Engineering Analysis

Syllabus

1- Introduction.
2- First Order Ordinary Differential Equations.
3- Applications on First Order Ordinary Differential Equations.
4- Second and Higher Order Ordinary Differential Equations.
5- Applications on Second and Higher Order Ordinary Differential Equations.
6- Simultaneous Linear Ordinary Differential Equations.
7- Fourier Series.
8- Partial Differential Equations.
9- Matrices and Determinants.

References

- Advanced Engineering Mathematics,
  by C. R. Wylie.

- Advanced Engineering Mathematics,
  by E. Kreyszig.

- Advanced Engineering Mathematics,
  by O'Neil.

- Advanced Mathematics for Engineers and Scientists,
  by M. R. Spiegel.

- Differential Equations,
  by F. Ayres.

1- Introduction

Definition of differential equations

A differential equation is an equation that contains one or more derivatives, such as

$$\frac{dy}{dx} = \cos x \qquad \text{and} \qquad y'' + y' - \ln x = 0 .$$

Classification of differential equations

A) By type:

* *Ordinary differential equation* (ODE): in which all derivatives are with respect to a single independent variable, such as

$$\frac{dy}{dx} + \ln x = x , \qquad dy + xdx = 0 , \qquad \text{and} \qquad \frac{dy}{dx} + \frac{dz}{dx} = 0 .$$

* *Partial differential equation* (PDE): in which at least one derivative is with respect to two or more independent variables, such as

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial y} = x \qquad \text{and} \qquad \frac{\partial w}{\partial x} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} .$$

B) By order:

The order of the differential equation is the order of the highest derivative appears in that equation, for example

$\left(\frac{dy}{dx}\right)^2 + \sin x = 0$ is a first-order ordinary differential equation (1st order ODE).

$\frac{\partial^3 u}{\partial x^3} + \frac{\partial u}{\partial y} = 0$ is a third-order partial differential equation (3rd order PDE).

C) By degree:

The degree of the differential equation is the power of the highest derivative appears in that equation, for example

$\left(\frac{dy}{dx}\right)^2 + \sin x = 0$ is a 2nd degree, 1st order ODE.

$y'' + y' - y^2 = e^x$ is a 1st degree, 2nd order ODE.

$(y''')^2 + 2(y')^4 = 2x$ is a 2nd degree, 3rd order ODE.

D) By linearity:

A differential equation is said to be "linear DE" if and only if each term of the equation which contains a dependent variable and\or its derivative is of linear form. In another words a differential equation is said to be "linear DE" if:

1- The dependent variable appears in a linear form.
2- All derivatives appear in a linear form.
3- There is no production of a dependent variable with one of its derivatives, or one of its derivatives with another derivative.

For example

$y''' + 2y' + y = x^2$ is a linear 1st degree, 3rd order ODE.

$\frac{dy}{dx} + y^2 = 1$ is a non-linear 1st degree, 1st order ODE.

$\frac{d^2y}{dx^2} + \sin y = 0$ is a non-linear 1st degree, 2nd order ODE.

$y^{iv} + (y')^2 = x$ is a non-linear 1st degree, 4th order ODE.

$\frac{\partial^3 u}{\partial x^3} = u.\frac{\partial u}{\partial y}$ is a non-linear 1st degree, 3rd order PDE.

$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}.\frac{dy}{dx} = 0$ is a non-linear 1st degree, 4th order ODE.

Solution of differential equations

The solution of a DE is a relation between the variables which is free of derivatives and satisfies that DE identically.

* *General solution*: The general solution of the n^{th} order DE is a relation between the variables involving n independent arbitrary constants which satisfy the DE. For example

For the DE $\dfrac{d^3y}{dx^3}=0$,

$y_1 = A$ is a solution to the above DE.

$y_2 = Bx$ is also a solution.

$y_3 = Cx^2$ is also a solution.

$\therefore\ y = y_1 + y_2 + y_3 = A + Bx + Cx^2$ is a general solution (G.S).

* *Particular solution*: The particular solution of a DE is one obtained from the general solution of that DE by assigning specific values to the arbitrary constants for example

For the DE $\dfrac{d^2y}{dx^2}=0$,

$y = A + Bx$ is a general solution (G.S) to the above DE.

$\therefore\ y = 2 + 3x$ is a particular solution (P.S) to the above DE.

Origin of differential equations

* *Geometric problems*. For example

If we want to find the family of curves which have a value equal to its slope then we must solve the DE $y = \dfrac{dy}{dx}$.

* *Physical problems*. For example

$\sum F = m.\dfrac{d^2x}{dt^2}$ (Newton's 2nd law)

$EI.y'' = -M$ (Flexural equation)