



**University of Basrah**  
**College of Engineering**  
**Department of Electrical Engineering**



# Basics of Electrical Engineering II

*First year*

Lecturer: Dr. Basim T. Kadhem

**University of Basrah**  
**College of Engineering**  
**Department of Electrical Engineering**  
**Class: First year**  
**Subject: Basic of Electrical Engg. I**  
**Lecturer: Dr.Basim Talib Kadhem**

**Theoretical: 3 hr/wk**  
**Tutorial: 1 hr/wk**  
**Practical:- -- hr/wk**

## **Content**

Chapter Four: Analysis of single phase  
A.C. Circuits

Chapter Five: Resonance

Chapter Six: Magnetic Circuit

Chapter Seven: Electrostatics and Capacitance

# Chapter Four

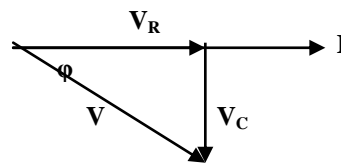
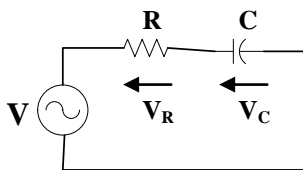
## Analysis of single phase A.C. Circuits

### 4.1 Introduction:

In chapter three the effect of resistance, inductance or capacitance had been taken into account alone in each case. In this chapter these components will be taken together. In order to limit this introductory study, only in those circuits the voltage and current waveforms are sinusoidal will be considered.

### 4.2 Series AC circuits:

#### 1- RC Series circuits:

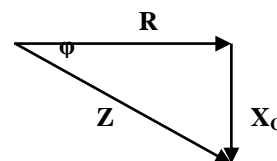


$$V = \sqrt{V_R^2 + V_C^2} \quad , \quad \vec{V} = \vec{V}_R + \vec{V}_C$$

$$V = V_R - jV_C$$

Where  $V_R = I R$  ,  $V_C = I X_C = I / (2\pi f C)$

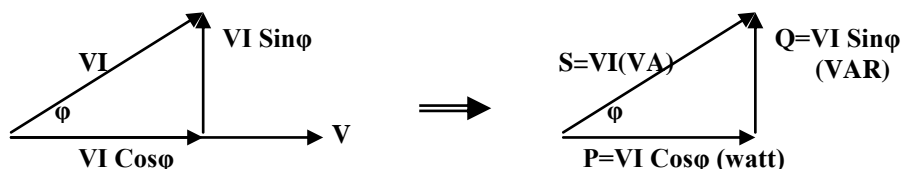
$$\tan \phi = V_C / V_R$$



$$Z = \sqrt{R^2 + X_C^2} \quad \Omega$$

$$\tan \phi = \frac{X_C}{R} \quad , \quad Z = R - jX_C$$

$\cos \phi =$  power factor



$$\tan \phi = Q/P$$

$$S = P + jQ$$

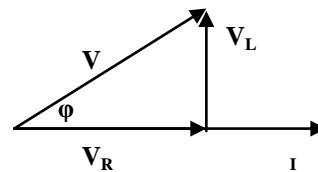
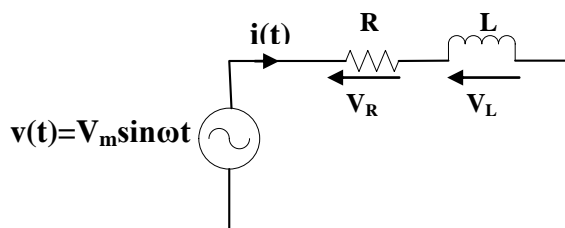
Where  $S = VI$  : in AC circuit is called the apparent power and represented by the symbol S (Volt-Amp.)

$P = VI \cos \phi$  : active power or real power

$$= I^2 R = V^2/R \quad (\text{watt})$$

$Q = VI \sin \phi$  : reactive power (VAR)

## 2- R-L series circuit:

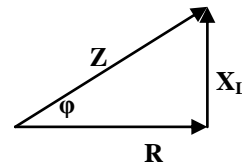


$V$  = r.m.s value of the applied voltage.

$I$  = r.m.s value of the circuit current.

$$V_R = I R$$

$$V_L = I X_L$$



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\tan \phi = \frac{V_L}{V_R}$$

$$V = V_R + jV_L$$

$\cos \phi = \text{P.F.}$

$$Z = R + jX_L$$

$$[Z] = \sqrt{R^2 + X_L^2}$$

$$i(t) = I_m \sin \omega t$$

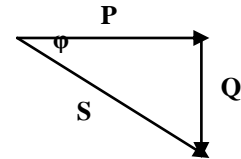
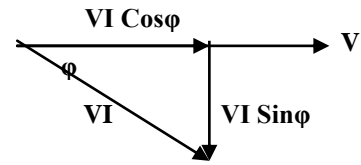
$$V(t) = V_m \sin(\omega t + \phi)$$

$$P(t) = i(t) * V(t) = I_m \sin \omega t * V_m \sin(\omega t + \phi)$$

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} V_m I_m \sin \omega t * \sin(\omega t + \phi) d\omega t$$

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} (\cos \omega t - \cos(2\omega t + \phi)) d\omega t$$

$$P_{av} = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi = VI \cos \phi$$

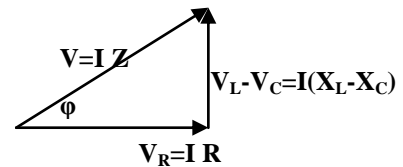
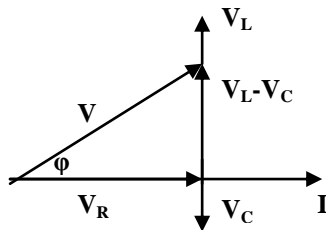
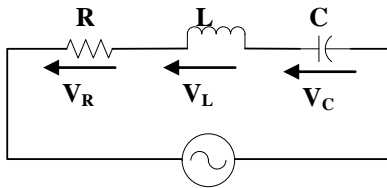


$$S = P - jQ = VI$$

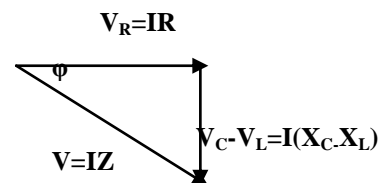
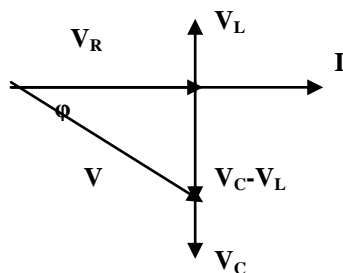
$$Q = VI \sin \phi = \text{reactive power (VAR)}.$$

