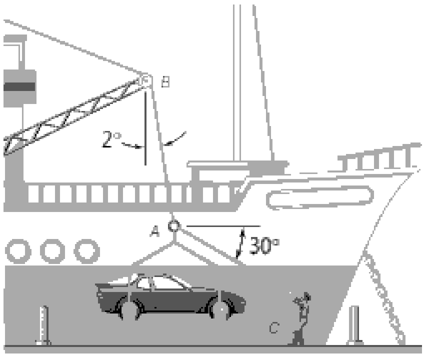
$$T_{AB} = 647\ \text{N} \qquad T_{AC} = 480\ \text{N}$$

If an analytic solution is desired, the equations of equilibrium

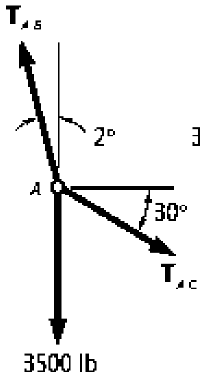
$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$

EX:

In a ship-unloading operation, a 3500-lb automobile is supported by a cable A rope is tied to the cable at A and pulled in order to center the automobile over its intended position. The angle between the cable and the vertical is 2°, while the angle between the rope and the horizontal is 30°. What is the tension in the rope?



Solution:



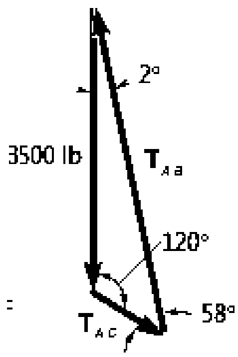
Free-Body Diagram. Point A is chosen as a free body, and the complete free-body diagram is drawn. T_{AB} is the tension in the cable AB, and T_{AC} is the tension in the rope.

Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Using the law of sines, we write

$$\frac{T_{AB}}{\sin 120^\circ} = \frac{T_{AC}}{\sin 2^\circ} = \frac{3500 \text{ lb}}{\sin 58^\circ}$$

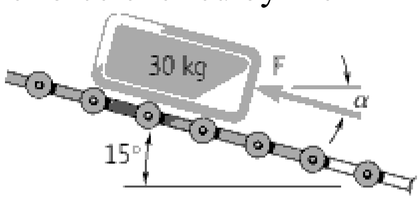
With a calculator, we first compute and store the value of the last quotient. Multiplying this value successively by sin 120° and sin 2°, we obtain

$$T_{AB} = 3570 \text{ lb} \qquad T_{AC} = 144 \text{ lb}$$

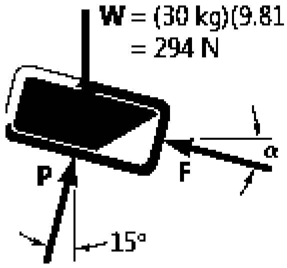


EX:

Determine the magnitude and direction of the smallest force F which will maintain the package shown in equilibrium. Note that the force exerted by the rollers on the package is perpendicular to the incline.



Solution:

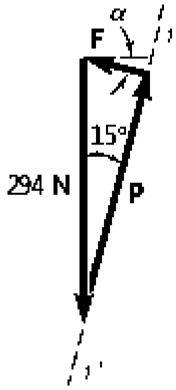


Free-Body Diagram. We choose the package as a free body, assuming that it can be treated as a particle. We draw the corresponding free-body diagram.

Equilibrium Condition. Since only three forces act on the free body, we draw a force triangle to express that it is in equilibrium. Line *1-1′* represents the known direction of **P**. In order to obtain the minimum value of the force **F**, we choose the direction of **F** perpendicular to that of **P**. From the geometry of the triangle obtained, we find

$$F = (294\ \text{N})\sin 15° = 76.1\ \text{N} \qquad a = 15°$$

F = 76.1 N b 15° ◀

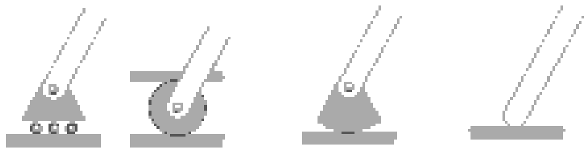
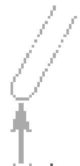
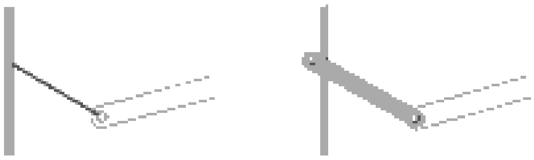
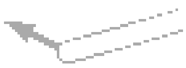
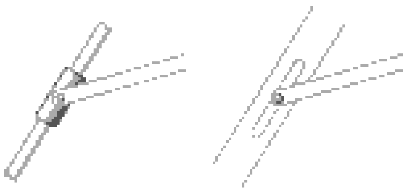
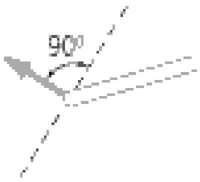
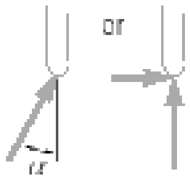
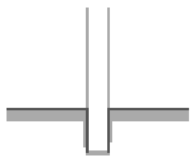
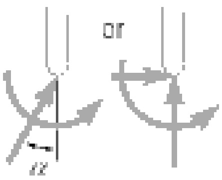


HW : Solve problem 2.F1-2.70 page 44 in ref.1

HW : Solve problem 2.127-2.132 page 69 in ref.1

EQUILIBRIUM IN TWO DIMENSIONS

Reactions at Supports and Connections for a Two-Dimensional Structure

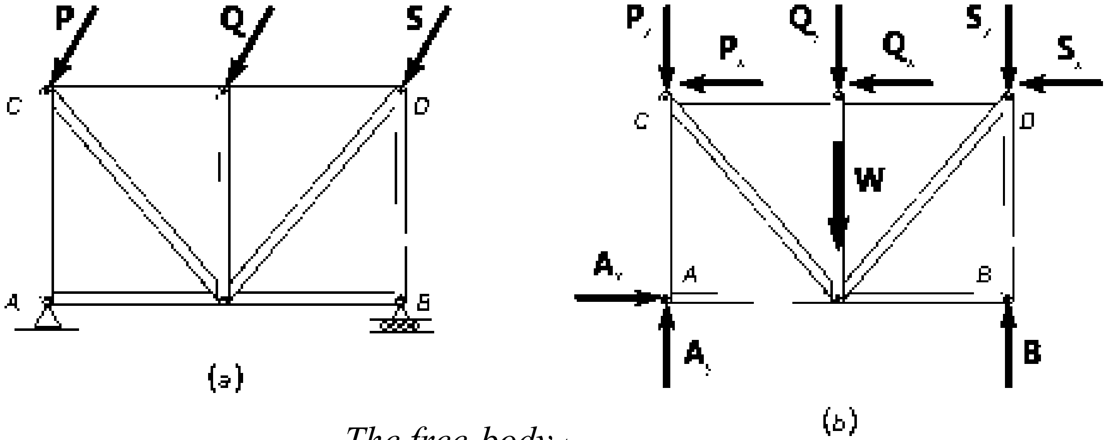
Support or Connection	Reaction	Number of Unknowns
 Rollers; Rocker; Frictionless surface	 Force with known line of action	1
 Short cable; Short link	 Force with known line of action	1
 Collar on frictionless rod; Frictionless pin in slot	 Force with known line of action	1
 Frictionless pin or hinge; Rough surface	 Force of unknown direction	2
 Fixed support	 Force and couple	3

Reactions at supports and connections (FBD)

EQUILIBRIUM OF A RIGID BODY IN TWO DIMENSIONS

we can write the equations of equilibrium for a two-dimensional structure in the more general form

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0$$



The free-body diagram of the truss

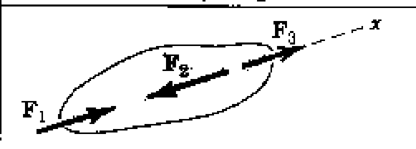
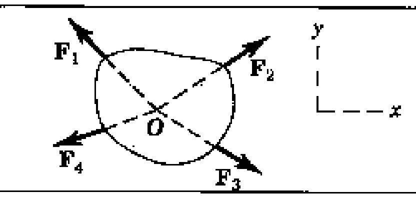
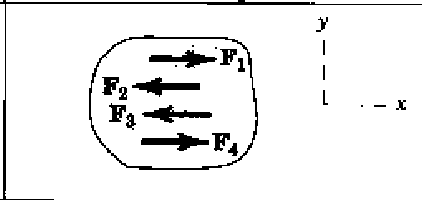
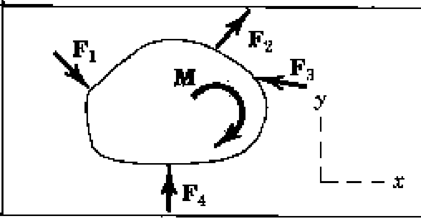
Mechanical system: body or group of bodies which can be conceptually isolated from all other bodies

System: single body, combination of bodies; rigid or non-rigid; combination of fluids and solids

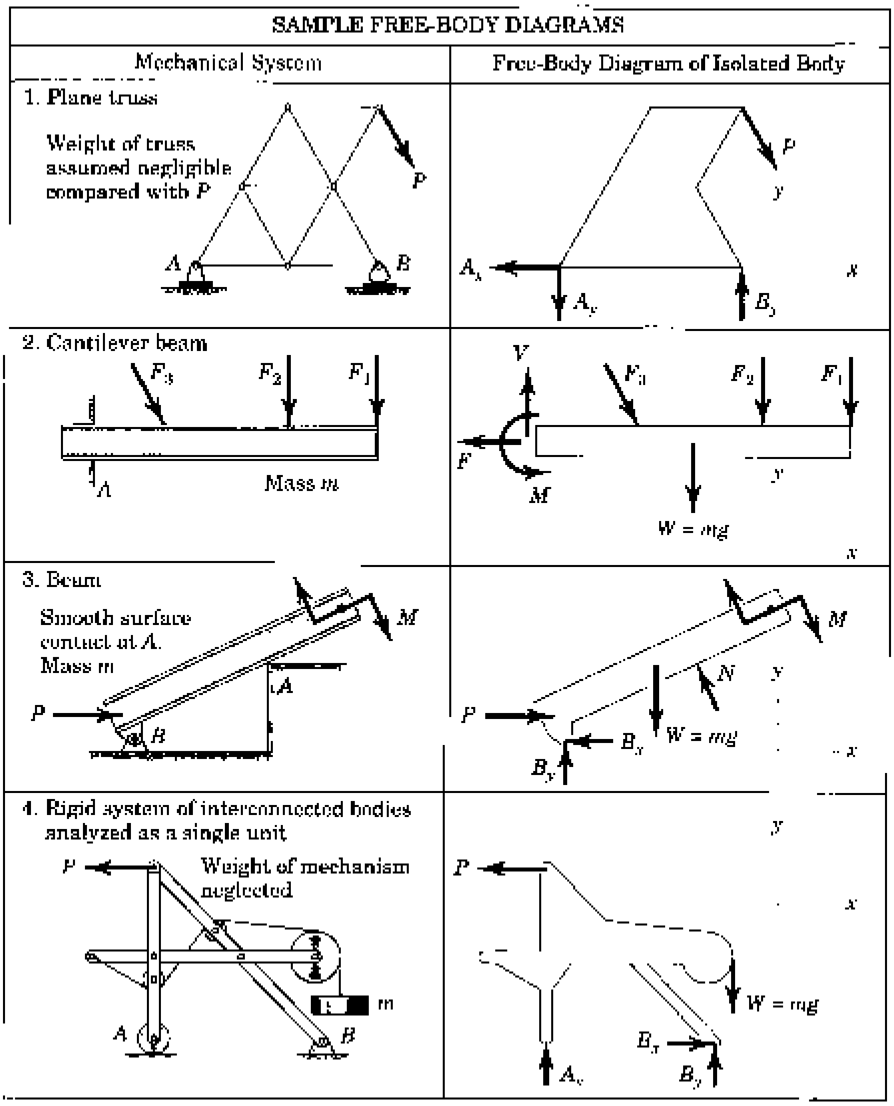
Free body diagram - FBD:

=> Body to be analyzed is isolated; Forces acting on the body are represented – action of one body on other, gravity attraction, magnetic force etc.

=> After FBD, equilibrium equns. can be formed

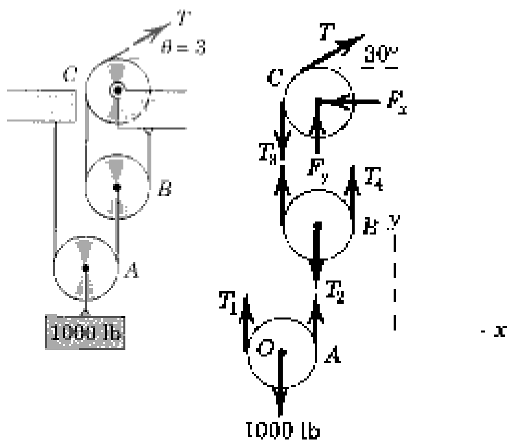
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \quad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \quad \Sigma M_z = 0$ $\Sigma F_y = 0$

Example of free-body diagram (FBD)



Ex:

Calculate the tension T in the cable (fig. below)



Sol:

$[\Sigma M_O = 0]$ $\quad T_1 r - T_2 r = 0 \quad T_1 = T_2$

$[\Sigma F_y = 0]$ $\quad T_1 + T_2 - 1000 = 0 \quad 2T_1 = 1000 \quad T_1 = T_2 = 500$ lb

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 250 \text{ lb}$$

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

$$T = T_3 \quad \text{or} \quad T = 250 \text{ lb} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the x- and y-directions requires

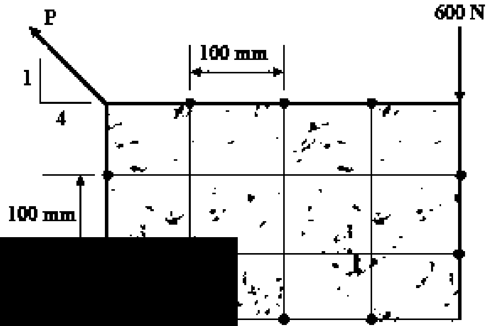
$[\Sigma F_x = 0]$ $\quad 250 \cos 30° - F_x = 0 \quad F_x = 217$ lb

$[\Sigma F_y = 0]$ $\quad F_y + 250 \sin 30° - 250 = 0 \quad F_y = 125$ lb

$[F = \sqrt{F_x^2 + F_y^2}]$ $\quad F = \sqrt{(217)^2 + (125)^2} = 250$ lb $\qquad$ *Ans.*

Example:-

Determine the force (P) shown in fig. knowing that the resultant of the two forces pass through the point (A).



Solution

$$R_x = F_1 . \cos\theta_1 + F_2 . \cos\theta_2$$

$$R * \cos\theta = -P * \frac{4}{\sqrt{17}}$$

$$R * \cos\theta = -0.9701 * P \ldots\ldots\ldots(1)$$

$$R_y = F_1 . \sin\theta_1 + F_2 . \sin\theta_2$$

$$R * \sin\theta = -600 + P * \frac{1}{\sqrt{17}}$$

$$R * \sin\theta = -600 + 0.2425 P \ldots\ldots\ldots(2)$$

$$\sum M_A = 0$$

$$600 * 300 + P * \frac{1}{\sqrt{17}} * 100 - P * \frac{4}{\sqrt{17}} * 300 = 0$$

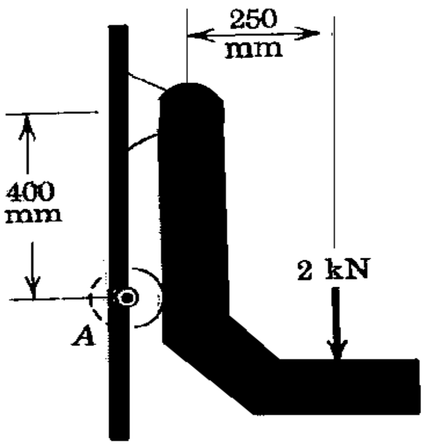
$$180000 + 24.25 P - 291 P = 0$$

$$180000 = 266.7 P$$

$$P = \frac{180000}{266.7} = 674.68 N$$

Ex :-

Find out the reaction on the cylinder (**A**) and the total force acting on the pin (**O**).



