Example 3: Given the following data. Find $y(0.23)$.

x	0.2	0.4	0.6	0.8	1.0
y	0.916	0.836	0.74	0.624	0.4

Solution:

By Gregory-Newton interpolation formula,

Since $h \neq 1 \Rightarrow$ Either we use the general formula or we can use the particular formula after rescaling the given points.

$x_o \neq 0 \Rightarrow$ Shifting is required.

x	$x_{rescaled}$	$x_{shifted}$	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0.2	1	0	0.916	- 0.08	- 0.016	- 0.004	- 0.084
0.4	2	1	0.836	- 0,096	- 0.02	- 0.088	
0.6	3	2	0.74	- 0.116	- 0.108		
0.8	4	3	0.624	- 0.224			
1.0	5	4	0.4				

$$y(x) = y(0) + x\Delta y_o + \frac{x(x-1)}{2!}\Delta^2 y_o + \frac{x(x-1)(x-2)}{3!}\Delta^3 y_o + \ldots\ldots.$$

$$y(x) = 0.916 + x(-0.08) + \frac{x(x-1)}{2}(-0.016) + \frac{x(x-1)(x-2)}{6}(-0.004) + \frac{x(x-1)(x-2)(x-3)}{24}(-0.084)$$

At $x_{old} = 0.23 \quad \Rightarrow \quad x_{new} = \frac{0.23}{0.2} - 1 = 0.15,$

$$y(x_{new}) = 0.916 + 0.15(-0.08) + \frac{0.15(0.15-1)}{2}(-0.016) + \frac{0.15(0.15-1)(0.15-2)}{6}(-0.004) +$$
$$+ \frac{0.15(0.15-1)(0.15-2)(0.15-3)}{24}(-0.084) = 0.907216.$$

$\therefore \quad y(0.23) = 0.907216.$

Interpolation with unequally spaced data

For unequally spaced data (h is different), the Lagrange interpolation polynomial may be used.

Example 1: (Final 2014) The accompanying table gives the velocity, of a moving body, at various times. Estimate the velocity at $t = 7$ s.

Time, t, s	1	2	3	8
Velocity, v, m/s	2	4.1	6.4	36.5

Solution:

Since h is different, we use Lagrange interpolation polynomial.

$$v(t)=\frac{(t-t_1)(t-t_2)\ldots(t-t_n)}{(t_o-t_1)(t_o-t_2)\ldots(t_o-t_n)}v(t_o)+\frac{(t-t_o)(t-t_2)\ldots(t-t_n)}{(t_1-t_o)(t_1-t_2)\ldots(t_1-t_n)}v(t_1)+\ldots$$

$$v(t)=\frac{(t-2)(t-3)(t-8)}{(1-2)(1-3)(1-8)}(2)+\frac{(t-1)(t-3)(t-8)}{(2-1)(2-3)(2-8)}(4.1)+\frac{(t-1)(t-2)(t-8)}{(3-1)(3-2)(3-8)}(6.4)+$$
$$+\frac{(t-1)(t-2)(t-3)}{(8-1)(8-2)(8-3)}(36.5).$$

$$\therefore \quad v(7)=\frac{(7-2)(7-3)(7-8)}{(1-2)(1-3)(1-8)}(2)+\frac{(7-1)(7-3)(7-8)}{(2-1)(2-3)(2-8)}(4.1)+$$
$$+\frac{(7-1)(7-2)(7-8)}{(3-1)(3-2)(3-8)}(6.4)+\frac{(7-1)(7-2)(7-3)}{(8-1)(8-2)(8-3)}(36.5)=26.5 \text{ m/s}.$$

Example 2: (Final 2015) The ratio of the work done in a project, as a function of time, is found as below. Estimate this ratio at $t = 2$ month.

Time, t, (month)	3	4	5
Work, W, (%)	5	14	37

Solution:

Since $h=1 \Rightarrow$ We can use the particular Gregory-Newton interpolation formula directly without rescaling.

$t_o \neq 0 \Rightarrow$ Shifting is required.

t	$t_{shifted}$	W	ΔW	$\Delta^2 W$
3	0	5	9	14
4	1	14	23	
5	2	37		

$$W(t) = W(0) + t\Delta W_o + \frac{t(t-1)}{2!}\Delta^2 W_o + ...$$

At $t_{old} = 2 \quad \Rightarrow \quad t_{new} = 2 - 3 = -1$,

$$W(t_{new}) = 5 + (-1)(9) + \frac{-1(-1-1)}{2}(14) = 10\% \qquad \text{Not O.k. .}$$

If a function cannot be well approximated by a polynomial, a useful device can be adopted by plotting a (log – log) graph. This reduces a large variety of functions to essentially straight lines or to smooth curves which are easy to interpolate.

$\therefore$ Use a (log – log) graph ,

$t^* = \ln t$	1.099	1.386	1.609
$W^* = \ln W$	1.609	2.639	3.611

Now, since *h* is different, we use Lagrange interpolation polynomial.

$$W^*(t^*) = \frac{(t^* - t^*_1)(t^* - t^*_2)\ldots(t^* - t^*_n)}{(t^*_o - t^*_1)(t^*_o - t^*_2)\ldots(t^*_o - t^*_n)} W^*(t^*_o) + \frac{(t^* - t^*_o)(t^* - t^*_2)\ldots(t^* - t^*_n)}{(t^*_1 - t^*_o)(t^*_1 - t^*_2)\ldots(t^*_1 - t^*_n)} W^*(t^*_1) + \ldots$$

$$W^*(t^*) = \frac{(t^* - 1.386)(t^* - 1.609)}{(1.099 - 1.386)(1.099 - 1.609)}(1.609) + \frac{(t^* - 1.099)(t^* - 1.609)}{(1.386 - 1.099)(1.386 - 1.609)}(2.639) + \frac{(t^* - 1.099)(t^* - 1.386)}{(1.609 - 1.099)(1.609 - 1.386)}(3.611).$$

At $t = 2 \quad \Rightarrow \quad t^* = \ln 2 = 0.693$,

$$W^*(t^*) = \frac{(0.693 - 1.386)(0.693 - 1.609)}{(1.099 - 1.386)(1.099 - 1.609)}(1.609) + \frac{(0.693 - 1.099)(0.693 - 1.609)}{(1.386 - 1.099)(1.386 - 1.609)}(2.639) + \frac{(0.693 - 1.099)(0.693 - 1.386)}{(1.609 - 1.099)(1.609 - 1.386)}(3.611) = 0.576664.$$

But $W^* = \ln W \quad \Rightarrow \quad W = e^{W^*} = e^{0.576664} = 1.78$.

$\therefore \quad W(2) = 1.78\%$.