### 3- RLC Series Circuits:

For lagging power factor :

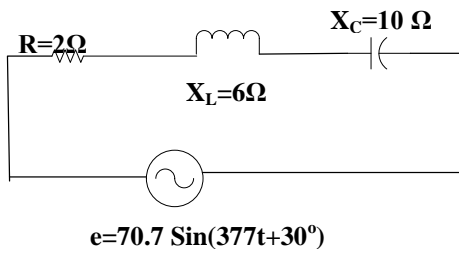


For leading power factor:



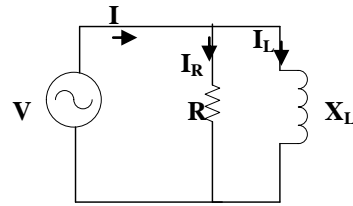
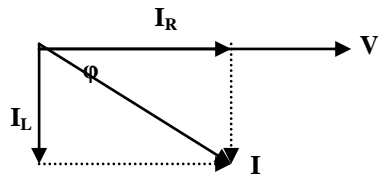
Example: For the circuit shown below, find:

- a- Z Total
- b- Draw the impedance diagram.
- c- Find C and L
- d- I,  $V_R$ ,  $V_L$ , and  $V_C$
- e-  $P_{av.}$
- f- Power factor



### 4.3 Parallel AC circuit:

#### 1- Parallel RL circuit:



$$I_R = V/R, \quad I_L = V/X_L, \quad I = I_R - jI_L$$

$$I = \sqrt{I_R^2 + I_L^2} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L}\right)^2} = V \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L}\right)^2} = VY$$

where Y is the admittance of the above circuit.

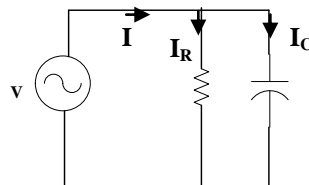
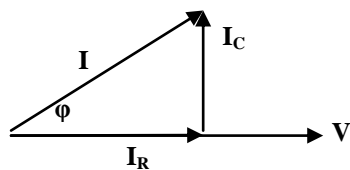
$$Y = \frac{1}{z} = G - jB$$

G: conductance and B: susceptance.

$$Z = R // jX_L = \frac{R * jX_L}{R + jX_L} * \frac{R - jX_L}{R - jX_L} = \frac{X_L^2 R}{R^2 + X_L^2} + j \frac{R^2 X_L}{R^2 + X_L^2}$$

= real + Im ajenary

#### 2- Parallel RC circuit:



$$I_R = V/R, \quad I_C = V/X_C, \quad I = I_R + jI_C$$

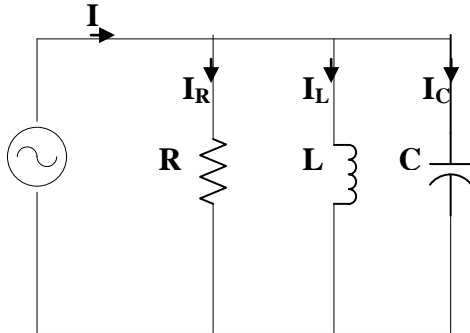
$$I = \sqrt{I_R^2 + I_C^2} = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_C}\right)^2} = V \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_C}\right)^2} = VY$$

where Y is the admittance of the above circuit.

$$Y = \frac{1}{z} = G + jB$$

$$Z = R // jX_C \quad , \quad I_R = I * \frac{-jX_C}{R - jX_C}$$

3- RLC circuit:

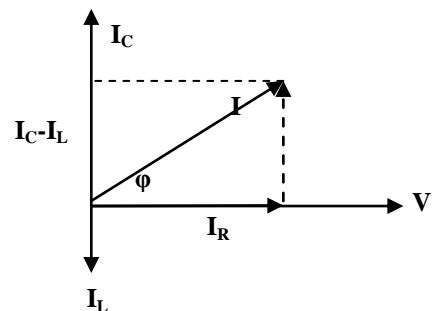


$$I = I_R + j(I_C - I_L)$$

$$\frac{V}{Z} = \frac{V}{R} + j\left(\frac{V}{X_C} - \frac{V}{X_L}\right)$$

$$\frac{1}{Z} = Y = G + j(B_C - B_L)$$

$$\phi = \tan^{-1} \frac{I_C - I_L}{I_R} = \tan^{-1} \frac{B_C - B_L}{G}$$



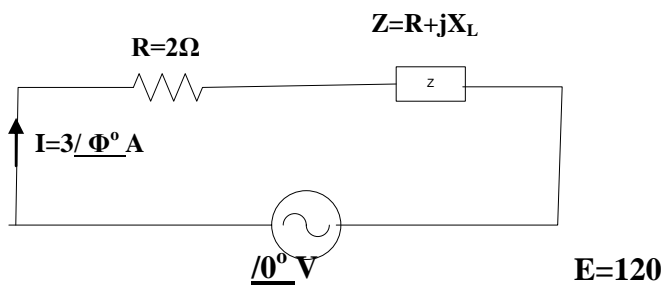
Example: Find the series element or elements that must be connected in series

with  $2\Omega$  resistor to satisfy the following conditions:

a- Average power to the circuit = 300 w.

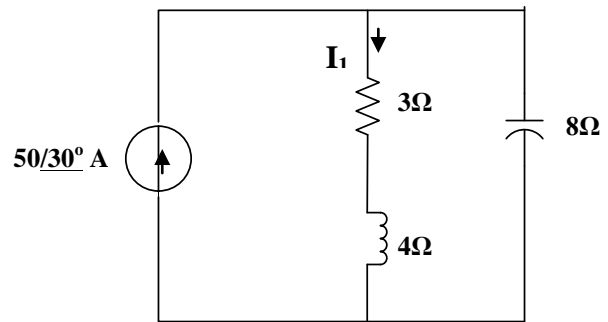


b- Circuit has a lagging power factor.