Solution:-

Σ M (O) = 0

$2 * 250 - R_A * 400 = 0$

$400\ R_A = 500$

$R_A = 1.25\ KN$

Σ Fy = 0

$Oy - 2 = 0$

$Oy = 2\ KN$

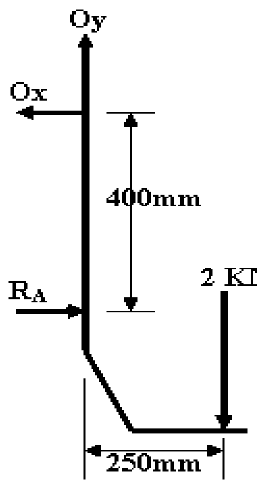
Σ Fx = 0

$R_A - Ox = 0$

$Ox = R_A = 1.25\ KN$

$$F = \sqrt{(Ox)^2 + (Oy)^2}$$

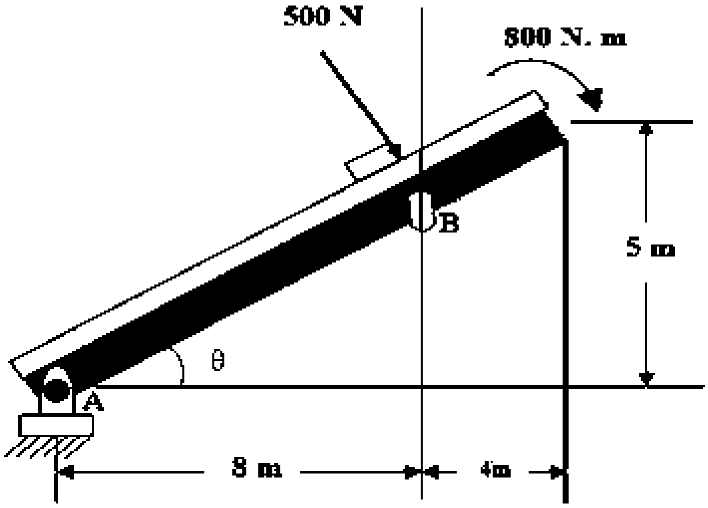
$$F = \sqrt{(1.25)^2 + (2)^2} = 2.35N$$



F.B.D

Ex (HW):-

Determine the reactions a



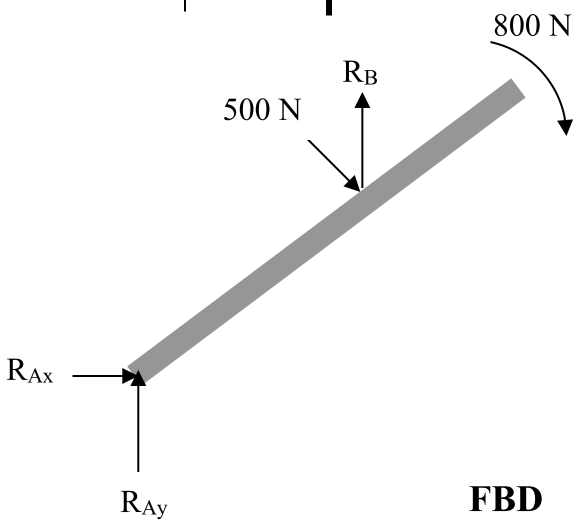
Solution:-

For equilibrium

$\sum R_x = 0 \quad , \quad \sum R_y = 0 \quad , \quad \sum M = 0$

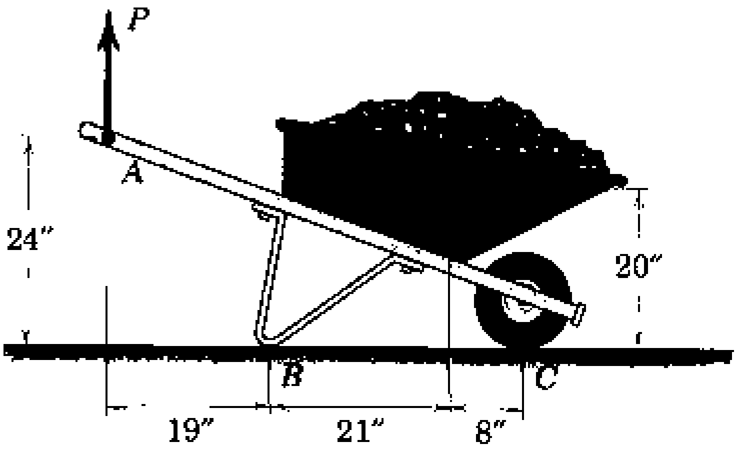
ANS:

R_b=641.7 N



FBD

EX(HW): Determine the magnitude P of the vertical force required to lift the wheelbarrow free of the ground at point B. The combined weight of the wheelbarrow and its load is 240 lb with center of gravity at G.



ANS.: P = 40 Ib

Read Example 4.1- 4.5 page 169 in Ref. 1

HW : Solve problem 4.f1- 4.90 page 173- 190 in ref.1

Review Problems

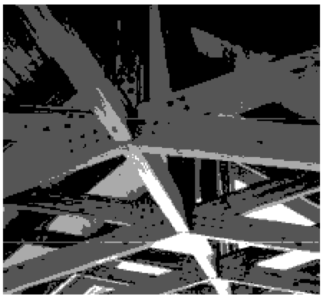
HW : Solve problem 4.142- 4.149 page 213 in ref.1

Tutorial; QUIZ

Analysis of Trusses,

→ Truss : a structure composed of slender members (wooden struts or metal bars) joined together at their end points.

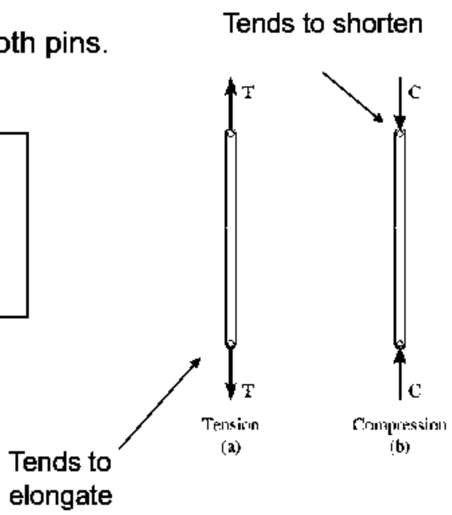





→ Assumptions for Design

→ All loadings are applied at the joints (in case the weight of the member is to be inlcuded, it is generally satisfactory to share it equally between the two ends of the member)

→ The members are joined together by smooth pins.



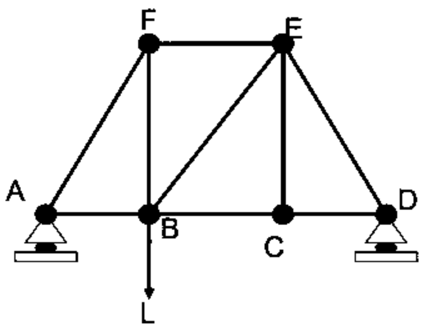
→Each truss member acts as a two force member. The forces at the end of the member must be directed along the axis of the member.

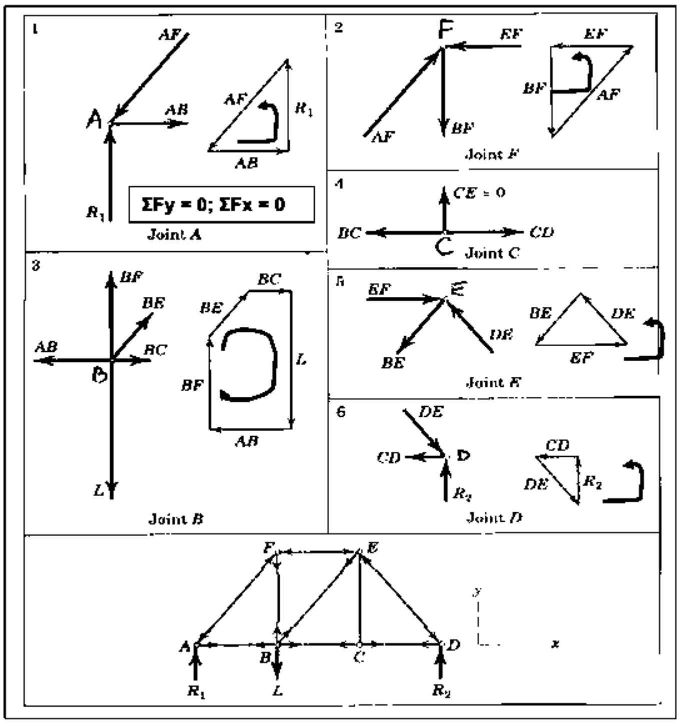
Two methods to analyze force in simple truss

Method of joints

- This method consists of satisfying the conditions of equilibrium for the forces acting on the connecting pin of each joint
- This method deals with equilibrium of concurrent forces and only two independent equilibrium equations are solved
- Newton's third law is followed

Example



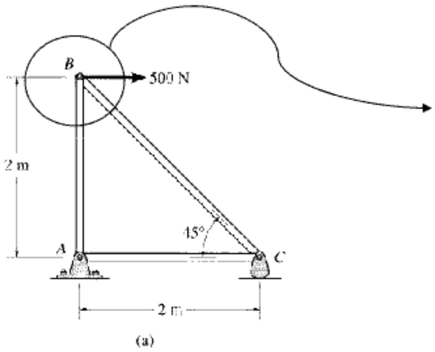


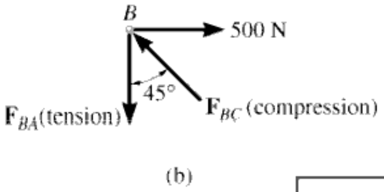
Finally sign can be changed if not applied correctly

→ We consider the equilibrium of a joint of the truss : A member force becomes an external force on the joint's free body diagram. → **Method of Joints**

→ First draw the F.B.D. of the joint.

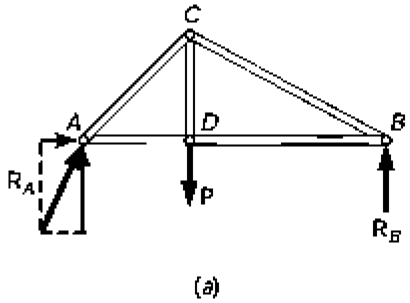
→ The force system acting at each joint is coplanar and concurrent. → Moment equilibrium is automatically satisfied.

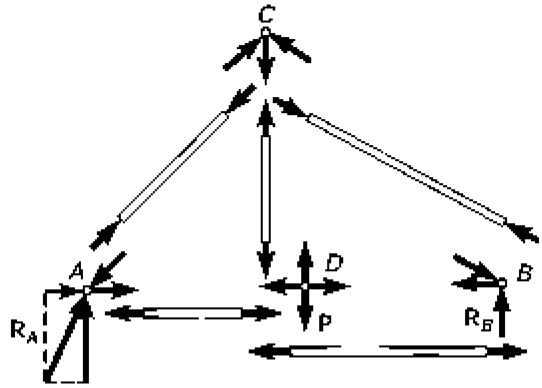


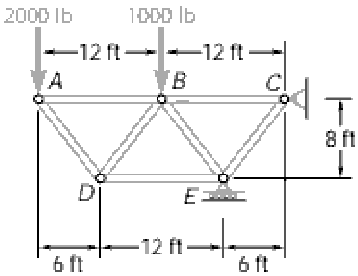


$$\sum F_X = 0$$
$$\sum F_Y = 0$$

The free-body diagram can be drawn for each pin and each member



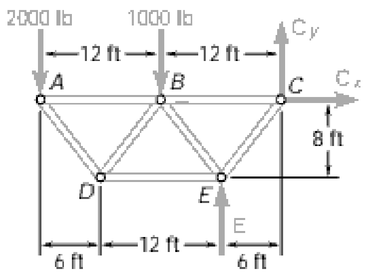




SAMPLE PROBLEM 6.1

Using the method of joints, determine the force in each member of the truss shown.

SOLUTION



Free-Body: Entire Truss. A free-body diagram of the entire truss is drawn; external forces acting on this free body consist of the applied loads and the reactions at C and E. We write the following equilibrium equations.

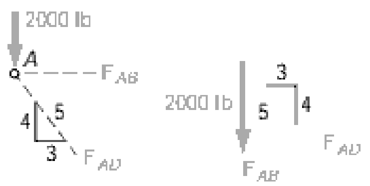
$+\circlearrowleft \Sigma M_C = 0$: $(2000 \text{ lb})(24 \text{ ft}) + (1000 \text{ lb})(12 \text{ ft}) - E(6 \text{ ft}) = 0$

$E = +10{,}000 \text{ lb}$ $\qquad$ $\mathbf{E} = 10{,}000 \text{ lb}\uparrow$

$\xrightarrow{+} \Sigma F_x = 0$: $\qquad$ $\mathbf{C}_x = 0$

$+\uparrow \Sigma F_y = 0$: $-2000 \text{ lb} - 1000 \text{ lb} + 10{,}000 \text{ lb} + C_y = 0$

$C_y = -7000 \text{ lb}$ $\qquad$ $\mathbf{C}_y = 7000 \text{ lb}\downarrow$

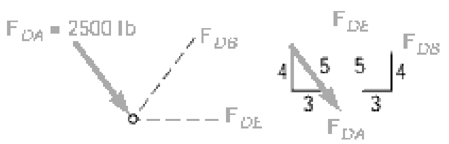


Free-Body: Joint *A*. This joint is subjected to only two unknown forces, namely, the forces exerted by members AB and AD. A force triangle is used to determine $\mathbf{F}_{AB}$ and $\mathbf{F}_{AD}$. We note that member AB pulls on the joint and thus is in tension and that member AD pushes on the joint and thus is in compression. The magnitudes of the two forces are obtained from the proportion

$$\frac{2000 \text{ lb}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$F_{AB} = 1500 \text{ lb } T$ ◀

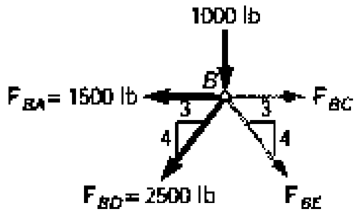
$F_{AD} = 2500 \text{ lb } C$ ◀



Free-Body: Joint *D*. Since the force exerted by member AD has been determined, only two unknown forces are now involved at this joint. Again, a force triangle is used to determine the unknown forces in members DB and DE.

$F_{DB} = F_{DA}$ $\qquad$ $F_{DB} = 2500 \text{ lb } T$ ◀

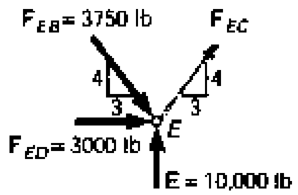
$F_{DE} = 2(\frac{3}{5})F_{DA}$ $\qquad$ $F_{DE} = 3000 \text{ lb } C$ ◀



Free-Body: Joint *B*. Since more than three forces act at this joint, we determine the two unknown forces $\mathbf{F}_{BC}$ and $\mathbf{F}_{BE}$ by solving the equilibrium equations $\Sigma F_x = 0$ and $\Sigma F_y = 0$. We arbitrarily assume that both unknown forces act away from the joint, i.e., that the members are in tension. The positive value obtained for F_{BC} indicates that our assumption was correct; member *BC* is in tension. The negative value of F_{BE} indicates that our assumption was wrong; member *BE* is in compression.

$$+\uparrow \Sigma F_y = 0: \quad -1000 - \tfrac{4}{5}(2500) - \tfrac{4}{5}F_{BE} = 0$$
$$F_{BE} = -3750 \text{ lb} \qquad F_{BE} = 3750 \text{ lb } C$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - 1500 - \tfrac{3}{5}(2500) - \tfrac{3}{5}(3750) = 0$$
$$F_{BC} = +5250 \text{ lb} \qquad F_{BC} = 5250 \text{ lb } T$$

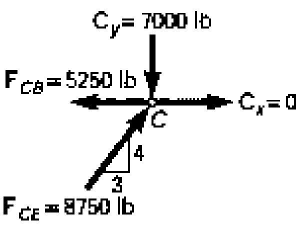


Free-Body: Joint *E*. The unknown force $\mathbf{F}_{EC}$ is assumed to act away from the joint. Summing *x* components, we write

$$\xrightarrow{+} \Sigma F_x = 0: \quad \tfrac{3}{5}F_{EC} + 3000 + \tfrac{3}{5}(3750) = 0$$
$$F_{EC} = -8750 \text{ lb} \qquad F_{EC} = 8750 \text{ lb } C$$

Summing *y* components, we obtain a check of our computations:

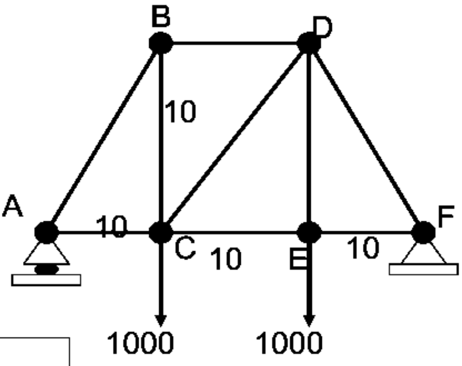
$$+\uparrow \Sigma F_y = 10{,}000 - \tfrac{4}{5}(3750) - \tfrac{4}{5}(8750)$$
$$= 10{,}000 - 3000 - 7000 = 0 \qquad \text{(checks)}$$



Free-Body: Joint *C*. Using the computed values of $\mathbf{F}_{CB}$ and $\mathbf{F}_{CE}$, we can determine the reactions $\mathbf{C}_x$ and $\mathbf{C}_y$ by considering the equilibrium of this joint. Since these reactions have already been determined from the equilibrium of the entire truss, we will obtain two checks of our computations. We can also simply use the computed values of all forces acting on the joint (forces in members and reactions) and check that the joint is in equilibrium:

$$\xrightarrow{+} \Sigma F_x = -5250 + \tfrac{3}{5}(8750) = -5250 + 5250 = 0 \qquad \text{(checks)}$$
$$+\uparrow \Sigma F_y = -7000 + \tfrac{4}{5}(8750) = -7000 + 7000 = 0 \qquad \text{(checks)}$$

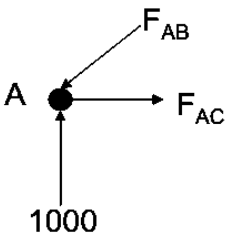
I. H. Shames

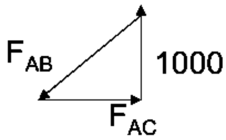


Determine the force transmitted by each member;

A, F = 1000 N

Pin A



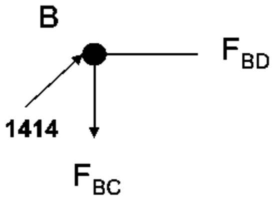


$\Sigma Fx = 0 \Rightarrow F_{AC} - 0.707F_{AB} = 0$

$\Sigma Fy = 0 \Rightarrow -0.707F_{AB} + 1000 = 0$

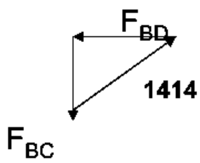
$\mathbf{F_{AB} = 1414\ N;\ F_{AC} = 1000\ N}$

Pin B

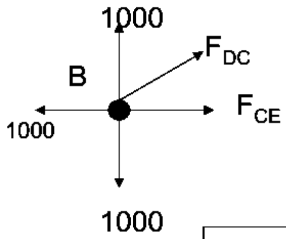


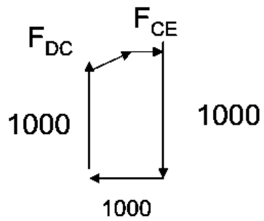
$\Sigma Fx = 0 \Rightarrow -F_{BD} + 1414COS45 = 0 \Rightarrow \mathbf{F_{BD} = 1000\ N}$

$\Sigma Fy = 0 \Rightarrow -F_{BC} + 1414\ COS45 = 0 \Rightarrow \mathbf{F_{BC} = 1000\ N}$



Pin C





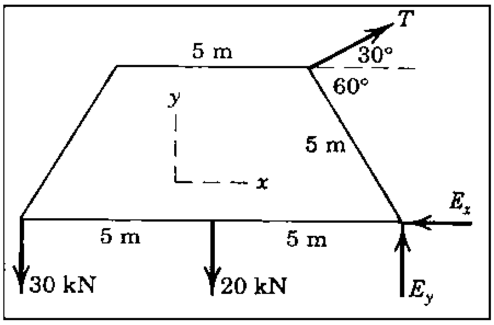
$\Sigma Fx = 0 \Rightarrow -1000 + F_{CE} + F_{DC}\ COS\ 45 = 0 \Rightarrow \mathbf{F_{CE} = 1000\ N}$