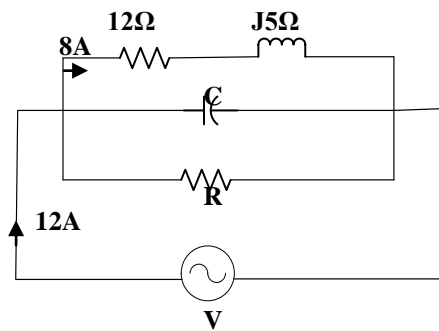


Example: Find the total admittance and impedance of the RLC parallel circuit, with (3, 6, 9)  $\Omega$  respectively and identify the values of conductance and susceptance.

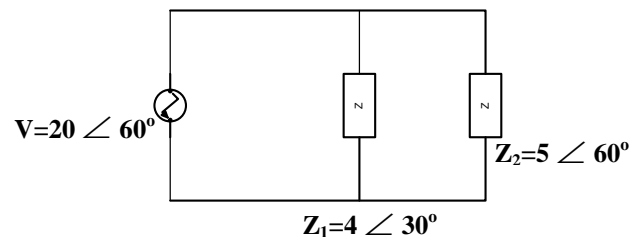
Example: For the circuit shown below, find  $I_1$ .



Example: For the circuit given below find C, R and V if the total current leads the supply voltage by  $30^\circ$ .



Example: Determine the power triangles for each branch of the circuit shown below:



Example:















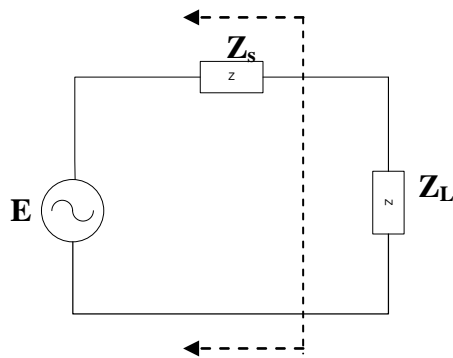
#### 4.4 Maximum power transfer:

$$Z_L = R + jX$$

$$Z_s = R_s + jX_s$$

$$I = \frac{E}{Z_s + Z_L} = \frac{E}{R + R_s + j(X + X_s)}$$

$$I = \frac{E}{[(R + R_s)^2 + j(X + X_s)^2]^{0.5}}$$



**Practical voltage source or Thevenin's equivalent.**

$$P_L = I^2 R = \frac{E^2 R}{(R + R_s)^2 + j(X + X_s)^2}$$

$P_L$  is a function of two variables  $R$  and  $X$ .

For maximum value of  $P_L$  :

$$\frac{dP_L}{dX} = 0 \quad \text{and} \quad \frac{\partial P_L}{\partial R} = 0$$

$$\frac{\partial P_L}{\partial X} = \frac{-E^2 R [2(X + X_s)]}{[(R + R_s)^2 + (X + X_s)^2]^2} = 0$$

$$X + X_s = 0 \quad X = -X_s$$

$$\frac{\partial P_L}{\partial R} = \frac{E^2[\{(R + R_s)^2 + (X + X_s)^2\} - R * 2(R + R_s)]}{[(R + R_s)^2 + (X + X_s)^2]^2} = 0$$

$$(R + R_s)^2 + (X + X_s)^2 = 2R(R + R_s)$$

$$R^2 + 2RR_s + R_s^2 + (-X_s + X_s)^2 = 2R^2 + 2RR_s$$

$$R^2 = R_s^2 \qquad R = R_s$$

Thus to produce maximum power in the load (maximum power transfer) the load impedance must be:

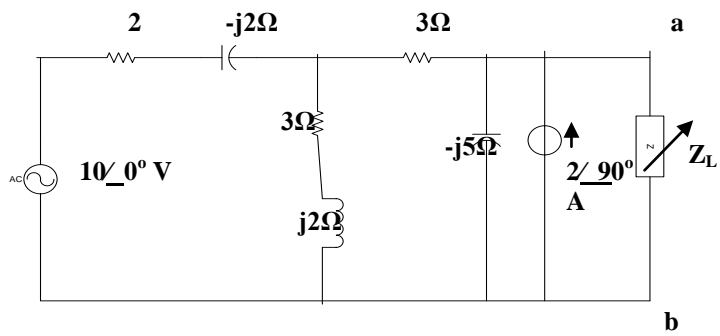
$$Z_L = R + jX = R_s - jX_s = Z_s^*$$

This means that the load impedance must be the complex conjugate of the source impedance.

$$I = \frac{E}{Z_s + Z_L} = \frac{E}{R_s + jX_s + R_s - jX_s} = \frac{E}{2R_s} = \frac{E}{2R}$$

$$P_{L\max} = I^2 R = \left(\frac{E}{2R}\right)^2 * R = \frac{E^2}{4R} = \frac{E^2}{4R_s}$$

Example: For the circuit shown below, calculate the max. power that can be transferred to  $Z_L$ .



#### 4.5 Power factor correction:

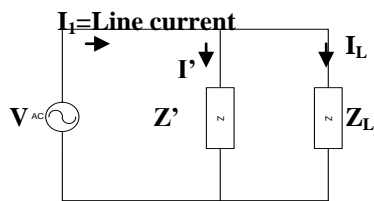
Most industrial and domestic electrical loads operate at a required fixed amount of real power  $P$ . The power factor thus becomes essentially important.

$$P = V I \cos\phi$$

$$I = \text{load current} = \frac{P}{V \cos\phi} = \frac{P}{V * pf}$$

From this equation it is clear that for constant  $P$  and  $V$ , the load current  $I$  can be reduced through increasing of  $\cos\phi$ .

To correct (or improve) the power factor, without affecting the real power in the load impedance  $Z_L$ , an impedance  $Z'$  is connected across the load impedance.



$Z'$  is a reactive impedance  
(pure L or pure C)

$$I_1 = I + I' = \frac{P}{V \cos\phi_1} = \frac{P}{V}$$

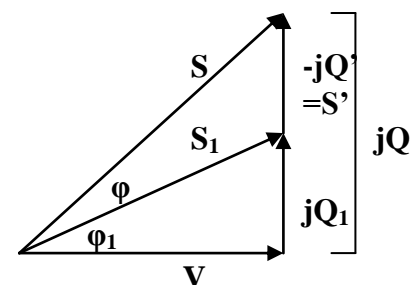
When  $\cos\phi_1$  approaches 1

This means that the angle  $\phi_1$  approaches zero.

The complex power of the overall system  $S_1$ .

$$S_1 = S + S'$$

$$= P + jQ + (-jQ') = P + j(Q + Q') = P + jQ_1$$



If  $S'$  is negative ( $S' = -jQ'$ ) then  $Z'$  must be capacitance, where:

$$Q' = \frac{V'}{X'_C} = V'^2 \omega C'$$

If  $S'$  is positive ( $S' = jQ'$ ) then  $Z'$  must be inductance, where:

$$Q' = \frac{V'}{X'_L} = \frac{V'}{\omega L'}$$

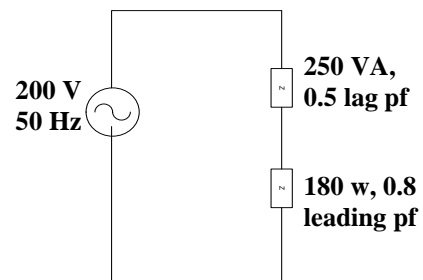
Example: A load with 0.8 lagging PF absorbs 60 W from a 100 V, 60 Hz power line. It is required to correct the power factor to 0.9 lagging.

Find:

- a)- The original and the final line current.
- b)- The value of the element to be added to achieve the required pf correction.

Example: For the circuit shown:

- a)- find the overall complex power and pf.
- b)- find the value and nature of the element required to correct the overall pf to unity
- c)- what is the percentage reduction in the line current due to this correction?



# Chapter Five

## Resonance

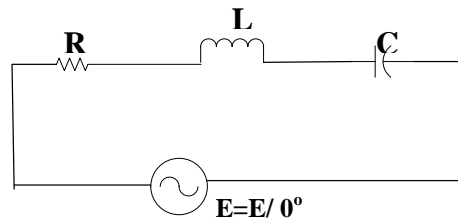
### 5.1 Introduction:

In the analysis of general a.c. circuit, one case was not completely argued, namely that in which the inductive reactance is equal to the capacitive reactance. When this occurs,  $V_L=V_C$  and consequently  $V$  and  $I$  are in phase with one another. This is an instance of a condition that is termed resonance.

### 5.2 Series resonance circuit:

The total impedance of this circuit at any frequency is:

$$Z_T=R+jX_L-jX_C$$



Series resonant will occur when

$$X_L=X_C$$

$Z_T=R$  which is the minimum value of the impedance.

$$X_L=X_C \quad , \quad \omega_s L=1/\omega_s C$$

$$\omega_s^2 = \frac{1}{LC} \quad , \quad \omega_s = \frac{1}{\sqrt{LC}}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

The quality factor  $Q=X_L/R$

To find  $Q$  at  $\omega_s$ :

$$\omega_s=2\pi f_s$$



$$Q_s = \omega_s L / R = (2\pi f_s L) / R = \frac{1}{2\pi\sqrt{LC}} (2\pi L) / R$$

$$Q_s = \frac{1}{R} \sqrt{\frac{L}{C}}$$

$$V_L = \frac{X_L E}{Z_T}$$

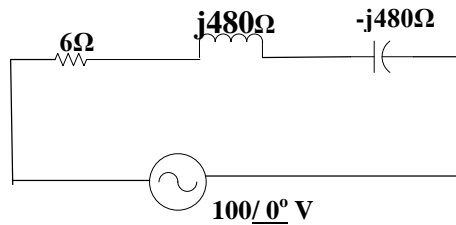
$$= \frac{X_L E}{R} \quad \text{at resonance.}$$

$$V_C = \frac{X_C E}{Z_T}$$

$$V_C = X_C \frac{E}{R} \quad \text{at resonance.}$$

$$X_L = X_C, \quad V_L = V_C = Q_s E$$

**Example:** For the circuit below, find  $Q_s$ ,  $V_C$  and  $V_L$  at resonance.

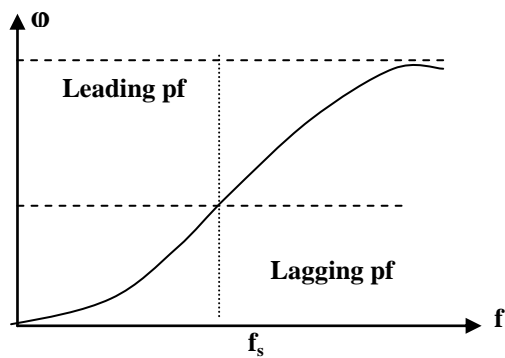
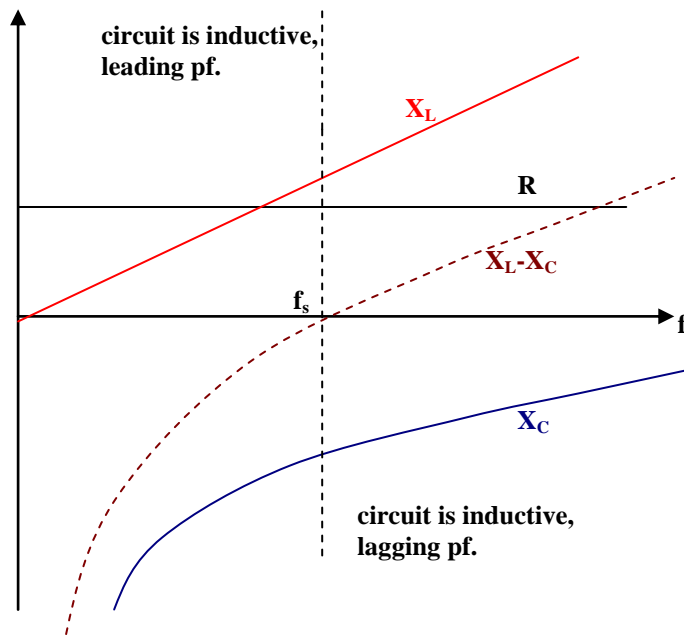


### 5.3 Versus frequency:

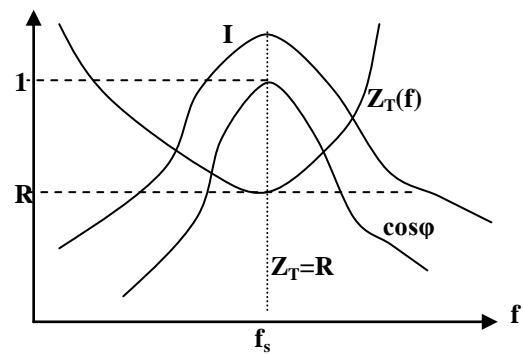
$$Z_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

$$|Z_T| = \sqrt{R^2 + (X_L - X_C)^2}$$

The total impedance versus frequency curve for the series resonant is:



Phase plot for the series resonant series Resonant circuit.