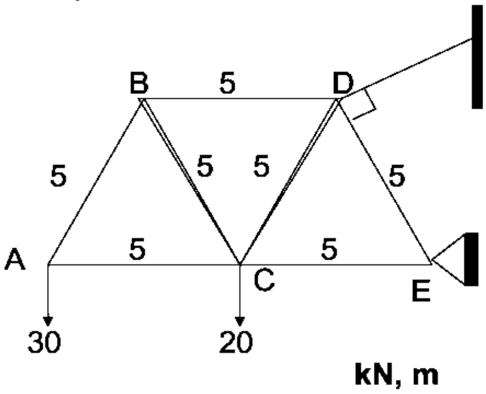
$\Sigma Fy = 0 \Rightarrow -1000+1000+ F_{DC}\ COS\ 45 = 0 \Rightarrow \mathbf{F_{DC} = 0}$

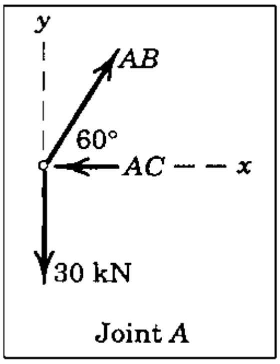
SIMILARLY D, E, F pins are solved

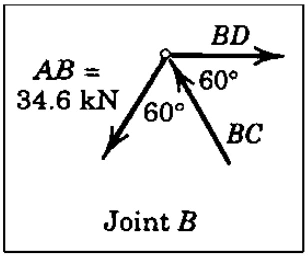
Meriem / Kraige (similar pbm. 6.1 in Beer/Johnston)

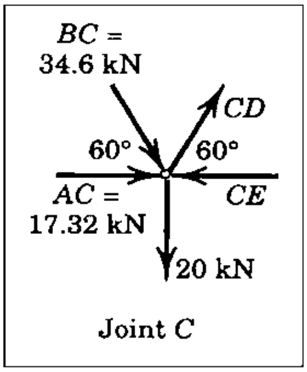
Find the force in each member of the loaded cantilever truss by method of joints

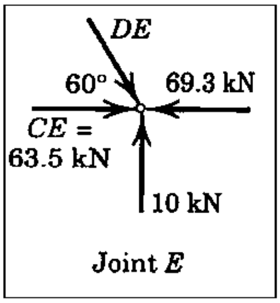










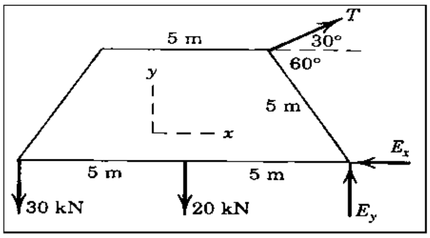


FBD of entire truss

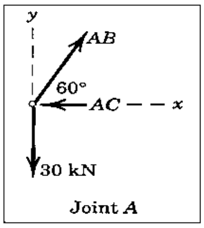
ΣME = 0 => 5T-20(5)-30 (10) = 0; **T = 80 kN**

ΣFx = 0 => 80 cos 30 – Ex = 0; **Ex = 69.28 kN**

ΣFy = 0 => Ey +80sin30-20-30 = 0 => **Ey = 10kN**

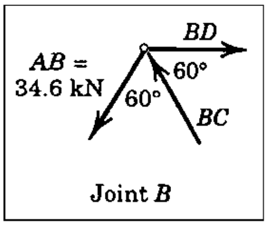


FBD of joints



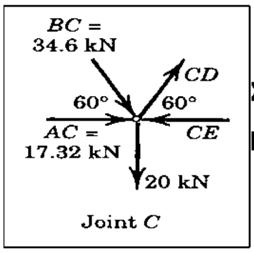
ΣFx = 0; ΣFy = 0

Find AB, AC forces



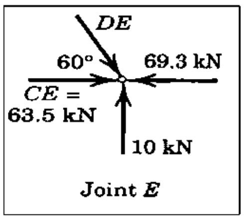
ΣFx = 0; ΣFy = 0

Find BC, BD forces



ΣFx = 0; ΣFy = 0

Find CD, CE forces



ΣFy = 0

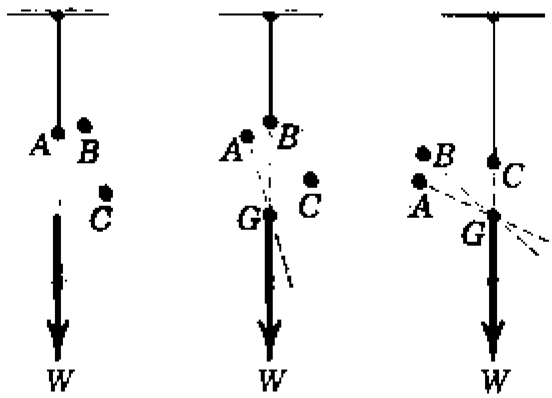
Find DE forces

ΣFx = 0 can be checked

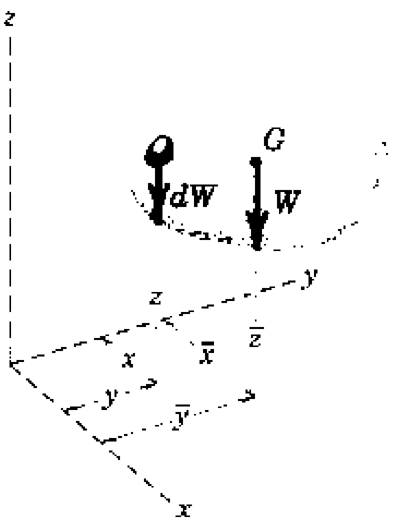
HW : Solve problem 6.1- 6.8 page 296 in ref.1

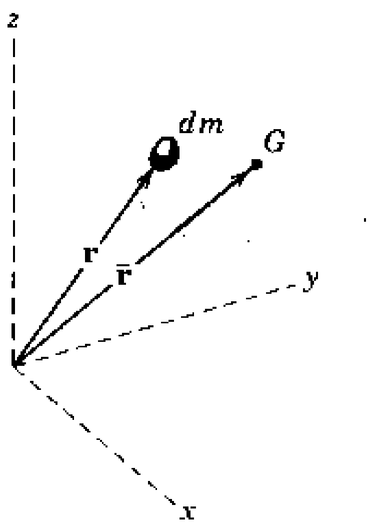
Center of Mass & Center of Gravity

Centroids



- Body of mass' m'.
- Body at equilibrium w.r.t. forces in the cord and resultant of gravitational forces at all particles 'W'.
- W is collinear with point A.
- Changing the point of hanging to B, C – Same effect.
- All practical purposes, LOA will be concurrent at single point G;
- G – center of gravity of the body.





Moment abt. Y axis = dw (x)

Sum of moments for small regions through out the body: $\int x\, dw$

Moment of 'w' force with Y axis = $w\bar{x}$

$$\int x\, dw = w\bar{x}$$

Sum of moments **Moment of the sum**

$\bar{X} = (\int x\, dw) / w$ $\quad \bar{Y} = (\int y\, dw) / w$ $\quad \bar{Z} = (\int z\, dw) / w$ **1**

$W = mg$

$\bar{X} = (\int x\, dm) / m$ $\quad \bar{Y} = (\int y\, dm) / m$ $\quad \bar{Z} = (\int z\, dm) / m$ **2**

In vector form, $\bar{r} = (\int r\, dm) / m$ **3**

$\rho = m/V;\ dm = \rho\, dv$

$\bar{X} = (\int x\, \rho\, dv) / \int \rho\, dv$

$\bar{Y} = (\int y\, \rho\, dv) / \int \rho\, dv$ **4**

$\bar{Z} = (\int z\, \rho\, dv) / \int \rho\, dv$

ρ = not constant through out body

Equns 2, 3, 4 are independent of 'g'; They depend only on mass distribution;
This define a co-ordinate point – center of mass
This is same as center of gravity as long as gravitational field is uniform and parallel

Centroids of lines, areas, volumes

Suppose if density is constant, then the expression define a purely geometrical property of the body; It is called as **centroid**

Centroid of volume

$\bar{X} = (\int x_c\, dv) / v$ $\quad \bar{Y} = (\int y_c\, dv) / v$ $\quad \bar{Z} = (\int z_c\, dv) / v$

Centroid of area

$\bar{X} = (\int x\, dA) / A$ $\quad \bar{Y} = (\int y\, dA) / A$ $\quad \bar{Z} = (\int z\, dA) / A$

Centroid of line

$\bar{X} = (\int x\, dL) / L$ $\quad \bar{Y} = (\int y\, dL) / L$ $\quad \bar{Z} = (\int z\, dL) / L$

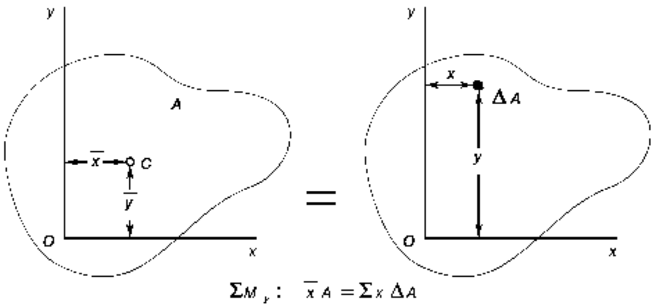


Fig. 5.3 Centroid of an area.

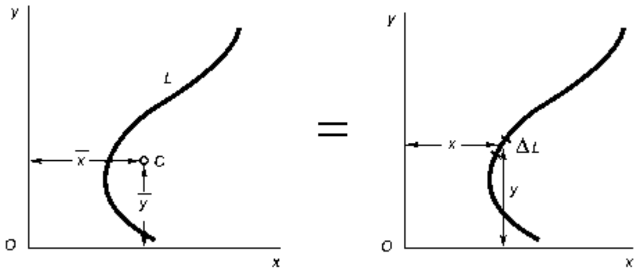
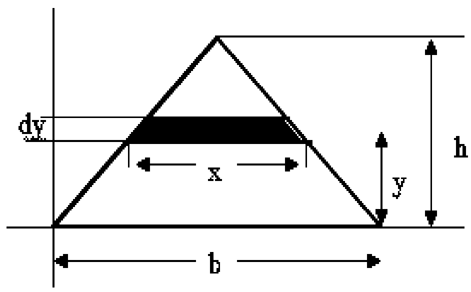


Fig. 5.4 Centroid of a line.

Ex:-

Locate the centroid of the triangle shown in figure.



Solution:-

$$\frac{b}{x} = \frac{h}{h-y};\rightarrow \therefore x = \frac{b}{h}*(h-y)$$

$$dA = x*dy \quad , \quad \bar{x} = \frac{b}{2}$$

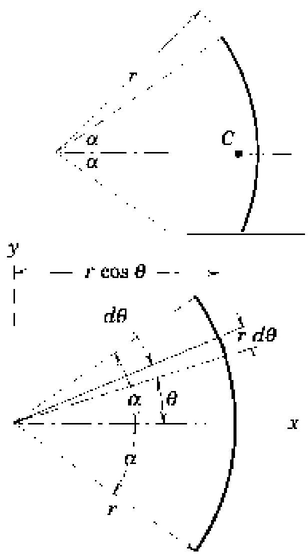
$$\bar{y} = \frac{\int y*dA}{\int dA} = \frac{\int_0^h y*x*dy}{\int_0^h x*dy};\Rightarrow \quad \bar{y} = \frac{\int_0^h y*\frac{b}{h}*(h-y)*dy}{\int_0^h \frac{b}{h}*(h-y)*dy}$$

$$\bar{y} = \frac{\int_0^h (y*b - \frac{y^2*b}{h})*dy}{\int_0^h (b - \frac{b*y}{h})*dy} \Rightarrow \quad \bar{y} = \frac{\left[\frac{y^2*b}{2} - \frac{y^3*b}{3h}\right]_0^h}{\left[b*y - \frac{y^2*b}{2h}\right]_0^h} = \frac{\frac{h^2*b}{2} - \frac{h^2*b}{3}}{bh - \frac{h*b}{2}}$$

$$\Rightarrow \bar{y} = \frac{h}{3}$$

EX:

Locate centroid of circular arc as shown in fig.



Solution. Choosing the axis of symmetry as the x-axis makes $\bar{y} = 0$. A differential element of arc has the length $dL = r\,d\theta$ expressed in polar coordinates, and the x-coordinate of the element is $r\cos\theta$.

Applying the first of Eqs. 5/4 and substituting $L = 2\alpha r$ give

$$\left[L\bar{x} = \int x\,dL\right] \qquad (2\alpha r)\bar{x} = \int_{-\alpha}^{\alpha} (r\cos\theta)\,r\,d\theta$$

$$2\alpha r\bar{x} = 2r^2\sin\alpha$$

$$\bar{x} = \frac{r\sin\alpha}{\alpha} \qquad \textit{Ans.}$$

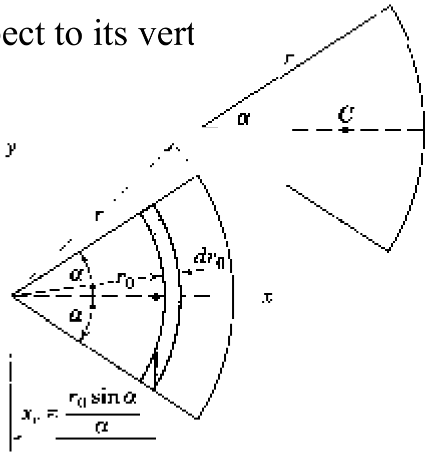
Ex:

Locate the centroid of the area of a circular sector with respect to its vert

Sol I: circular arc

$dA = 2r_0\alpha\, dr_0.$

$x_c = r_0 \sin\alpha/\alpha,$



Solution I

$$[A\bar{x} = \int x_c\, dA] \qquad \frac{2\alpha}{2\pi}(\pi r^2)\bar{x} = \int_0^r \left(\frac{r_0 \sin\alpha}{\alpha}\right)(2r_0\alpha\, dr_0)$$

$$r^2\alpha\bar{x} = \tfrac{2}{3}r^3 \sin\alpha$$

$$\bar{x} = \frac{2}{3}\frac{r \sin\alpha}{\alpha}$$

$dA = (r/2)(r\, d\theta),$ triangular element

$x_c = \tfrac{2}{3}r\cos\theta.$

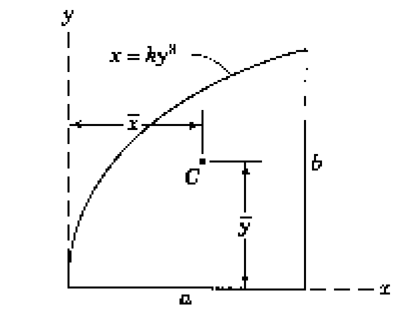
$$[A\bar{x} = \int x_c\, dA] \qquad (r^2\alpha)\bar{x} = \int_{-\alpha}^{\alpha} (\tfrac{2}{3}r\cos\theta)(\tfrac{1}{2}r^2\, d\theta)$$

$$r^2\alpha\bar{x} = \tfrac{2}{3}r^3 \sin\alpha$$

and as before $$\bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$$

EX:

Locate the centroid of the area under the curve ***x =k y***^{3} from x=0 to x= a.



Sol I:

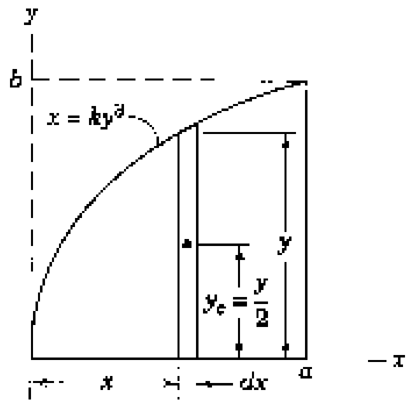
d A= y dx

$$[A\bar{x} = \int x_c\, dA] \qquad \bar{x}\int_0^a y\, dx = \int_0^a xy\, dx$$

Substituting $y = (x/k)^{1/3}$ and $k = a/b^3$ and integrating give

$$\frac{3ab}{4}\bar{x} = \frac{3a^2b}{7} \qquad \bar{x} = \tfrac{4}{7}a$$

y_c =y/2

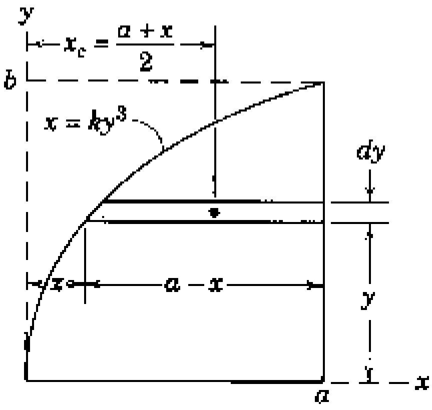


$$[A\bar{y} = \int y_c\, dA] \qquad \frac{3ab}{4}\bar{y} = \int_0^a \left(\frac{y}{2}\right) y\, dx$$

Substituting $y = b(x/a)^{1/3}$ and integrating give

$$\frac{3ab}{4}\bar{y} = \frac{3ab^2}{10} \qquad \bar{y} = \tfrac{2}{5}b$$

Sol. II:



Solution II. The horizontal element of area shown in the lower figure may be employed in place of the vertical element. The x-coordinate to the centroid of the rectangular element is seen to be $x_c = x + \frac{1}{2}(a - x) = (a + x)/2$, which is simply the average of the coordinates a and x of the ends of the strip. Hence,

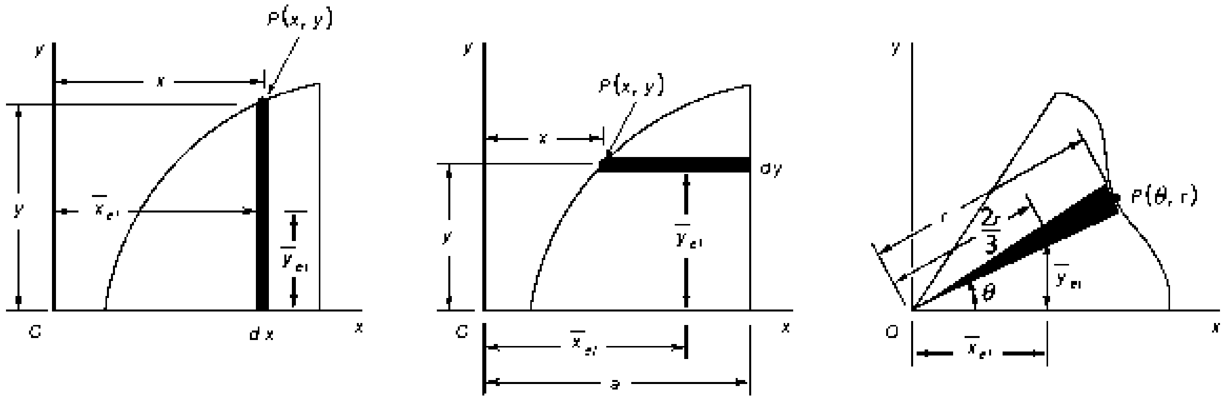
$$[A\bar{x} = \int x_c \, dA] \qquad \bar{x}\int_0^b (a - x)\, dy = \int_0^b \left(\frac{a + x}{2}\right)(a - x)\, dy$$

The value of $\bar{y}$ is found from

$$[A\bar{y} = \int y_c \, dA] \qquad \bar{y}\int_0^b (a - x)\, dy = \int_0^b y(a - x)\, dy$$

where $y_c = y$ for the horizontal strip. The evaluation of these integrals will check the previous results for $\bar{x}$ and $\bar{y}$.