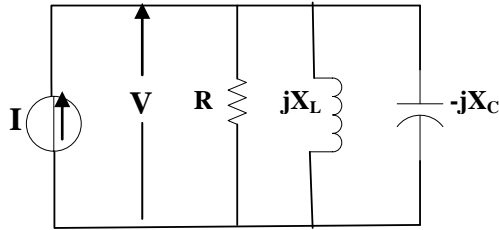
$Z_T, I, \text{pf}$  Vs frequency for the Resonant circuit.

## 5.4 Parallel resonant circuit:

$$Y_T = G + \frac{1}{j\omega L} + \frac{1}{-j1/\omega C}$$

$$= G + \frac{1}{j\omega L} + j\omega C$$

$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = |Y_T| \angle 0^\circ$$



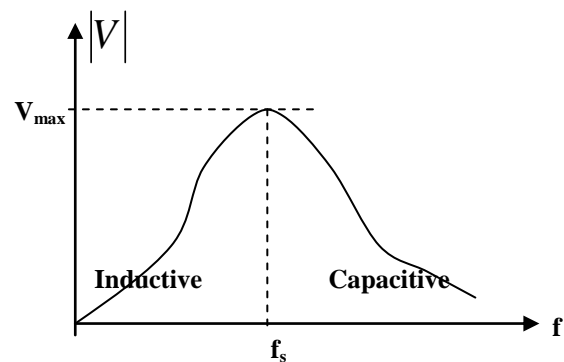
$$Y_T = \frac{1}{V} = \frac{1}{Z_T}$$

At resonance :  $-\frac{1}{j\omega L} + j\omega C = 0$

$$\omega_s^2 = \frac{1}{LC} \quad \cdot \quad \omega_s = \frac{1}{\sqrt{LC}} \text{ rad/sec.}$$

$$f_s = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \text{ Hz}$$

At  $f=f_s$  ;  $Z=R$  ;  $V_{\max}=IR$



The quality factor  $Q = R / X_L$

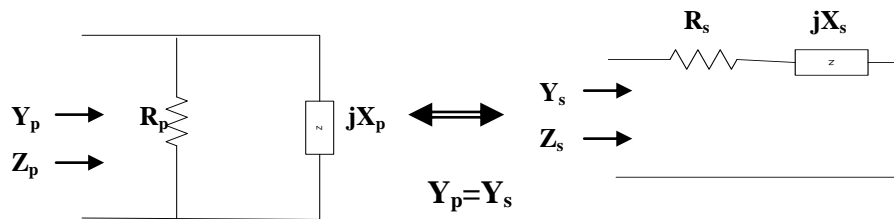
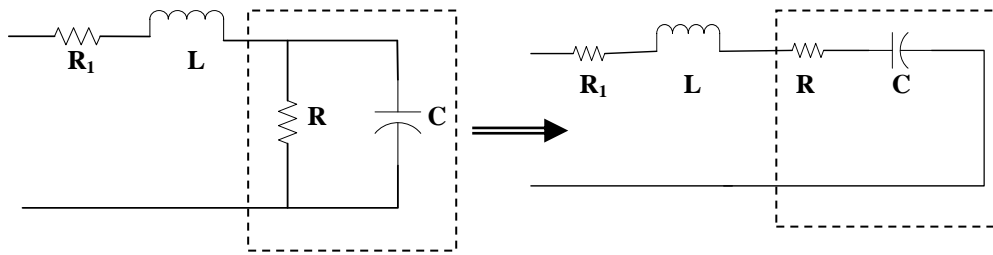
Example: In a parallel RLC circuit,  $R=250 \Omega$ ,  $L=2 \text{ mH}$ , and  $C=20 \mu\text{F}$ .

The circuit is driven by a  $0.1 \text{ A}$  current source operating at the current resonance frequency. Find:

a)-  $\omega_s$ ,  $Q$ .

b)- the voltage across the circuit and the current in each branch.

## 5.5 Resonance in series-parallel RLC circuit:



$$\frac{1}{R_p} + \frac{1}{jX_p} = \frac{1}{R_s + jX_s}$$

$$\frac{1}{R_p} - j \frac{1}{X_p} = \frac{R_s - jX_s}{R_s^2 + X_s^2}$$

$$R_p = \frac{R_s^2 + X_s^2}{R_s} \quad X_p = \frac{R_s^2 + X_s^2}{X_s}$$

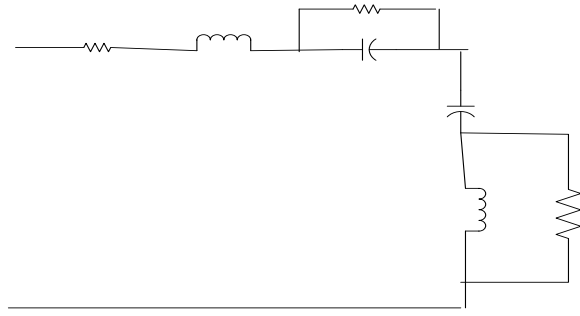
$$\frac{R_p}{X_p} = \frac{X_s}{R_s} \quad \text{i.e., } Q_p = Q_s = Q$$

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = R_s(1 + Q^2) \quad , \quad X_p = \frac{R_s^2 + X_s^2}{X_s} = X_s \left( \frac{1}{Q^2 + 1} \right)$$

$$R_p = R_s Q^2 \quad ; \quad R_s = R_p / Q^2$$

$$X_p \approx X_s \quad \text{i.e.} \quad L_p = L_s \quad \text{or} \quad C_p = C_s$$

Example: Find the approximate series resonance circuit.



Example:

# Chapter Six

## Magnetic Circuit

### **6.1 Introduction:**

Electromagnetism is the study of magnetic fields set up by the passage of electrical currents through a system of conductors.

**6.2 Magnetic Field:** The space surrounding a current carrying conductor in which the current effect can be observed is called the magnetic field of the conductor.

The force on the compass needle causing deflect is called the magnetic force. The relation between the direction of magnetic field and direction of current can be defined by the right-hand rule.

### **6.3 Laws of magnetic forces:**

- 1- Like poles repel each other but unlike poles attract each other.
- 2- Force between two magnetic poles is directly proportional to the product of their poles strengths and inversely proportional to the square of the distance between them.

$$F \propto \frac{m_1 m_2}{d^2} \quad \text{where } m_1 \text{ and } m_2 \text{ are poles strengths and } d \text{ is distance}$$

between the

two poles.