Note:



$\bar{x}_{el} = x$ | $\bar{x}_{el} = \frac{a+x}{2}$ | $\bar{x}_{el} = \frac{2r}{3}\cos\theta$

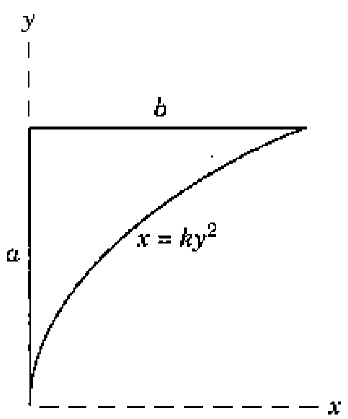
$\bar{y}_{el} = y/2$ | $\bar{y}_{el} = y$ | $\bar{y}_{el} = \frac{2r}{3}\sin\theta$

$dA = y\,dx$ | $dA = (a - x)\,dy$ | $dA = \frac{1}{2}r^2\,d\theta$

Problem for student:

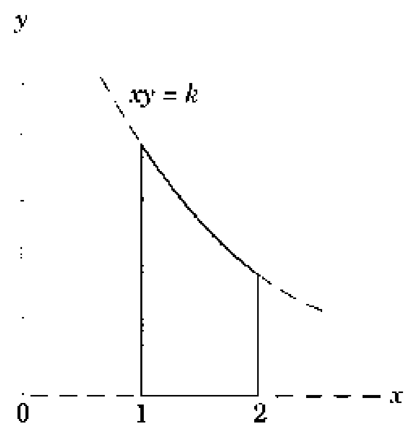
Determine the coordinates of the centroid of the shaded area.

Ans. $\bar{x} = \frac{3}{10}b$, $\bar{y} = \frac{3}{4}a$



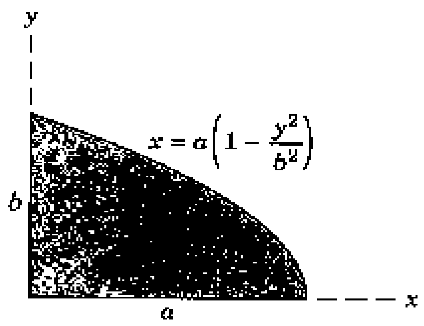
Determine the coordinates of the centroid of the shaded area.

Ans. $\bar{x} = 1.443$, $\bar{y} = 0.361k$



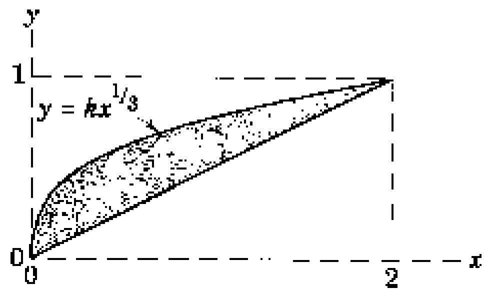
Locate the centroid of the shaded area.

$\bar{x} = 2a/5, \bar{y} = 3b/8$



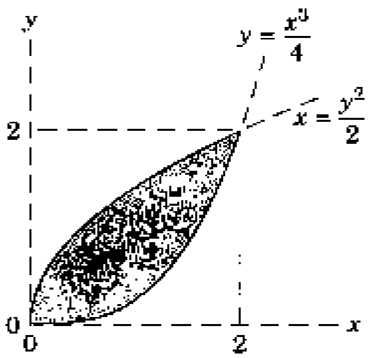
Determine the x- and y-coordinates of the centroid of the shaded area.

Ans. $\bar{x} = 0.762, \bar{y} = 0.533$



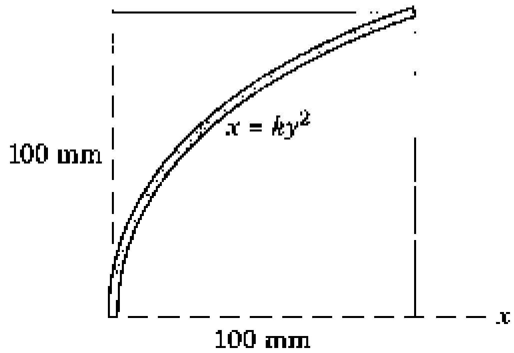
Locate the centroid of the shaded area between the two curves.

Ans. $\bar{x} = \frac{24}{25}, \bar{y} = \frac{6}{7}$



The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y-coordinate of the mass center of the rod. (*Reminder:* A differential arc length is $dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + (dx/dy)^2}\,dy$.)

Ans. $\bar{y} = 57.4$ mm



Centroid of Composite Figures

1) Lines:-

$$\bar{x} = \frac{\sum L_i * x_i}{\sum L_i} \quad , \quad \bar{y} = \frac{\sum L_i * y_i}{\sum L_i},$$

2) Areas:-

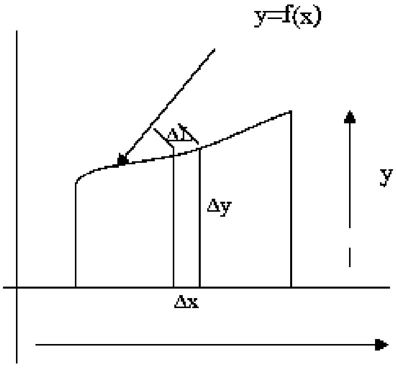
$$\bar{x} = \frac{\sum A_i * x_i}{\sum A_i} \quad , \quad \bar{y} = \frac{\sum A_i * y_i}{\sum A_i}$$

Note:

for Lines

X = (∫ x dL) / L   Y = (∫ y dL) / L

In which

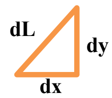


$$(dL)^2 = (dx)^2 + (dy)^2$$

$$\Rightarrow (\frac{dL}{dx})^2 = (\frac{dx}{dx})^2 + (\frac{dy}{dx})^2$$

$$\therefore dL = \sqrt{1 + (\frac{dy}{dx})^2} * dx$$

$$or \qquad dL = \sqrt{1 + (\frac{dx}{dy})^2} * dy$$



First Moments of Areas And Lines

first moment of the area A with respect to the y axis and is denoted by Q_y. Similarly, the integral (y dA) defines the first moment of A with respect to the x axis and is denoted by Q_x.

$$Q_y = \int x\, dA \qquad Q_x = \int y\, dA$$

That lead to

$$Q_y = \bar{x}A \qquad Q_x = \bar{y}A$$

for the composite area:

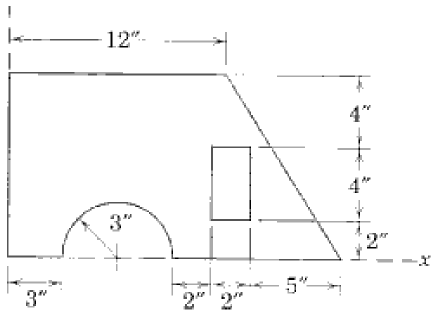
$$Q_y = \bar{X}(A_1 + A_2 + \cdots + A_n) = \bar{x}_1 A_1 + \bar{x}_2 A_2 + \cdots + \bar{x}_n A_n$$
$$Q_x = \bar{Y}(A_1 + A_2 + \cdots + A_n) = \bar{y}_1 A_1 + \bar{y}_2 A_2 + \cdots + \bar{y}_n A_n$$

or, for short,

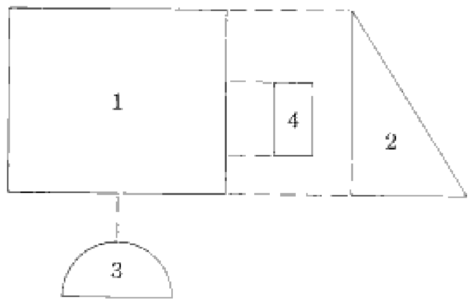
$$Q_y = \bar{X}\Sigma A = \Sigma \bar{x} A \qquad Q_x = \bar{Y}\Sigma A = \Sigma \bar{y} A$$

EX:

Locate the centroid of the shaded area.



Solution. The composite area is divided into the four elementary shapes shown in the lower figure. The centroid locations of all these shapes may be obtained from Table D/3. Note that the areas of the "holes" (parts 3 and 4) are taken as negative in the following table:



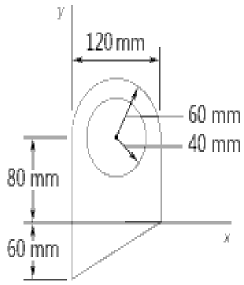
PART	A in.^{2}	$\bar{x}$ in.	$\bar{y}$ in.	$\bar{x}A$ in.^{3}	$\bar{y}A$ in.^{3}
1	120	6	5	720	600
2	30	14	10/3	420	100
3	−14.14	6	1.273	−84.8	−18
4	−8	12	4	−96	−32
TOTALS	127.9			959	650

The area counterparts to Eqs. 5/7 are now applied and yield

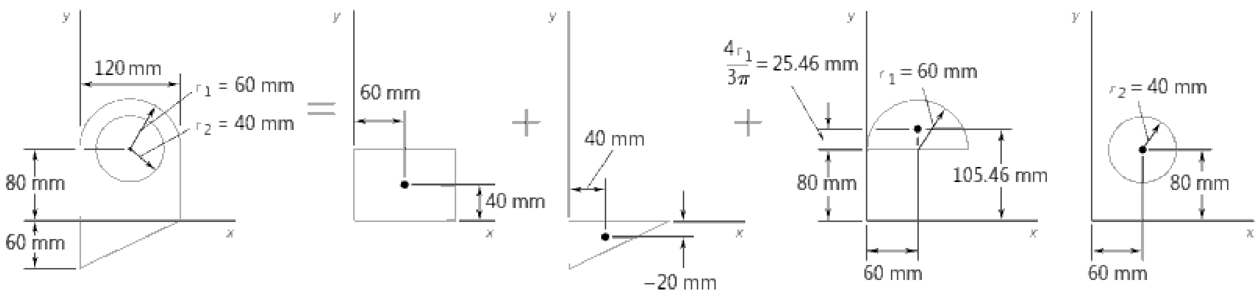
$$\left[\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}\right] \qquad \bar{X} = \frac{959}{127.9} = 7.50 \text{ in.} \qquad \textit{Ans.}$$

$$\left[\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A}\right] \qquad \bar{Y} = \frac{650}{127.9} = 5.08 \text{ in.} \qquad \textit{Ans.}$$

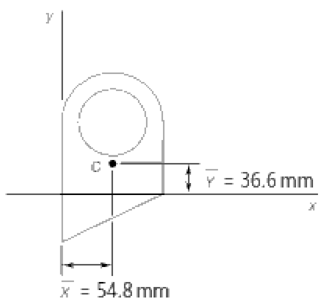
<u>Ex:</u> For the plane area shown, determine (a) the first moments with respect to the x and y axes, (b) the location of the centroid.



<u>Sol:</u>



Component	A, mm^2	$\bar{x}$, mm	$\bar{y}$, mm	$\bar{x}A$, mm^3	$\bar{y}A$, mm^3
Rectangle	$(120)(80) = 9.6 \times 10^3$	60	40	$+576 \times 10^3$	$+384 \times 10^3$
Triangle	$\frac{1}{2}(120)(60) = 3.6 \times 10^3$	40	−20	$+144 \times 10^3$	-72×10^3
Semicircle	$\frac{1}{2}p(60)^2 = 5.655 \times 10^3$	60	105.46	$+339.3 \times 10^3$	$+596.4 \times 10^3$
Circle	$-p(40)^2 = -5.027 \times 10^3$	60	80	-301.6×10^3	-402.2×10^3
	$\Sigma A = 13.828 \times 10^3$			$\Sigma\bar{x}A = +757.7 \times 10^3$	$\Sigma\bar{y}A = +506.2 \times 10^3$



a. First Moments of the Area. Using Eqs. (5.8), we write

$$Q_x = \Sigma\bar{y}A = 506.2 \times 10^3 \text{ mm}^3 \qquad Q_x = 506 \times 10^3 \text{ mm}^3 \blacktriangleleft$$
$$Q_y = \Sigma\bar{x}A = 757.7 \times 10^3 \text{ mm}^3 \qquad Q_y = 758 \times 10^3 \text{ mm}^3 \blacktriangleleft$$

b. Location of Centroid. Substituting the values given in the table into the equations defining the centroid of a composite area, we obtain

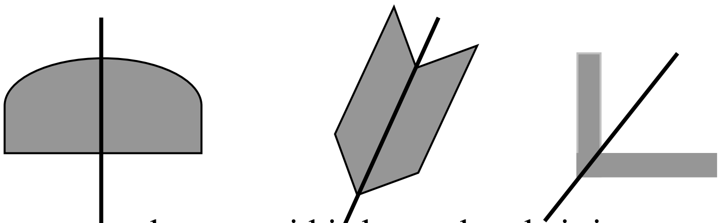
$$\bar{X}\Sigma A = \Sigma\bar{x}A: \qquad \bar{X}(13.828 \times 10^3 \text{ mm}^2) = 757.7 \times 10^3 \text{ mm}^3$$
$$\bar{X} = 54.8 \text{ mm} \blacktriangleleft$$

$$\bar{Y}\Sigma A = \Sigma\bar{y}A: \qquad \bar{Y}(13.828 \times 10^3 \text{ mm}^2) = 506.2 \times 10^3 \text{ mm}^3$$
$$\bar{Y} = 36.6 \text{ mm} \blacktriangleleft$$

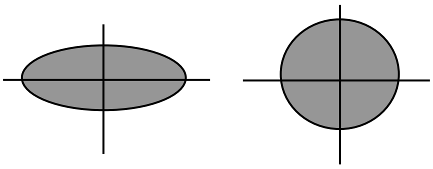
Point of Symmetry and Axes of Symmetry

Some Time the Position of centroid of a plane figure or curve can be seen by inspection for example if a figure has:-

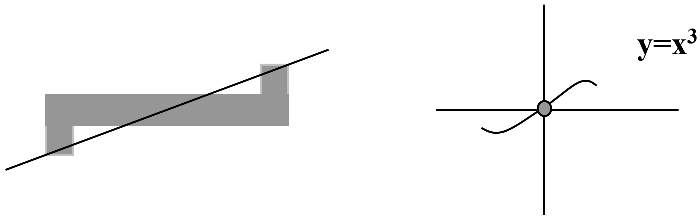
1- A line of symmetry its centroid is located on that line.



2- Two lines of symmetry the centroid is located at their intersection

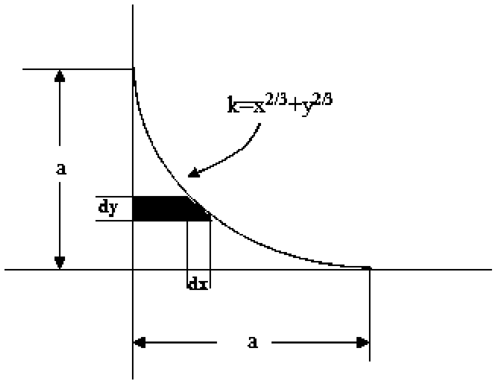


3- A point of symmetry which represents in this case the centroid of line of the figure:-



Ex:-

Determined the coordinate of the centroid for the arc below, which lie in the first quadrant.



Solution:-

$$x^{2/3} + y^{2/3} = k$$

$$y(a) = 0$$

$$\Rightarrow a^{2/3} + 0 = k$$

$$\therefore k = a^{2/3}$$

$$\therefore x^{2/3} + y^{2/3} = a^{2/3}$$

$$dl = \sqrt{1 + (\frac{dy}{dx})^2} * dx = \sqrt{(\frac{dx}{dy})^2 + 1} * dy$$

$$x_c = x \qquad , y_c = y$$

$$\frac{2}{3}*x^{-1/3}+\frac{2}{3}*y^{-1/3}*\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=-(\frac{y}{x})^{1/3}$$

$$\int x_c*dl=\int_0^a x*\sqrt{1+(\frac{dy}{dx})^2}*dx$$

$$=\frac{3}{5}*a^2$$

$$\int dl=\int_0^a\sqrt{1+(\frac{dy}{dx})^2}*dx=\frac{3}{2}a \qquad \Rightarrow \bar{x}=\frac{\frac{3}{5}*a^2}{\frac{3}{2}*a}=\frac{2}{5}*a$$

------------ complete for $\bar{y}$ **(H.W)**

Read Example 5.1- 5.3 page 229 in Ref. 1

Read Example 5.4 page 240 in Ref. 1

HW 9: Solve problem 5.1- 5.21 page 234 in ref.1

HW 10: Solve problem 5.34 5.46 page 245 in ref.1

Moments of Inertia (Second Moment of Area)

I –Introduction

Many engineering formulas, such as those relating stresses involve the mathematical expression of the form; $\int_A r^2 dA$ this integral is named (Moment of inertia **or** second moment of inertia).

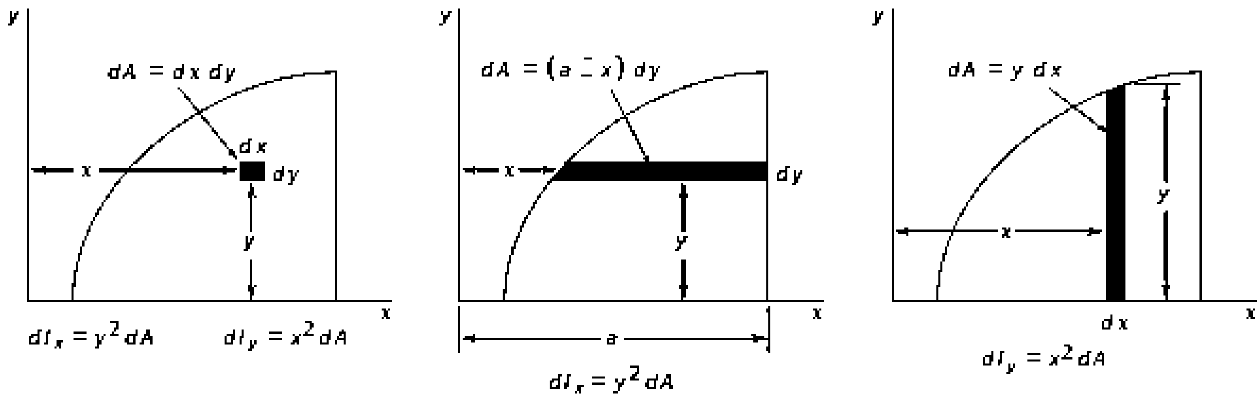
II- Rectangular moment of inertia

$$I_x = \int_A y^2 dA = \iint y^2 dxdy$$

$$I_y = \int_A x^2 dA = \iint x^2 dxdy$$

I_x = Moment of inertia with respect to x-axis
I_y = Moment of inertia with respect to y-axis

These may be calculated by single integration in this regard two possibilities exist.

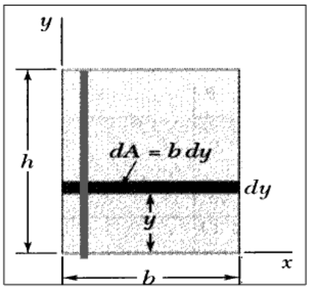


Notes:

- To compute I_x, the strip is chosen parallel to the x axis, so that all of the points of the strip are at the same distance y from the x axis (Fig. above b);
- To compute I_y, the strip is chosen parallel to the y axis so that all of the points of the strip are at the same distance x from the y axis (Fig. above);

Moment of Inertia of a Rectangular Area.

As an example, let us determine the moment of inertia of a rectangle with respect to its base (Fig. below). Dividing the rectangle into strips parallel to the x axis, we obtain

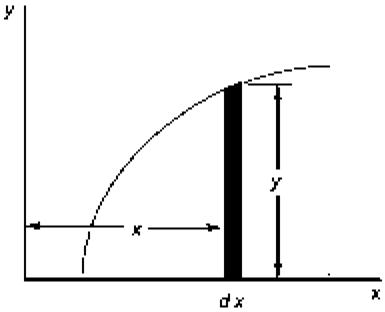


$$I_x = \int y^2 dA = \int_0^h y^2 b dy = \tfrac{1}{3} b h^3 \qquad *)$$

$$I_Y = \int x^2 dA = \int_0^b x^2 h dx = 1/3\ b^3 h$$

Computing I_x and I_y Using the Same Elemental Strips.

The formula just derived can be used to determine the moment of inertia dI_x with respect to the x axis of a rectangular strip which is parallel to the y axis, such as the strip shown in Fig. above. Setting ***b*= *dx*** and ***h*= *y*** in formula (**), we write



$$dI_x = \tfrac{1}{3} y^3 dx$$

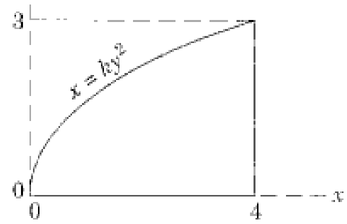
On the other hand, we have

$$dI_y = x^2 dA = x^2 y dx$$

The same element can thus be used to compute the moments of inertia I_x and I_y of a given area.

Example:-

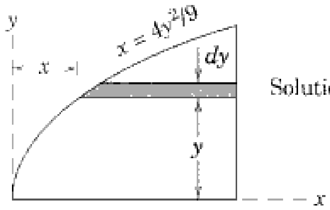
Determine the moment of inertia of the area under the parabola about the x-axis. Solve by using (a) a horizontal strip of area and (b) a vertical strip of area.



Solution. The constant $k = \frac{4}{9}$ is obtained first by substituting $x = 4$ and $y = 3$ into the equation for the parabola.

(a) Horizontal strip. Since all parts of the horizontal strip are the same distance from the x-axis, the moment of inertia of the strip about the x-axis is $y^2\,dA$ where $dA = (4 - x)\,dy = 4(1 - y^2/9)\,dy$. Integrating with respect to y gives us

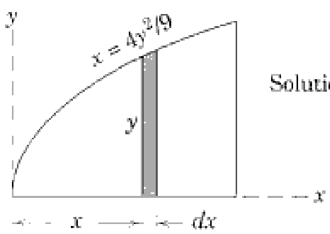
$$\left[I_x = \int y^2\,dA\right] \quad I_x = \int_0^3 4y^2\left(1 - \frac{y^2}{9}\right) dy = \frac{72}{5} = 14.40\ (\text{units})^4 \qquad Ans.$$



Solution (a)

(b) Vertical strip. Here all parts of the element are at different distances from the x-axis, so we must use the correct expression for the moment of inertia of the elemental rectangle about its base, which, from Sample Problem A/1, is $bh^3/3$. For the width dx and the height y the expression becomes

$$dI_x = \tfrac{1}{3}(dx)y^3$$



Solution (b)

To integrate with respect to x, we must express y in terms of x, which gives $y = 3\sqrt{x}/2$, and the integral becomes

① $$I_x = \tfrac{1}{3}\int_0^4 \left(\frac{3\sqrt{x}}{2}\right)^3 dx = \frac{72}{5} = 14.40\ (\text{units})^4 \qquad Ans.$$

Helpful Hint

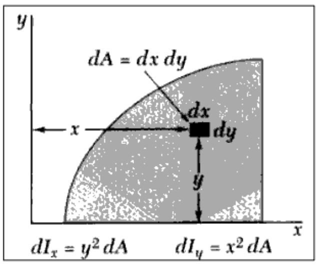
① There is little preference between Solutions (a) and (b). Solution (b) requires knowing the moment of inertia for a rectangular area about its base.

Note for review:-

$$dIx = 1/3\ dx\ (y^3) = 1/3\ y^3\ dx$$

$$dIy = x^2 dA = x^2 y\ dx \text{ or } 1/3\ x^3 dy$$

Computing I_x and I_y for quarter circle



Second moments or *moments of inertia* of an area with respect to the *x* and *y* axes,

$$I_x = \int y^2 dA \qquad I_y = \int x^2 dA$$

III Polar moment of inertia

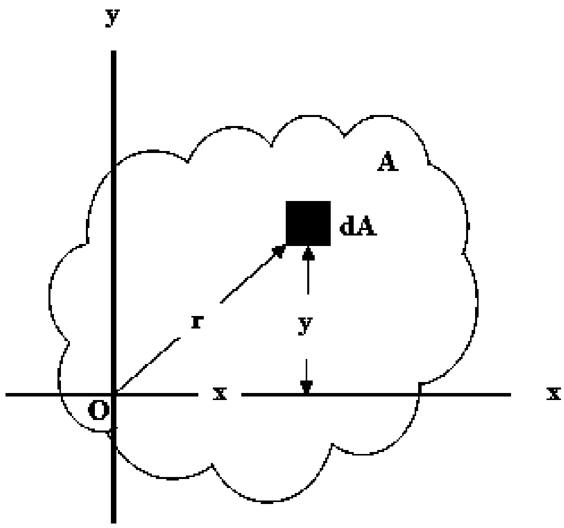
Def. the moment of inertia about an axis perpendicular to the plane of the figure is called (the polar moment of inertia).

$$J_o = \int_A r^2 dA = \int\int r^2 dxdy$$

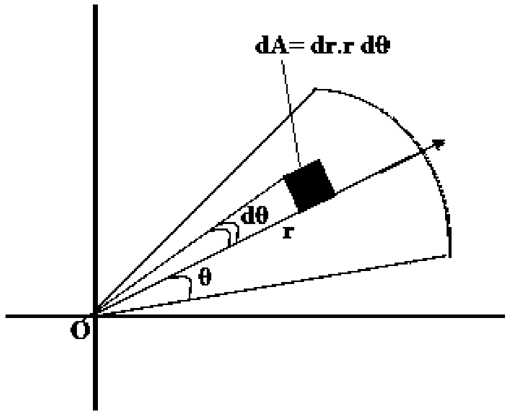
But, $r^2 = x^2 + y^2$

$$\therefore J_o = \int_A (x^2 + y^2) dA = \int_A x^2 dA + \int_A y^2 dA$$

$$\therefore J_o = I_x + I_y$$



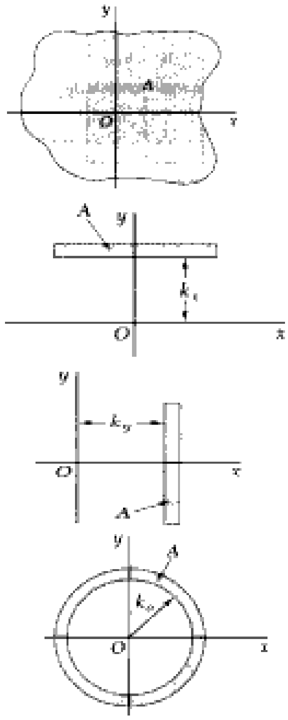
In polar coordinates



$$J_o = \int_A r^2 dA$$

$$= \int_\theta \int_r r^2 (r dr d\theta)$$

IIII Radius of Gyration



- Consider area A with moment of inertia I_x. Imagine that the area is concentrated in a thin strip parallel to the x axis with equivalent I_x.

$$I_x = k_x^2 A \qquad k_x = \sqrt{\frac{I_x}{A}}$$

$k_x =$ *radius of gyration* with respect to the x axis

$$I_y = k_y^2 A \qquad k_y = \sqrt{\frac{I_y}{A}}$$

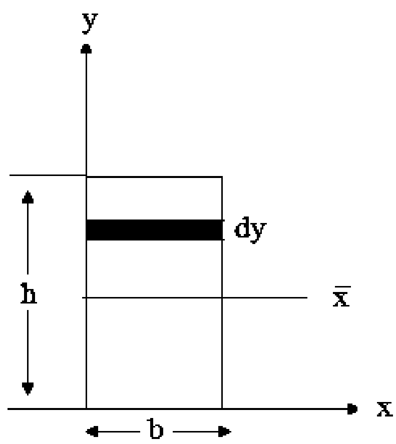
$$J_O = k_O^2 A \qquad k_O = \sqrt{\frac{J_O}{A}}$$

$$k_O^2 = k_x^2 + k_y^2$$

Ex:

Determine the moment of inertia about $\bar{x}$ axis

Solution:-



$$I_{\bar{x}} = \int y^2 dA$$

$$dA = b dy$$

$$I_{\bar{x}} = \int_{-h/2}^{h/2} y^2 b dy = b\left[\frac{y^3}{3}\right]_{-h/2}^{h/2}$$

$$I_{\bar{x}} = \frac{b}{3} * \left[(\frac{h}{2})^3 - (-\frac{h}{2})^3 \right]$$

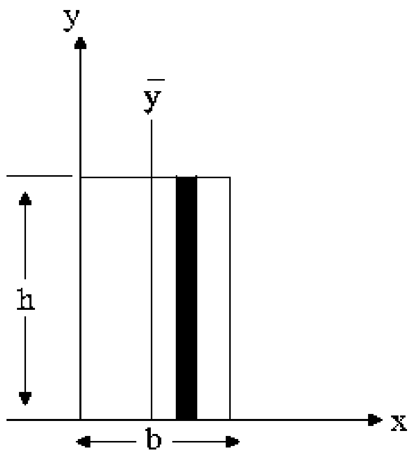
$$= \frac{b}{3} * \left[\frac{h^3}{8} + \frac{h^3}{8} \right]$$

$$= \frac{b}{3} * \left[\frac{2 * h^3}{8} \right] = \frac{b * h^3}{12}$$

Ex:

Determine the moment of inertia about $\bar{x}$ axis

Solution:-



$$I_{\bar{y}} = \int x^2 dA$$

$$dA = hdx$$

$$I_{\bar{y}} = \int_{-b/2}^{b/2} x^2 h dx = h \left[\frac{x^3}{3} \right]_{-b/2}^{b/2}$$

$$I_{\bar{y}} = \frac{h}{3} * \left[(\frac{b}{2})^3 - (-\frac{b}{2})^3 \right]$$

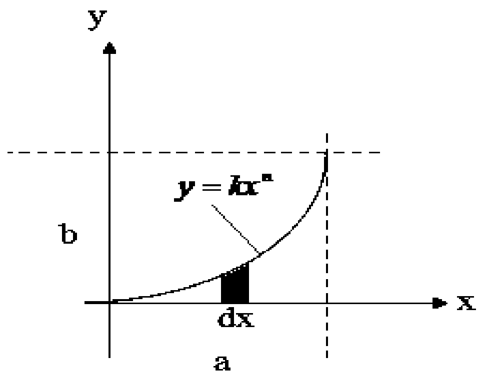
$$= \frac{h}{3} * \left[\frac{b^3}{8} + \frac{b^3}{8} \right]$$

$$= \frac{h}{3} * \left[\frac{2 * b^3}{8} \right] = \frac{h * b^3}{12}$$

Ex:-

Find I_x , I_y and J_o for the curve $y = kx^n$

Solution:-



$y = kx^n$

$y(a) = b \Rightarrow b = k * a^n$

$\therefore k = \frac{b}{a^n}$

$\therefore y = \frac{b}{a^n} x^n$

$I_x = \int y^2 * dA$

$$= \int_0^a y^2 * y * dx = \int_0^a y^3 * dx \qquad = \int_0^a (\frac{b}{a^n} x^n) * dx = \frac{1}{3} * \frac{ab^3}{3n+1}$$

$$I_y = \int x^2 * dA \qquad = \int_0^a x^2 * y * dx = \int_0^a x^2 (\frac{b}{a^n} x^n) * dx = \frac{ba^3}{n+3}$$

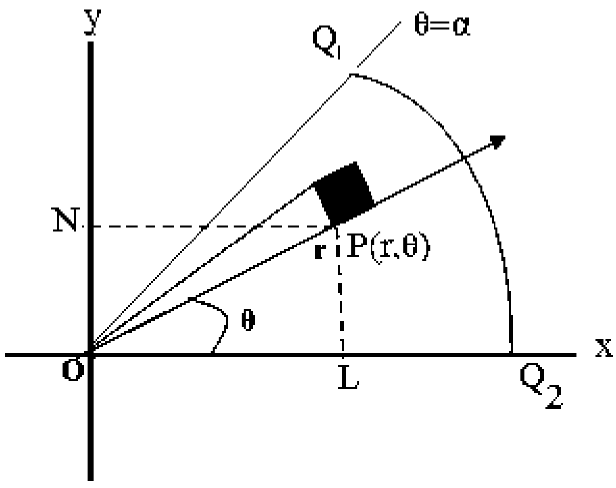
$$J_o = I_x + I_y = \frac{1}{3} * \frac{ab^3}{3n+1} + \frac{ba^3}{n+3}$$

Ex:-

Find the moment of inertia and J_o of a sector of a circle with radius (a) subtending an angle (α) at the center.

Solution:-

The sector is shown in figure, we take an elementary area at p(r,θ) of area dr.edθ



Clearly

$PL = r \sin\theta$

$PN = r \cos\theta$

Then M.I. about Ox is:

$$I_x = \int_{r=0}^{a} \int_{\theta=0}^{\alpha} dr.rd\theta (PL)^2$$

$$I_x = \int_{r=0}^{a} \int_{\theta=0}^{\alpha} dr.rd\theta.r^2 \sin^2\theta$$

$$= \frac{a^4}{4}\int_0^{\alpha} \frac{1}{2}(1-\cos 2\theta)d\theta$$

$$= \frac{a^4}{4}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\alpha} = \frac{a^4}{4}\left(\frac{\alpha}{2} - \frac{\sin 2\alpha}{4}\right)$$

Similarly, M.I. about Oy is :-

$$I_y = \int_{r=0}^{a} \int_{\theta=0}^{\alpha} dr.rd\theta(PN)^2$$

$$I_y = \int_{r=0}^{a} \int_{\theta=0}^{\alpha} dr.rd\theta.r^2 \cos^2\theta$$

$$= \frac{a^4}{4}\int_0^{\alpha} \frac{1}{2}(1+\cos 2\theta)d\theta$$

$$= \frac{a^4}{4}\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right]_0^{\alpha} = \frac{a^4}{4}\left(\frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right)$$

$$J_o = I_x + I_y$$

$$= \frac{a^4}{4}\left(\frac{\alpha}{2} - \frac{\sin 2\alpha}{4}\right) + \frac{a^4}{4}\left(\frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right)$$

$$= \frac{a^4}{4}\left[\frac{\alpha}{2} - \frac{\sin 2\alpha}{4} + \frac{\alpha}{2} + \frac{\sin 2\alpha}{4}\right]$$

$$\Rightarrow J_o = \frac{a^4.\alpha}{4}$$

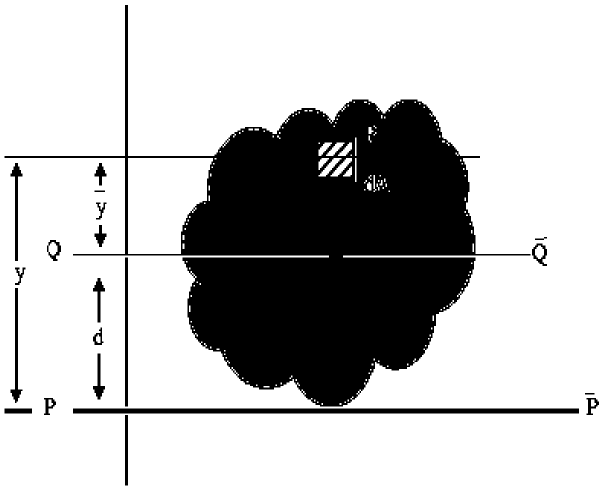
Read Example 9.1- 9.3 page 476 in Ref. 1

HW 11: Solve problem 9.1- 9.30 page 480 in ref.1

V- Parallel Axes Theorem (the Transfer Formula)

The moment of inertia of an area with respect to only axis equal to the moment of inertia with respect to a centroid at parallel axis plus the product of the area times the square of the distance between them.