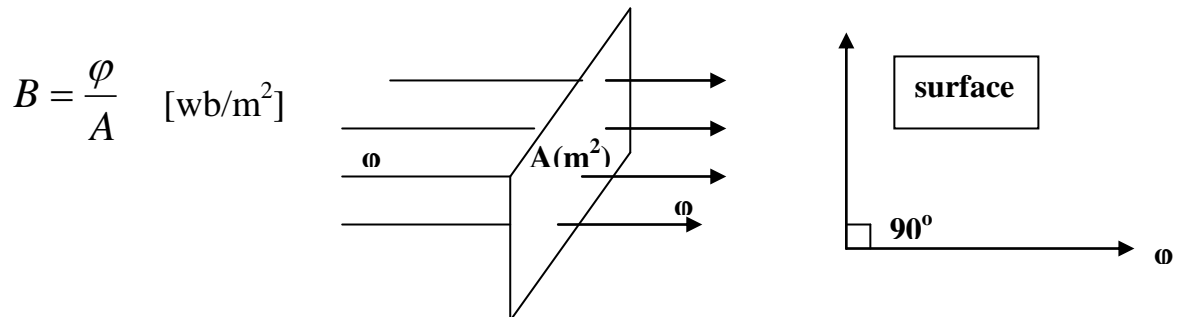
$$F = k \frac{m_1 m_2}{d^2} \quad \text{where } k \text{ is constant.}$$

### **6.4 Magnetic flux and flux density:**

The lines of magnetic field is termed as the magnetic flux. It has the symbols “ $\phi$ ” and the unit is (wb) where:

1 wb=10<sup>8</sup> lines.

The magnetic flux density is the flux that is passing uniformly and normally at right angle of any surface of area 'A' as shown below:



Where:

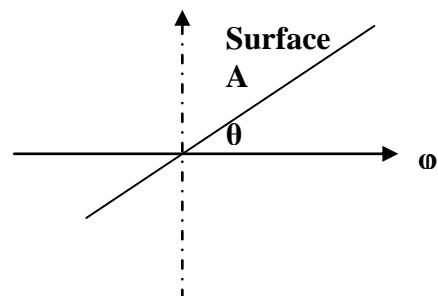
B= is the magnetic flux density [wb/m<sup>2</sup>] or [Tesla].

φ=magnetic flux [wb].

A=Area in [m<sup>2</sup>] normal to the flux.

If the plane is not taken at right angle to the direction of the field then:

$$\varphi = BA \sin \theta$$



Note: if the plane is parallel to the direction of the flux, no flux pass through the plane.

### **6.5 Force on current – carrying conductor :**

$$F=B l I \text{ [Newton]}$$

where:

F : Force in [N].

B: Flux density in [wb/m<sup>2</sup>].

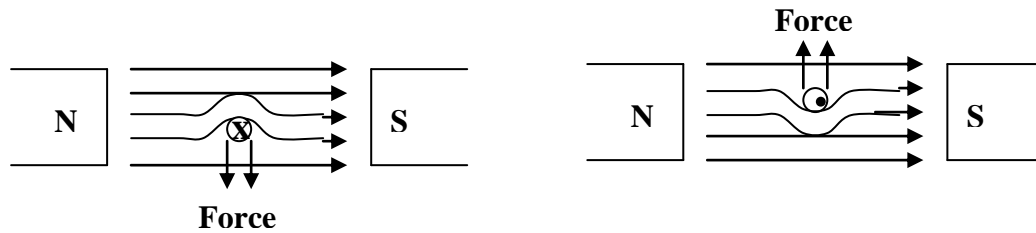
l : effective length of the conductor [m].

I : current passing through the conductor [A].

If the angle between the conductor and the field lines is θ then :



$$F = BIl \sin \theta \text{ [N]}$$



### **6.6 Faraday Laws :**

If a conductor is moved through a magnetic field so that it cuts magnetic lines of flux, a voltage will be induced across the conductor.

If a coil of N turns is placed in the region of a changing flux, a voltage will be induced across the coil as determined by Faraday's law:

$$e = N \frac{d\phi}{dt}$$

### **6.7 Lenz's Laws :**

The law states that the direction of induce emf of such that the current produced by it sets up a magnetic field opposing the motion of change producing it.

$$EMF \text{ induced} = -N \frac{d\phi}{dt} \quad [\text{volt}].$$

Example: A coil of resistance  $100\Omega$  is placed in a magnetic field of 1 mwb. The coil has 100 turns and a galvanometer of  $400\Omega$  resistance is connected in series with it. Find the average emf and current of the coil moved  $1/10$  sec from the given field to a field of a 0.2 mwb.

## 6.8 Magnetic field intensity or magnetizing force :

Magnetic field intensity at point in a magnetic field is the force acting on a unit pole (pole of 1 wb) placed at that point. It has a symbol (H) and unit of (N/wb).

H is a vector quantity.

## 6.9 Relation between B and H :

$$B/H = \text{constant} = \mu$$

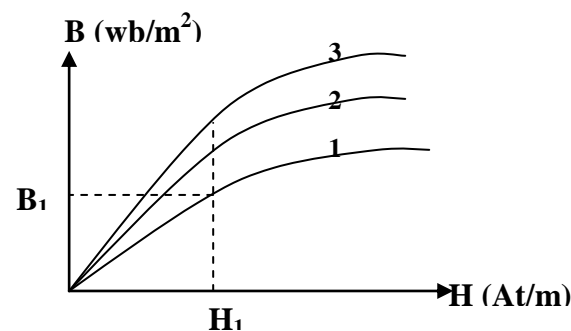
The ratio B/H in a material is always constant and is equal to the absolute permeability ( $\mu$ ) which is a measure of material conductivity for magnetic flux. Greater  $\mu$  means greater conductivity of flux. The air or vacuum is the poorest magnetic medium.

$$\mu = \mu_0 \mu_r$$

$\mu$  := Absolute permeability of the material.

$\mu_0$  := Absolute permeability of the air or vacuum =  $4\pi \times 10^{-7}$  [H/m]

$\mu_r$  := Relative permeability of the material = 1 for air or vacuum.



**B-H Curve for different materials.**

## 6.10 Reluctance :

The resistance for electrical circuit by the equation:

$$R = \rho \frac{l}{A} \quad [\Omega]$$

The reluctance of a material which allows magnetic flux lines to pass through material is determined by the following equation:

$$R = \frac{l}{\mu A} \quad [\text{At/wb}]$$

where:

R := the magnetic reluctance.

l := the length of the magnetic path.