Proof



$$I_{p\bar{p}} = \int_A y^2 dA$$

$$= \int_A (\bar{y} + d)^2 dA$$

$$== \int_A \left[\bar{y}^2 + 2\bar{y}d + d^2\right] dA$$

$$= \int_A \bar{y}^2 dA + 2d\int_A \bar{y}dA + d^2 \int_A dA$$

the term $2d\int_A \bar{y}dA$ is the moment of area about centroid and by definition of centroid, it is zero. Then,

$$I_{p\bar{p}} = I_{Q\bar{Q}} + Ad^2$$

$$or$$

$$\left[I_x = I_{\bar{x}} + Ad^2\right]$$

A:- Area

d:- distance between $Q\bar{Q}$ and $P\bar{P}$

Ex:

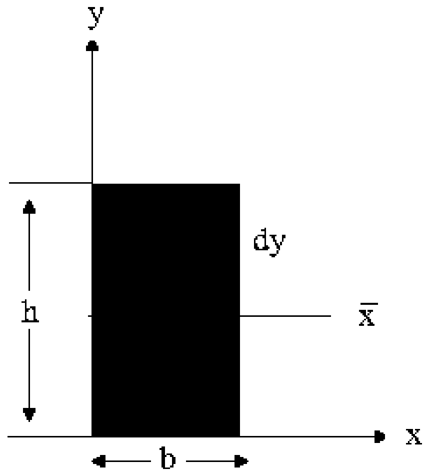
If $I_{\bar{x}} = \frac{bh^3}{12}$ for the rectangle shown find I_x

Solution:-

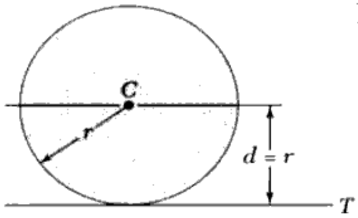
$$I_x = I_{\bar{x}} + bh(\frac{h}{2})^2$$

$$= \frac{bh^3}{12} + bh\frac{h^2}{4}$$

$$= \frac{bh^3}{3}$$



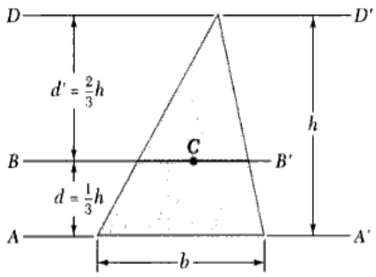
Application 1:



Moment of inertia I_T of a circular area with respect to a tangent to the circle T,

$$I_T = \bar{I} + Ad^2 = \tfrac{1}{4}\pi r^4 + \left(\pi r^2\right) r^2$$
$$= \tfrac{5}{4}\pi r^4$$

Application 2:



Moment of inertia of a triangle with respect to a centroidal axis,

$$I_{AA'} = \bar{I}_{BB'} + Ad^2$$
$$I_{BB'} = I_{AA'} - Ad^2 = \tfrac{1}{12} bh^3 - \tfrac{1}{2} bh \left(\tfrac{1}{3} h\right)^2$$
$$= \tfrac{1}{36} bh^3$$

$I_{DD'} = I_{BB'} + ad'^2 = 1/36\ bh^3 + 1/2bh\ (2/3h)^2 = ¼\ bh^3$

VI Moment of Inertia for the Composite Area

The moment of inertia of composite area about a particular axis is simply the same of the moment of inertia of its component parts about the some axis, using the transfer formula when necessary.

Note:-

The radius of gyration of composite area is not equal to the sum of the redil of the component area ,but it is given by

$$r = \sqrt{\frac{I}{A}}$$

Where

r:- The radius of gyration ; I:- The total moment of inertia; A:- The total area

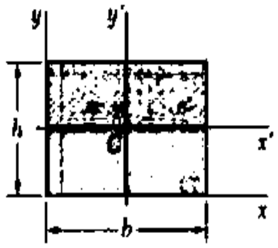
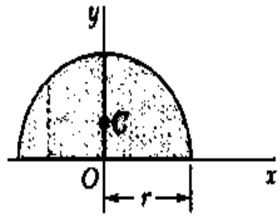
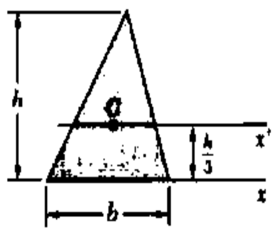
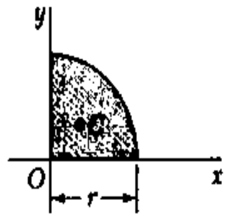
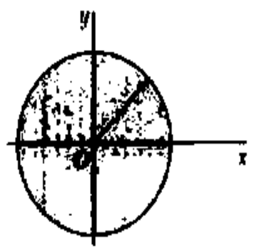
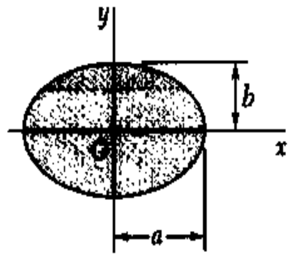
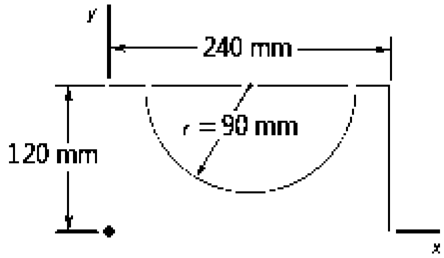
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$	Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$	Quarter circle		$I_x = I_y = \frac{1}{16}\pi r^4$ $J_O = \frac{1}{8}\pi r^4$
Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$	Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

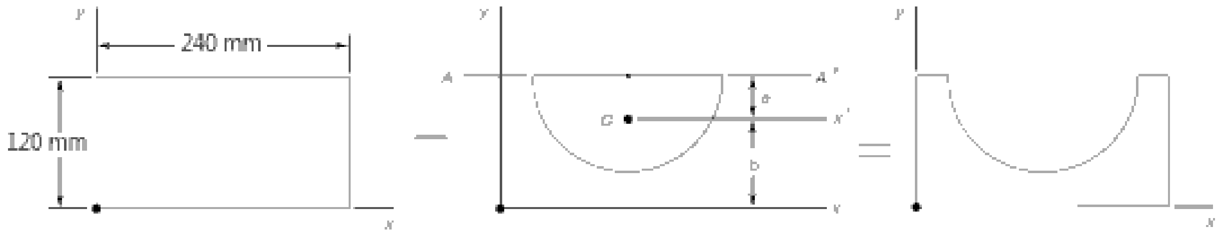
Fig. 9.12 Moments of inertia of common geometric shapes.[ref. 1 pp.483]

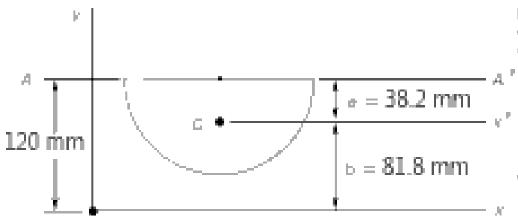
EX:

Determine the moment of inertia of the shaded area with respect to the x axis.

Sol:







Moment of Inertia of Rectangle. Referring to Fig. 9.12, we obtain

$$I_x = \tfrac{1}{3}bh^3 = \tfrac{1}{3}(240 \text{ mm})(120 \text{ mm})^3 = 138.2 \times 10^6 \text{ mm}^4$$

Moment of Inertia of Half Circle. Referring to Fig. 5.8, we determine the location of the centroid C of the half circle with respect to diameter AA'.

$$a = \frac{4r}{3\pi} = \frac{(4)(90 \text{ mm})}{3\pi} = 38.2 \text{ mm}$$

The distance b from the centroid C to the x axis is

$$b = 120 \text{ mm} - a = 120 \text{ mm} - 38.2 \text{ mm} = 81.8 \text{ mm}$$

Referring now to Fig. 9.12, we compute the moment of inertia of the half circle with respect to diameter AA'; we also compute the area of the half circle.

$$I_{AA'} = \tfrac{1}{8}\pi r^4 = \tfrac{1}{8}\pi(90 \text{ mm})^4 = 25.76 \times 10^6 \text{ mm}^4$$
$$A = \tfrac{1}{2}\pi r^2 = \tfrac{1}{2}\pi(90 \text{ mm})^2 = 12.72 \times 10^3 \text{ mm}^2$$

Using the parallel-axis theorem, we obtain the value of $\bar{I}_{x'}$:

$$I_{AA'} = \bar{I}_{x'} + Aa^2$$
$$25.76 \times 10^6 \text{ mm}^4 = \bar{I}_{x'} + (12.72 \times 10^3 \text{ mm}^2)(38.2 \text{ mm})^2$$
$$\bar{I}_{x'} = 7.20 \times 10^6 \text{ mm}^4$$

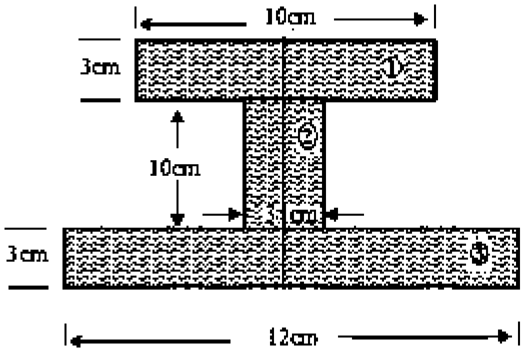
Again using the parallel-axis theorem, we obtain the value of I_x:

$$I_x = \bar{I}_{x'} + Ab^2 = 7.20 \times 10^6 \text{ mm}^4 + (12.72 \times 10^3 \text{ mm}^2)(81.8 \text{ mm})^2$$
$$= 92.3 \times 10^6 \text{ mm}^4$$

Moment of Inertia of Given Area. Subtracting the moment of inertia of the half circle from that of the rectangle, we obtain

$$I_x = 138.2 \times 10^6 \text{ mm}^4 - 92.3 \times 10^6 \text{ mm}^4$$
$$I_x = 45.9 \times 10^6 \text{ mm}^4 \quad ◄$$

<u>Ex:</u> Find the moment of inertia for the shaded area about its centroid



<u>Solution:-</u>

$\bar{x} = 0$

$calculate, \bar{y}$

$A_{i,1} = 30, y_{i,1} = 14.5$

$A_{i,2} = 30, y_{i,2} = 8$

$A_{i,3} = 36, y_{i,3} = 1.5$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A_i} = \frac{30*14.5 + 30*8 + 36*1.5}{30+30+36}$$

$\Rightarrow \bar{y} = 7.6cm$

$$I_{\bar{x}} = (I_{\bar{x}} + Ad_1^2)_1 + (I_{\bar{x}} + Ad_2^2)_2 + (I_{\bar{x}} + Ad_3^2)_3$$

$$= (\frac{10*3^3}{12} + 30*6.9^2) + (\frac{3*10^3}{12} + 30*0.4^2) + (\frac{12*3^3}{12} + 36*6.1^2)$$

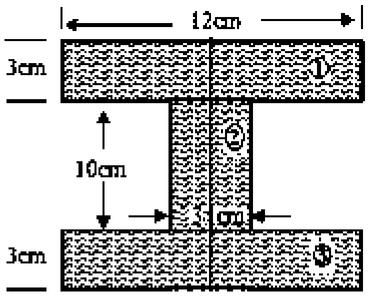
$\therefore I_{\bar{x}} = 3049.66\text{cm}^4$

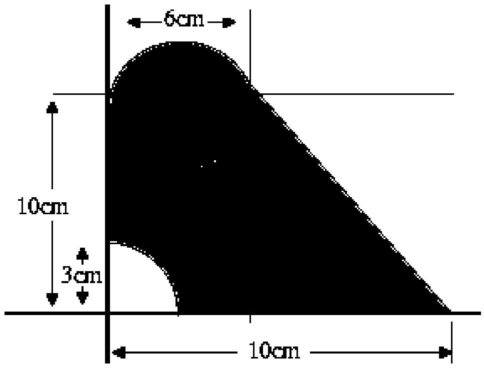
$$I_{\bar{y}} = (I_{\bar{y}} + Ad_1^2)_1 + (I_{\bar{y}} + Ad_2^2)_2 + (I_{\bar{y}} + Ad_3^2)_3$$

$$= (\frac{3*10^3}{12}) + (\frac{10*3^3}{12}) + (\frac{3*12^3}{12})$$

$\therefore I_{\bar{y}} = 704.5\text{cm}^4$

<u>HW</u> Find the moment of inertia for the shaded area about its centroid





Read Example 4.1- 4.5 page 169 in Ref. 1

HW 12: Solve problem 9.31- 9.60 page 492 in ref.1

Friction

When a body slides on another, the tangential forces generated near contacting surfaces are called **Friction Forces.**

- Sliding of one contact surface to other – friction occurs and it is opposite to the applied force.
- Reduce friction in bearings, power screws, gears, aircraft propulsion, missiles through the atmosphere, fluid flow etc.
- Maximize friction in brakes, clutches, belt drives etc.

Friction

Dry friction
(coulomb friction)

- Occurs when un-lubricated surfaces are in contact during sliding
- friction force always oppose the sliding motion

Fluid friction

- Occurs when the adjacent layers in a fluid (liquid, gas) are moving at different velocities
- This motion causes friction between fluid elements
- Depends on the relative velocity between layers
- No relative velocity – no fluid friction
- depends on the viscosity of fluid – measure of resistance to shearing action between the fluid layers

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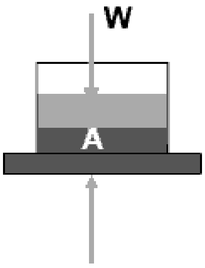
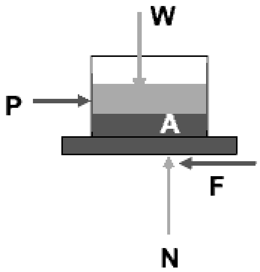
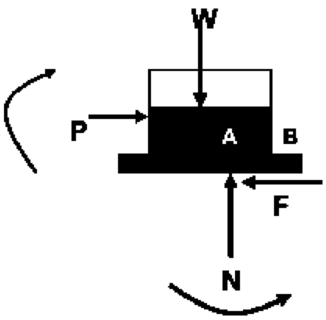
Dry friction: Laws of dry friction



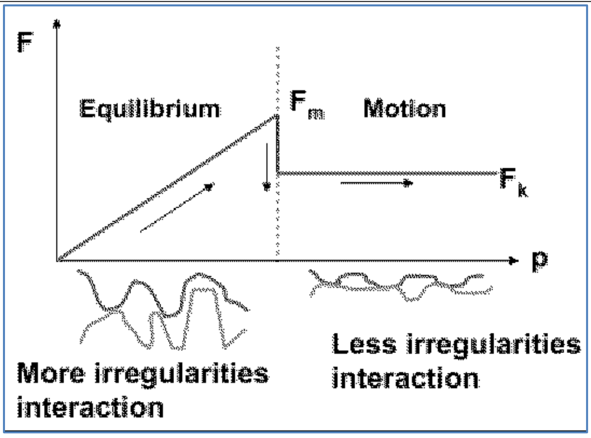
- W – weight; N – Reaction of the surface
- Only vertical component

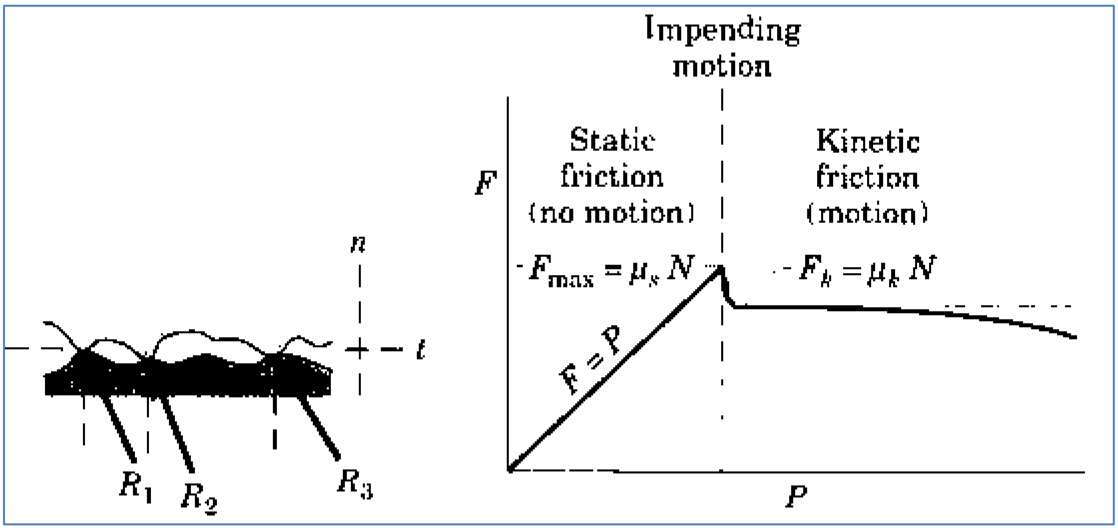


- P – applied load
- F – static friction force : resultant of many forces acting over the entire contact area
- Because of irregularities in surface & molecular attraction



- 'P' is increased; 'F' is also increased and continue to oppose 'P'
- This happens till maximum 'Fm' is reached – Body tend to move till F_m is reached
- After this point, block is in motion
- Block in motion: 'F_m' reduced to 'F_k – lower value – kinetic friction force' and it remains same – related to irregularities interaction
- 'N' reaches 'B' from 'A' – Then tipping occurs abt. 'B'





EXPERIMENTAL EVIDENCE:

F_m proportional to N

$F_m = \mu_s N$; μ_s – static friction co-efficient

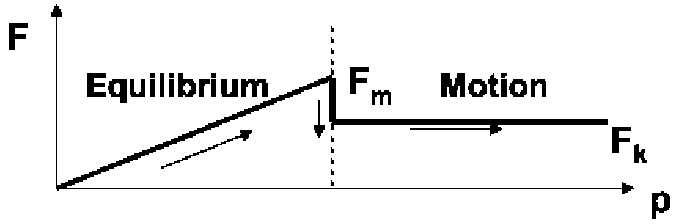
Similarly, $F_k = \mu_k N$; μ_k – kinetic friction co-efficient

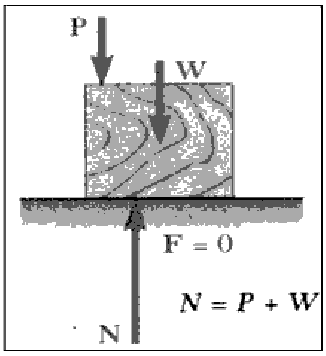
μ_s and μ_k depends on the nature of surface; not on contact area of surface. usually **μ_s** more than **μ_k**.
where:
μ_s and **μ_k** : coefficient of static, kinetic friction;

F_s, F_k, : Static , kinetic Friction force; **N** : Normal force

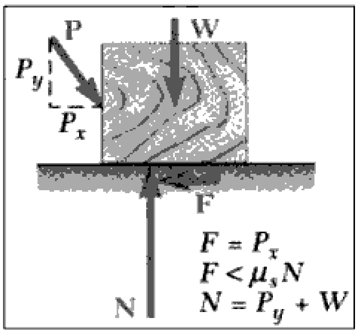
Four different situation can occur when a rigid body is in contact with a horizontal surface.