A := the cross - sectional area of the magnetic path.

$\mu$  := the permeability.

### **6.11 Ohm's law for magnetic circuit:**

For electric circuit:

$$I = \frac{V}{R}$$

The Ohm's law for magnetic circuit:

$$\phi = \frac{F}{R}$$

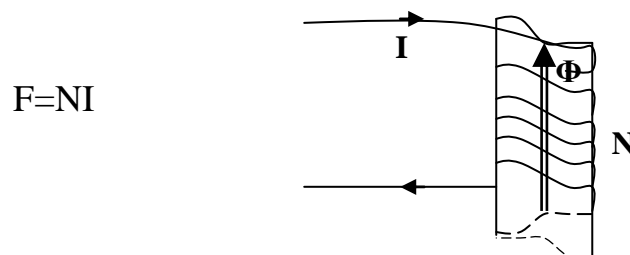
where:

F := the magneto-motive force.

$\Phi$  := the flux.

R := the reluctance.

The magneto-motive force is proportional to the product of the number of turns around the core and the current through the turns of wire.



### 6.12 Magnetizing force or magnetic field intensity:

The magnetomotive force per unit length is called the magnetizing force (H).

$$H = \frac{F}{l} = \frac{NI}{l} \quad (\text{At/m})$$

$$Hl = NI \quad (\text{work law})$$

### 6.13 Ampere's circuit law:

The similarity between the analysis of electric and magnetic circuits:

	Electric circuit	Magnetic circuit
Cause	E	F
Effect	I	$\Phi$
Opposition	R	R

$$\sum V = 0 \quad (\text{Kirchhoff's voltage law}).$$

$$\sum F = 0 \quad (\text{for magnetic circuit}).$$

$$F = NI \quad (\text{source of mmf}).$$

$$F = \Phi R \quad (\text{the mmf drop across a portion of a magnetic circuit}).$$

$$F = Hl$$

Consider the magnetic circuit shown in the Figure constructed from three different ferromagnetic materials.

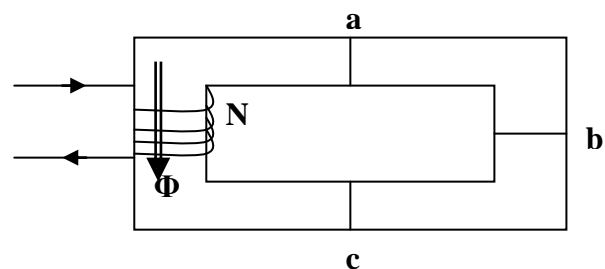
Applying Ampere's circuit law:

$$\sum F = 0$$

$$NI + H_{ac}l_{ac} + H_{cb}l_{cb} + H_{ba}l_{ba} = 0$$

$$\text{Rise} + \text{drop} + \text{drop} + \text{drop} = 0$$

$$NI - H_{ac}l_{ac} - H_{cb}l_{cb} - H_{ba}l_{ba} = 0$$

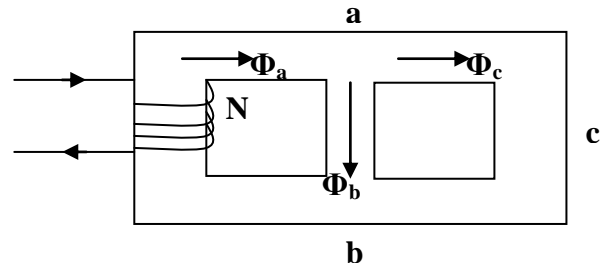


### 6.14 The flux $\Phi$ :

$$\Phi_a = \Phi_b + \Phi_c \quad (\text{at junction a})$$

$$\Phi_b + \Phi_c = \Phi_a \quad (\text{at junction b})$$

Similar to Kirchoff's current law in electric circuit.

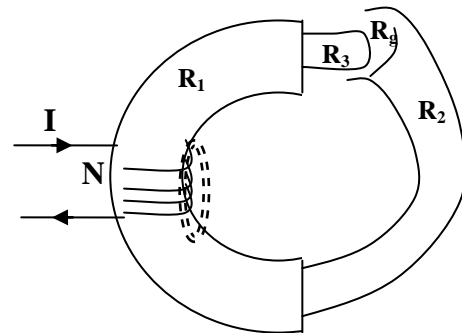


### 6.15 Series magnetic circuits:

In series magnetic circuits, same Flux flows through each part of The circuit.

$$R_T = R_1 + R_2 + R_3 + R_g$$

$$\text{Total mmf} = \Phi R_T$$



$$\text{Total mmf} = \frac{\phi l_1}{A_1 \mu_o \mu_{r1}} + \frac{\phi l_2}{A_2 \mu_o \mu_{r2}} + \frac{\phi l_3}{A_3 \mu_o \mu_{r3}} + \frac{\phi l_g}{A_g \mu_o \mu_{rg}}$$

$$= \frac{B_1 l_1}{\mu_o \mu_{r1}} + \frac{B_2 l_2}{\mu_o \mu_{r2}} + \frac{B_3 l_3}{\mu_o \mu_{r3}} + \frac{B_g l_g}{\mu_o \mu_{rg}}$$

$$= H_1 l_1 + H_2 l_2 + H_3 l_3 + H_g l_g$$

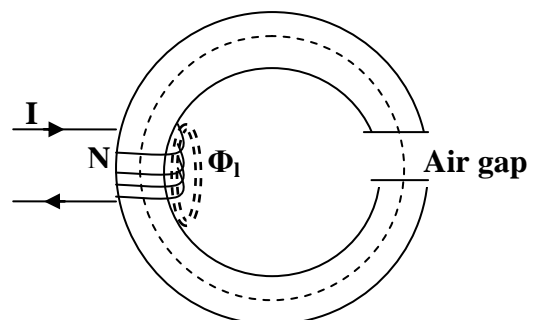
$$= (AT)_1 + (AT)_2 + (AT)_3 + (AT)_g$$

### 6.16 Leakage flux :

Let  $\Phi_i$  total flux produced in the iron.