We have horizontal and vertical force equilibrium equns. and F = μ N

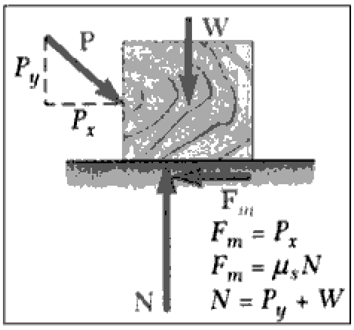




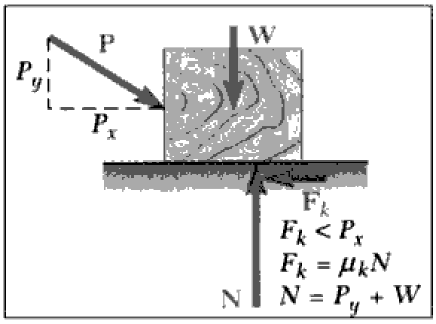
- No friction, $(P_x = 0)$



- No motion, $(P_x < F_m)$

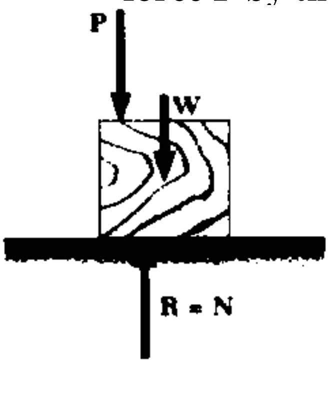


- Motion impending, $(P_x = F_m)$

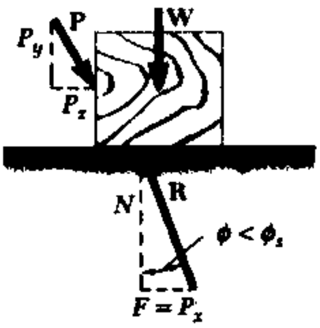


- Motion, $(P_x > F_m)$

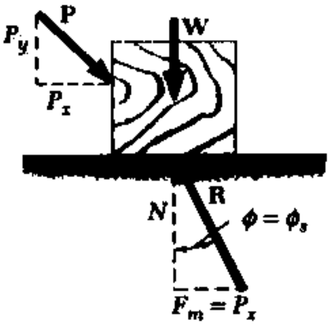
It is sometimes convenient to replace normal force N and friction force F by their resultant R:



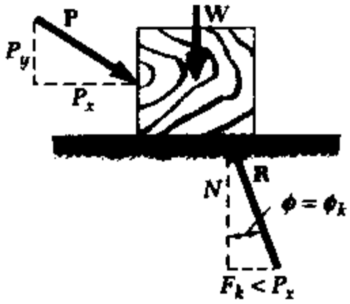
- No friction



- No motion



- Motion impending



- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$
$$\tan\phi_s = \mu_s$$

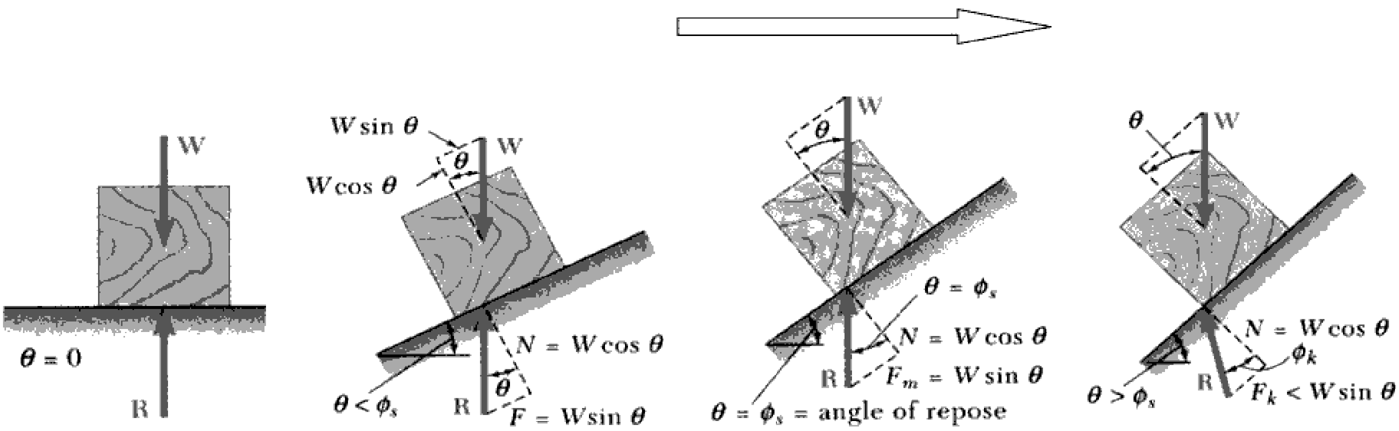
$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$
$$\tan\phi_k = \mu_k$$

Φs – angle of static friction – maximum angle (like F_m)

Φ_k – angle of kinetic friction; $\Phi_k < \Phi_s$

Consider block of weight W resting on board with variable inclination angle θ

ANGLE OF INCLINATION IS INCREASING



- No friction
- No motion
- Motion impending

  Angle of inclination = angle of repose; $\theta = \varphi_s$
- Motion

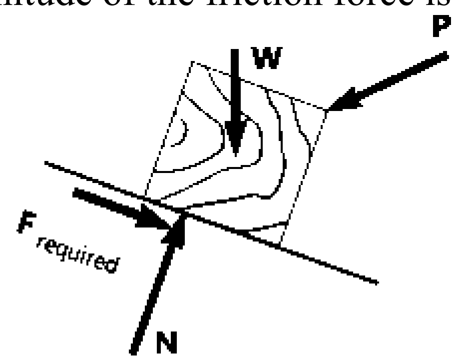
  R – Not vertical

Problems Involving Dry Friction

Most problems involving friction fall into one of the following three groups:-

In the first group of problems,

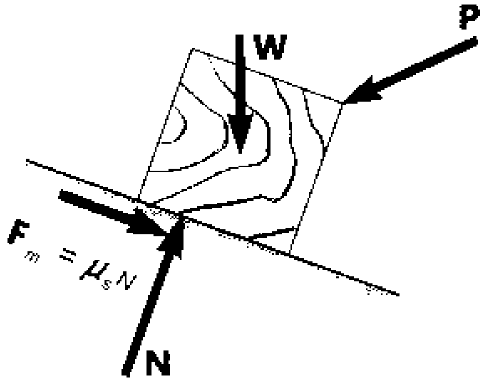
- all applied forces are given and the coefficients of friction are known; we are to determine whether the body considered will remain at rest or slide.
- The friction force F *required to maintain equilibrium* is unknown (its magnitude is not equal to $m_s\ N$) and should be determined, together with the normal force N, by drawing a free-body diagram and solving the equations of equilibrium (Fig. below). The value found for the magnitude F of the friction force is then compared with the maximum value $Fm = m_s\ N$. If F is smaller than or equal to F_m ,the body remains at rest. If the value found for F is larger than F_m, equilibrium cannot be maintained and motion takes place; the actual magnitude of the friction force is then $F_k = m_k\ N$.



In the second group of problems,

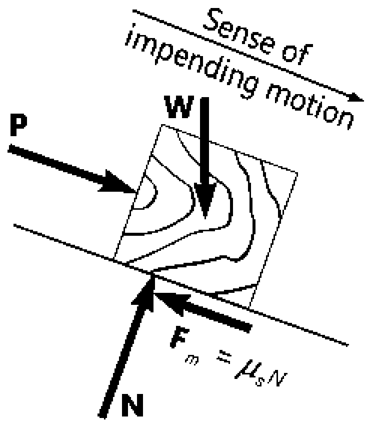
- All applied forces are given and the motion is known to be impending; we are to determine the value of the coefficient of static friction. Here again, we determine the friction force and the normal force by drawing a free-body diagram and solving the equations of equilibrium (Fig. below). Since we know that the value found for F is the maximum value F_m, the coefficient of friction may be found by writing and solving the equation

$F_m = m_s N$.



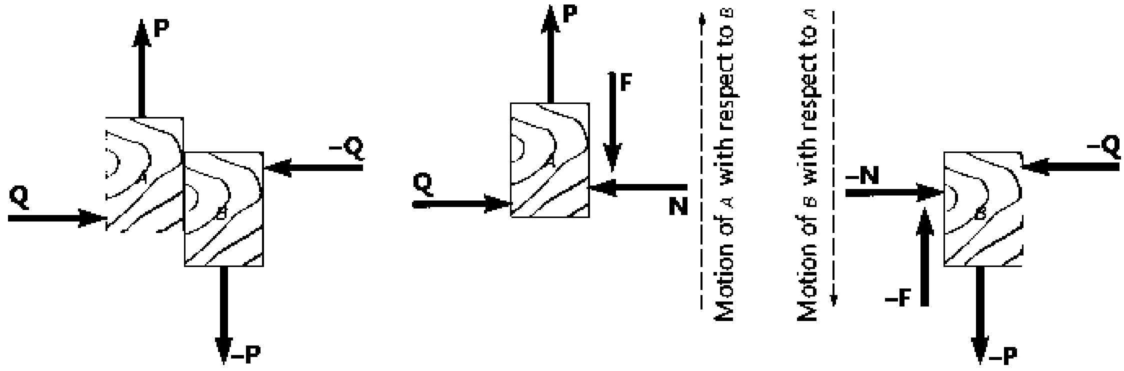
In the third group of problems,

- The coefficient of static friction is given, and it is known that the motion is impending in a given direction; we are to determine the magnitude or the direction of one of the applied forces. The friction force should be shown in the free body diagram with a sense opposite to that of the impending motion and with a magnitude $F_m = m_s N$ (Fig. below). The equations of equilibrium can then be written, and the desired force determined.



Note:

When two bodies A and B are in contact (Fig. below), the forces of friction exerted, respectively, by A on B and by B on A are equal and opposite (Newton's third law). In drawing the free body diagram of one of the bodies, it is important to include the appropriate friction force with its correct sense.



The problem you have to solve may fall in one of the following three categories: (the first step in your solution is to draw a free-body diagram of the body under consideration)

1. **All the applied forces and the coefficients of friction are known, and you must determine whether equilibrium is maintained.** Note that in this situation the friction force is unknown and *cannot be assumed to be equal* to $m_s N$.

   a. **Write the equations of equilibrium to determine N and F.**

   b. **Calculate the maximum allowable friction force, $F_m = M_s N$.** If $F \leq F_m$, equilibrium is maintained. If $F > F_m$, motion occurs, and the magnitude of the friction force is $F_k = m_k N$ [Sample Prob. 8.1].

2. **All the applied forces are known, and you must find the smallest allowable value of M_s for which equilibrium is maintained.** You will assume that motion is impending and determine the corresponding value of m_s.

   a. **Write the equations of equilibrium to determine N and F.**

   b. **Since motion is impending, $F = F_m$.** Substitute the values found for N and F into the equation $F_m = m_s N$ and solve for m_s.

3. **The motion of the body is impending and μ_s is known; you must find some unknown quantity,** such as a distance, an angle, the magnitude of a force, or the direction of a force.

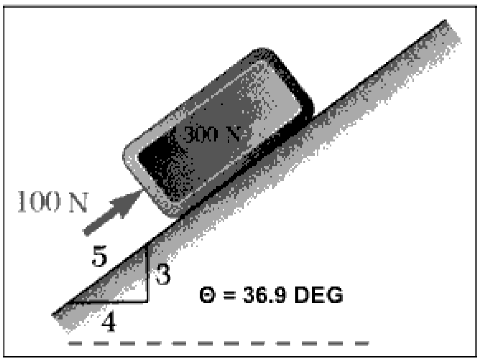
   a. **Assume a possible motion of the body** and, on the free-body diagram, draw the friction force in a direction opposite to that of the assumed motion.

   b. **Since motion is impending, $F = F_m = \mu_s N$.** Substituting for m_s its known value, you can express F in terms of N on the free-body diagram, thus eliminating one unknown.

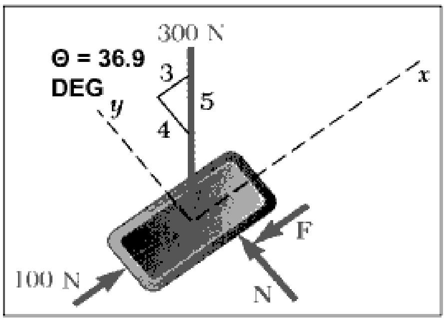
   c. **Write and solve the equilibrium equations for the unknown you seek** [Sample Prob. 8.3].

Ex:-

Beer/Johnston



A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.



$$\sum F_x = 0: \quad 100\text{ N} - \tfrac{3}{5}(300\text{ N}) - F = 0$$

$$F = -80\text{ N}$$

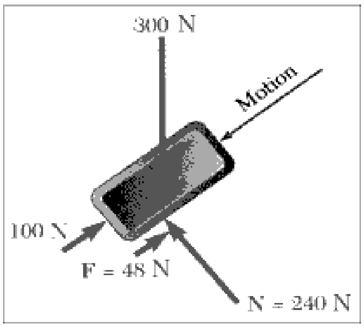
$$\sum F_y = 0: \quad N - \tfrac{4}{5}(300\text{ N}) = 0$$

$$N = 240\text{ N}$$

$F_m = \mu_s N = 0.25\ (240) = 60$ N

$F_m < F$

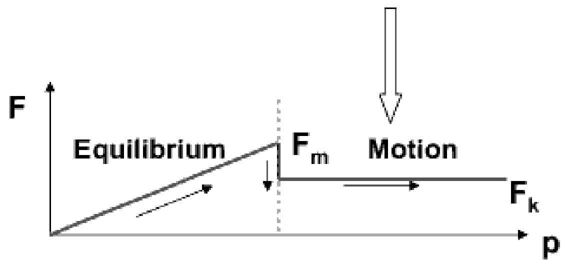
The block will slide down the plane.



- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k N$$
$$= 0.20(240\text{ N})$$

$$F_{actual} = 48\text{ N}$$



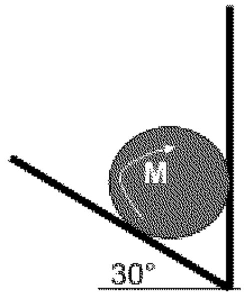
EX:

Meriam/Kraige; 6/8

Cylinder weight: 30 kg; Dia: 400 mm

Static friction co-efft: 0.30 between cylinder and surface

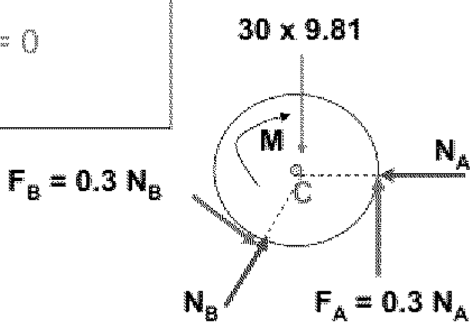
Calculate the applied CW couple M which cause the cylinder to slip



$\Sigma Fx = 0 = -N_A + 0.3N_B \cos 30 - N_B \sin 30 = 0$

$\Sigma Fy = 0 \Rightarrow -294.3 + 0.3N_A + N_B \cos 30 - 0.3N_B \sin 30 = 0$

Find N_A & N_B by solving these two equns.

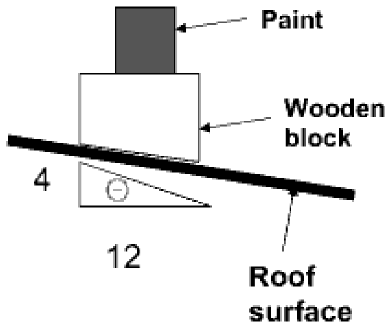


$\Sigma M_C = 0 => 0.3\ N_A\ (0.2) + 0.3\ N_B\ (0.2) - M = 0$

Put N_A & N_B; Find 'M'

NA = 237 N & NB = 312 N; M = 33 Nm

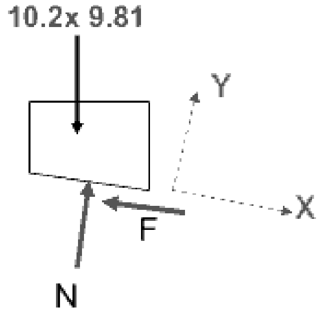
Impending relative motion when two or three bodies in contact with each other



Meriam/Kraige; 6/5

Wooden block: 1.2 kg; Paint: 9 kg

Determine the magnitude and direction of (1) the friction force exerted by roof surface on the wooden block, (2) total force exerted by roof surface on the wooden block



$\Theta = \tan^{-1}(4/12) = 18.43°$

(2) Total force = 10.2 x 9.81 = 100.06 N UP ↑

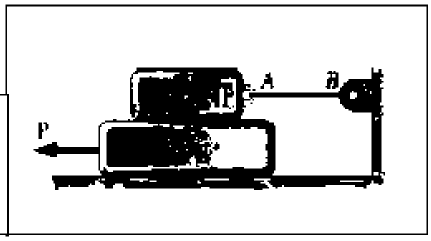
(1) $\Sigma Fx = 0 \Rightarrow -F + 100.06 \sin 18.43 \Rightarrow F = 31.6$ N

$\Sigma Fy = 0 \Rightarrow N = 95$ N

EX:

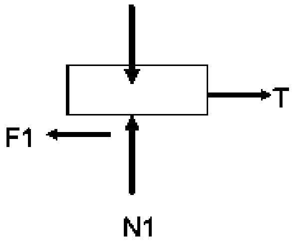
Beer/Johnston

The coefficients of friction are $\mu_s = 0.40$ and $\mu_k = 0.30$ between all surfaces of contact. Determine the force P for which motion of the 30-kg block is impending if cable AB (a) is attached as shown, (b) is removed.

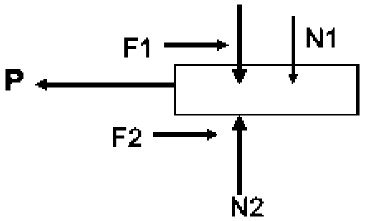


(a)

For 20 kg block



For 30 kg block



$\Sigma F_y = 0$: $N_1 - 196.2\text{ N} = 0$

$N_1 = 196.2\text{ N}\uparrow$

$\Sigma F_y = 0$: $N_2 - 196.2\text{ N} - 294.3\text{ N} = 0$

$N_2 = 490.5\text{ N}\uparrow$

$F_1 = \mu_s N_1 = 0.4(196.2\text{ N})$ $F_1 = 78.48\text{ N}\leftarrow$

$F_2 = \mu_s N_2 = 0.4(490.5\text{ N}) = 196.2\text{ N}$

$\Sigma F_x = 0$: $-P + 78.48\text{ N} + 196.2\text{ N} = 0$

$P = 275\text{ N}\leftarrow$

(B)

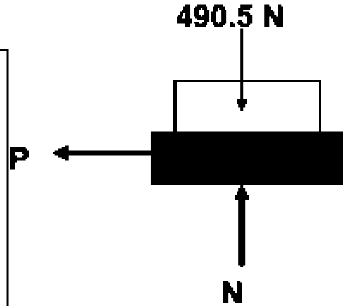
(b) Without cable AB, top and bottom blocks will move together

$+\uparrow \Sigma F_y = 0$: $N - 490.5\text{ N} = 0$, $N = 490.5\text{ N}$

Impending slip: $F = \mu_s N = 0.40(490.5\text{ N}) = 196.2\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0$: $-P + 196.2\text{ N} = 0$

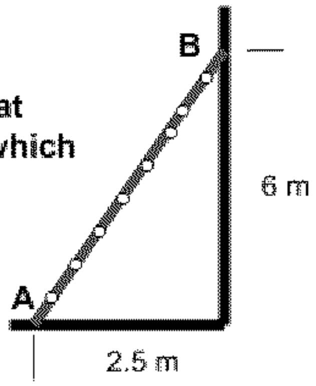
$P = 196.2\text{ N}\leftarrow$



EX:

Beer/Johnston

A 6.5-m ladder AB of mass 10 kg leans against a wall as shown. Assuming that the coefficient of static friction on μ_s is the same at both surfaces of contact, determine the smallest value of μ_s for which equilibrium can be maintained.

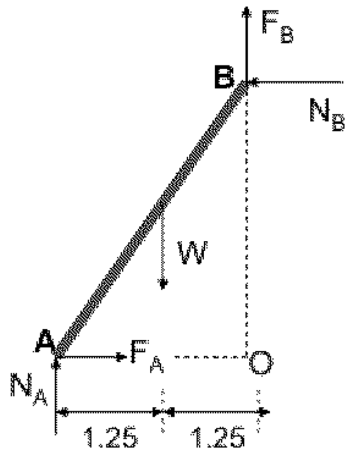


Slip impends at both A and B, $F_A = \mu_s N_A$, $F_B = \mu_s N_B$

$\Sigma Fx=0 \Rightarrow F_A - N_B = 0$, $N_B = F_A = \mu_s N_A$

$\Sigma Fy=0 \Rightarrow N_A - W + F_B = 0$, $N_A + F_B = W$

$N_A + \mu_s N_B = W$; $W = N_A(1+\mu_s^2)$



$\Sigma Mo = 0 \Rightarrow (6)\,N_B - (2.5)\,(N_A) + (W)\,(1.25) = 0$

$6\mu_s N_A - 2.5\,N_A + N_A(1+\mu_s^2)\,1.25 = 0$

$\mu_s = -2.4 \pm 2.6 \Rightarrow$ Min $\mu_s = 0.2$

Read Example **(8.2-8.3) pp. 420 in ref. [1].**

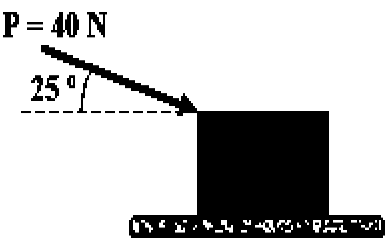
HW 13: Solve problems 8. F1- 8.47 page 423 in ref.1

HW 14: Solve review problems 8. 134 - 8.139 page 463 in ref.1

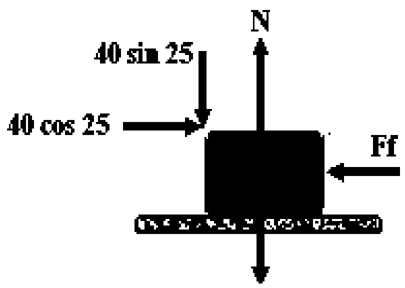
Additional Examples

Example (1):-

A block with 200 N weights rests on a rough horizontal plane , is subjected to the force (P = 40 N) which inclined (25 ₒ) . Determine the **coefficient of friction**.



Solution:-



F.B.D

$$\sum F_x = o$$

$$F_f = 40 * \cos 25$$

$$\sum F_y = 0$$

$$N - 40 * \sin 25 - 200 = 0$$

$$N = 40 * \sin 25 + 200 = 217 N$$

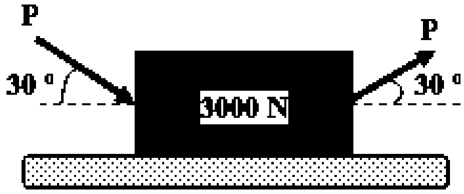
$$F_f = \mu * N$$

$$36.25 = \mu * 217$$

$$\Rightarrow \mu = \frac{36.25}{217} = 0.17$$

Example (2):-

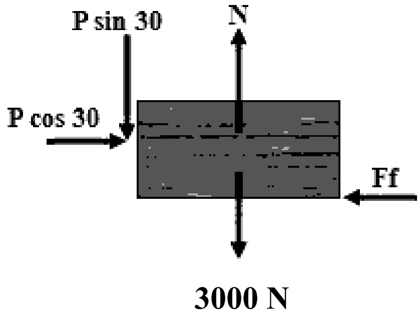
A wooden block (3000 N) weight , the **coefficient of friction** between the block and the floor is (0.35) , determine whether pushing or pulling process by the force (P) is suitable to make the block tend to move to the right with a least force (P).



Solution:-

First case

In case of pushing



F.B.D

$\sum F_x = o$

$p * \cos 30 - F_f = 0$

$F_f = 0.866P ----------(1)$

$\sum F_y = o$

$N - P * \sin 30 - 3000 = 0$

$N = P\sin 30 + 3000 -----(2)$

$F_f = \mu * N -----------(3)$

Sub. eq(1) and eq(2) in eq(3)

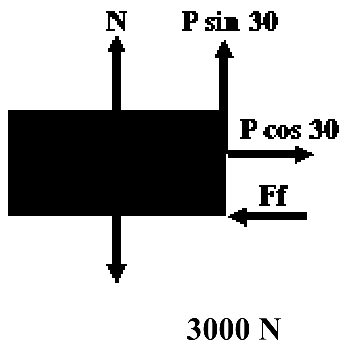
$0.866P = 0.35(0.5P + 3000)$

$0.691P = 1050$

$P = \frac{1050}{0.691} = 1519.4N$

Second case

In case of Pulling



F.B.D

$\sum F_x = o$

$P\cos 30 = F_f$

$\Rightarrow F_f = 0.866P -----------(1)$

$\sum F_y = o$

$N + P\sin 30 - 3000 = 0$

$N = -P\sin 30 + 3000$

$\Rightarrow N = 3000 - P\sin 30 --------(2)$

$F_f = \mu * N --------------(3)$

Sub. eq(1) and eq(2) in eq(3)

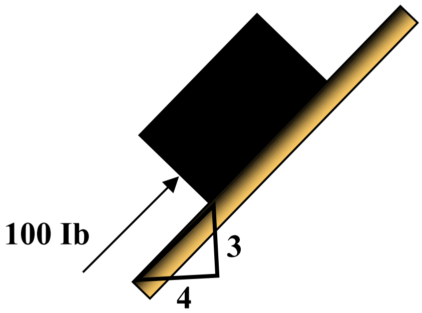
$$0.866P = 0.35*(3000-0.5P)$$

$$1.041P = 1050$$

$$P = \frac{1050}{1.041} = 1008.6N$$ $\Longrightarrow$ **the pulling is easier than the pushing**

Example (3):-

A 100 Ib force acts as shown on a 300 Ib block placed on an inclined plane. The coefficient of friction between the block and the plane are μ_s=0.25 and μ_k = 0.20. Determine whether the block is in equilibrium, and find the value of the friction force.



Solution:-

Force required for equilibrium

$$\sum F_x = 0$$

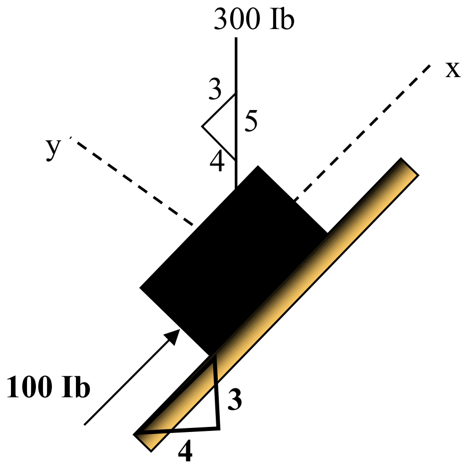
$$100 - \frac{3}{5}*300 - = 0$$

$$\Rightarrow F = -80Ib$$

$$\sum F_y = 0$$

$$N - \frac{4}{5}*300 = 0$$

$$\Rightarrow N = +240Ib$$



Maximum Friction Force

$$F_{Max.} = F_s = \mu_s * N$$

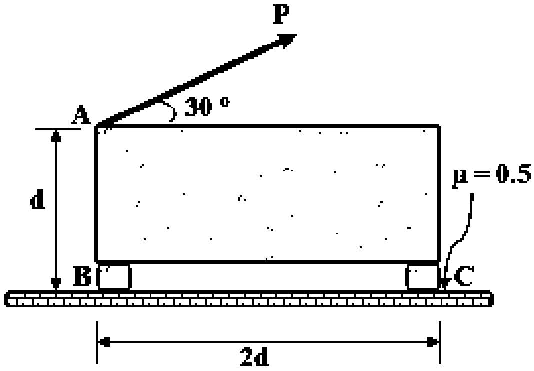
$$\Rightarrow F_s = 0.25*240 = 60Ib$$

Actual value of Friction Force

$$F_{actual} = F_k = \mu_k * N \Rightarrow F_k = 0.2*240 = 48Ib$$

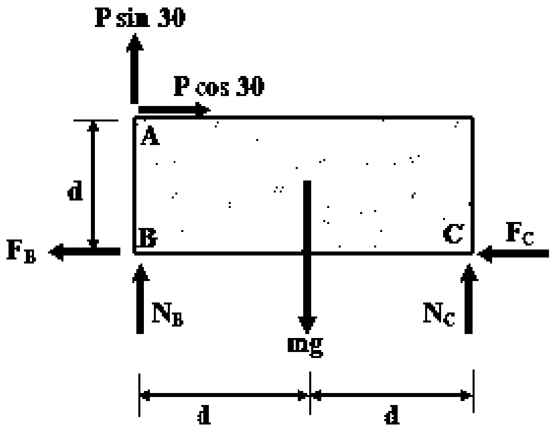
Example (4):-

The magnitude of the force (P) is slowly increased .Does the homogeneous box of mass (m) slips or tip first? State the value of (P) which would cause each occurrence. Neglect any effect of the size of the small feet.



Solution:-

Slips



$$\sum F_x = 0$$

$$-F_B - F_C + P * \cos 30 = 0$$

$$F_B = \mu * N_B \qquad , F_C = \mu * N_C$$

$$-\mu * N_B - \mu * N_C + P * \cos 30 = 0$$

$$P * \cos 30 = \mu * N_B + \mu * N_C ------(1)$$

$$\sum F_y = 0$$

$$N_B + N_C - m * g + P * \sin 30 = 0$$

$$N_B = -N_C + m * g - P * \sin 30 = 0 ----(2)$$

$$subst.(2) in (1)$$

$$P * \cos 30 = \mu * (mg - N_c - P * \sin 30) + \mu * N_C$$

$$P * \cos 30 = \mu * mg - \mu * N_c - \mu * P * \sin 30 + \mu * N_C$$

$$P * \cos 30 = \mu * mg - \mu * P * \sin 30$$

$$P * \cos 30 + \mu * P * \sin 30 = \mu * mg$$

$$\Rightarrow P = \frac{\mu * mg}{\cos 30 + \mu * \sin 30} \Rightarrow P = \frac{0.5 * mg}{0.866 + 0.5 * 0.5} = 0.44 mg$$

Tips

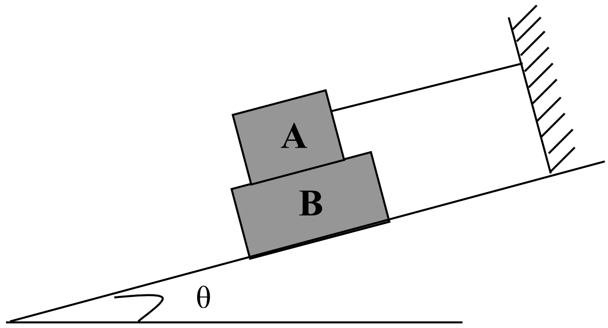
$$\sum M_C$$

$$P * \cos 30 * d + P * \sin 30 * 2 * d - mg * d = 0$$

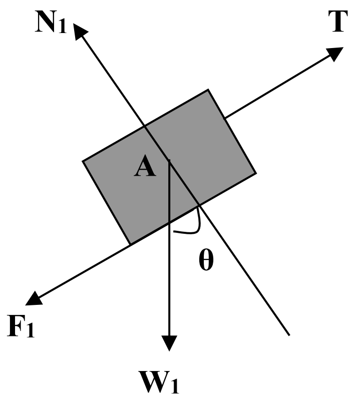
$$P = \frac{mg}{\cos 30 + 2 * \sin 30} = 0.53mg \Rightarrow \because P_{slip} < P_{tip} \therefore$$ **<u>Slipping will occur</u>**

<u>Example (5):-</u>

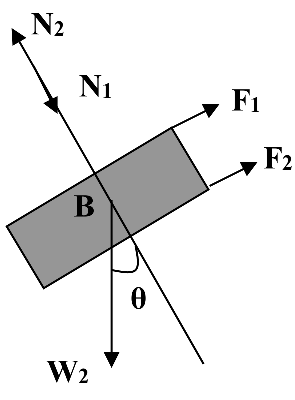
Blocks A and B of weights (300 kN) and (900 kN) respectively are placed over an inclined plane of inclination θ. Upper block A is tied by a string parallel to plane. If (μ=1/3) is the coefficient of friction for all contact surfaces, then find θ for the downward impending motion of block B.



<u>Solution:-</u>



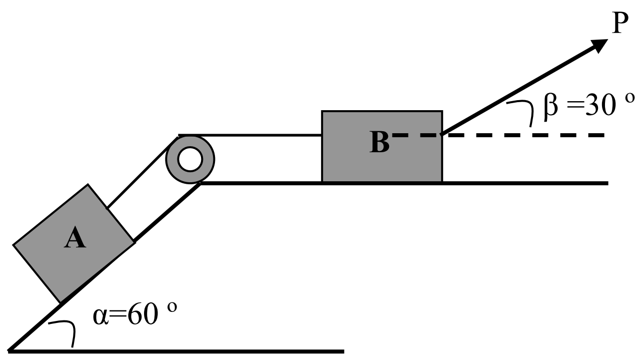
$$N_1 = W_1 \cos\theta$$
$$F_1 = \mu * N_1 = \mu * W_1 \cos\theta$$
$$T = F_1 + W_1 \sin\theta$$



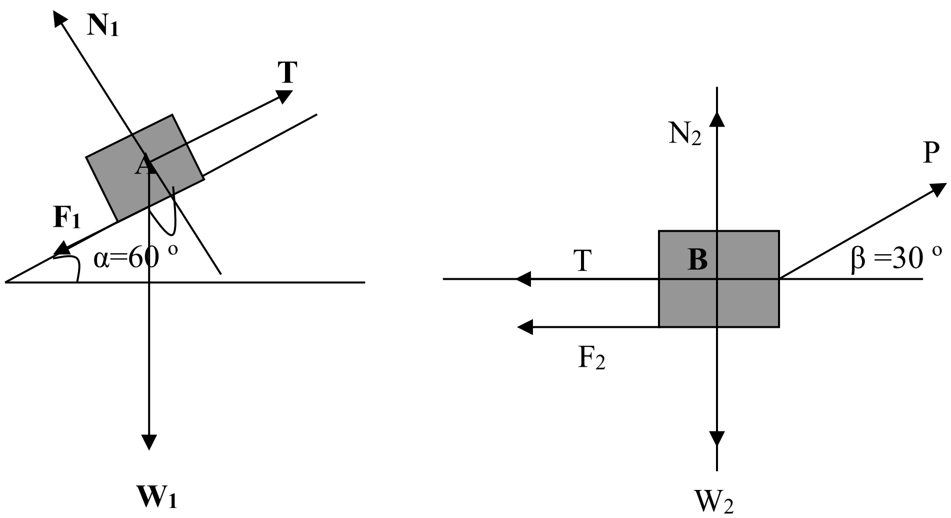
$$N_2 = N_1 + W_2 \cos\theta$$
$$F_2 = \mu * N_2 = \mu * (W_1 \cos\theta + W_2 \cos\theta)$$
$$F_1 + F_2 = W_2 \sin\theta$$
$$\mu * W_1 \cos\theta + \mu(W_1 \cos\theta + W_2 \cos\theta) = W_2 \sin\theta$$
$$\frac{\mu * (2 * W_1 + W_2)}{W_2} = \tan\theta$$
$$\Rightarrow \tan\theta = \frac{(1/3) * (300 * 2 + 900)}{900}$$
$$= \frac{5}{9} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{9}\right) = 29.05^o$$

Example(6):-

Blocks A and B of weights (750 kN) and (500 kN) respectively are placed on the planes as shown in figure bellow. A force P is applied to pull A up the plane. The connecting string passes over a smooth pulley. If A and B bear the coefficient of friction (μ=1/5), then find P for the impending motion.



Solution:-



$$N_1 = W_1 \cos\alpha$$
$$F_1 = \mu * N_1 = \mu * W_1 \cos\alpha$$
$$T = F_1 + W_1 \sin\alpha$$
$$= \mu * W_1 \cos\alpha + W_1 \sin\alpha$$

$$P * \sin\beta + N_2 = W_2$$

$$F_2 = \mu * N_2$$

$$= \mu * (W_2 - P * \sin\beta)$$

$$T + F_2 = P * \cos\beta$$

$$\cos\beta * P = \left[\mu * W_1 * \cos\alpha + W_1 * \sin\alpha + \mu * (W_2 - P * \sin\beta)\right]$$

$$\Rightarrow P = \frac{W_1 * (\mu * \cos\alpha + \sin\alpha) + \mu * W_2}{\cos\beta + \mu * \sin\beta}$$

Taking

W_1=750 kN , W_2= 500 kN

α=60 º ، β= 30 º , μ=1/5

P=853.52 kN ⟹

Tutorial; QUIZ