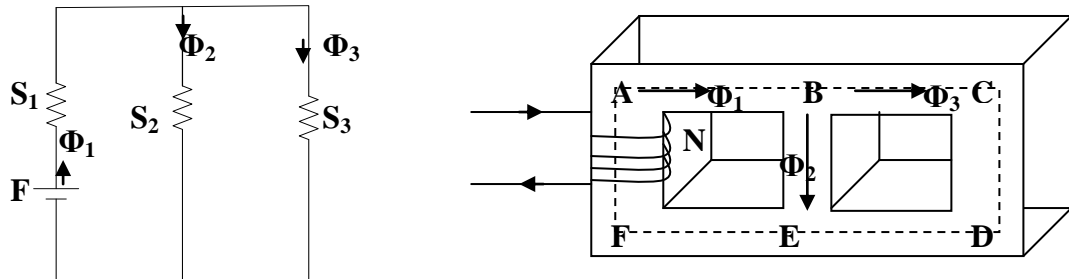
$\Phi_g$  useful flux across the air gap.

$$\text{Leakage flux} = \Phi_l = \Phi_i - \Phi_g$$



$$\text{Leakage coefficient } \lambda = \frac{\text{total flux}}{\text{useful flux}} = \frac{\phi_i}{\phi_g}$$

### 6.17 Parallel magnetic circuit :



A parallel magnetic circuit is that one which has more than one path for flux.

$$\Phi_1 = \Phi_2 + \Phi_3$$

$S_1$  := Reluctance of path EFAB.

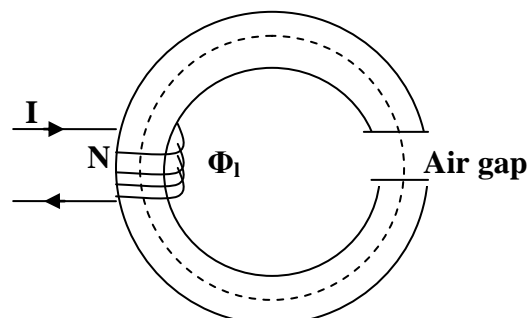
$S_2$  := Reluctance of path BE.

$S_3$  := Reluctance of path BCDE.

$$\text{Total HT} = F = \Phi_1 S_1 + \Phi_2 S_2 = \Phi_1 S_1 + \Phi_3 S_3$$

Example: An iron ring of cross-sectional area  $6 \text{ cm}^2$  is wound with a wire of 100 turns. The ring has an air gap of 2 mm. Calculate the magnetizing current required

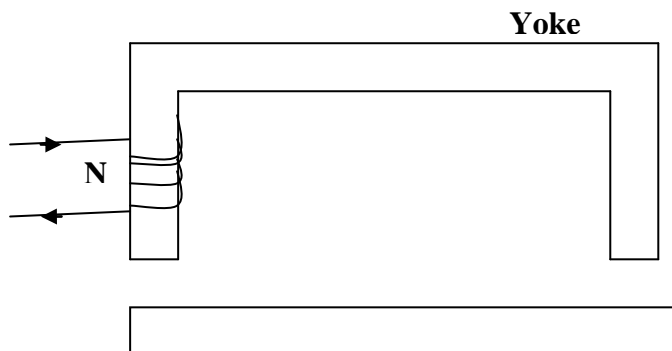
to produce a flux of 0.1 mwb. The mean length of magnetic path is 30 cm. the relative permeability of iron is 470.



Example: In the previous example if the flux in the gap is 0.1 mwb and the leakage factor =1.25, calculate the magnetizing current.

Example: The cross section of a simple relay is shown in figure, shown together with the characteristic of the yoke and armature material. Calculate the ampere-turns required on the coil for a flux density of 0.1 wb/m<sup>2</sup> and an airgap of 6.5 cm<sup>2</sup>.

H(AT/m)	0	100	200	300	400	500	600	700	800	900
B(wb/m <sup>2</sup> )	0	0.4	0.65	0.83	0.95	1.04	1.09	1.12	1.14	1.15



## 6.18 Self inductance :

The self inductance of a magnetic circuit can be derived in terms of the reluctance as follows:

$$B = \mu_o \mu_r H = \mu_o \mu_r \frac{NI}{l}$$
$$\phi = BA = \mu_o \mu_r \frac{NIA}{l} \quad \dots (1)$$

$$e = N \frac{d\phi}{dt} = \frac{dN\phi}{dt} = L \frac{di}{dt} = \frac{dLi}{dt}$$
$$N\phi = Li \quad , \quad L = \frac{N\phi}{I} \quad \dots (2)$$

From (1) in (2):

$$L = \frac{N}{I} \mu_o \mu_r \frac{NIA}{l} = N^2 * \frac{\mu_o \mu_r A}{l}$$

$$L = \frac{N^2}{S} \quad [\text{H}]$$

## 6.19 Self inductance of a long solenoid :

A long solenoid is a coil with length  $l$  and a diameter  $d$  and  $N$  turns.

The field strength is assumed to be uniform within  $H$  value.

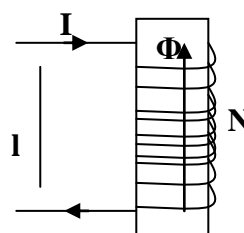
$$F = Hl = NI$$

$$H = \frac{NI}{l}$$

$$B = \mu_o \mu_r H = \mu_o \mu_r \frac{NI}{l}$$

$$\phi = BA = \mu_o \mu_r \frac{NIA}{l}$$

$$L = \frac{N\phi}{I} = \frac{N}{I} \mu_o \mu_r \frac{NIA}{l} = \frac{N^2}{S}$$



The solenoid is an important winding arrangement. It is found in relays, inductors, small transformers, etc.



Example: A solenoid 800mm long and 20mm in diameter is uniformly wound with a coil of 1000 turns. Determine the self inductance of the coil assuming that it is air-cored. Also determine the flux density within the solenoid when the coil current is 1 A.

### 6.20 Self inductance of a uniform wound toroid :

A toroid is a ring as shown in the figure.

Apply the work law to a path having a radius  $r$  and lying within the core.

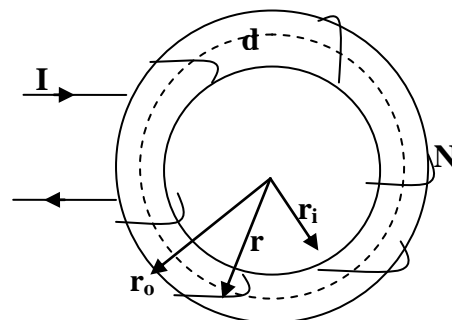
At every point on the path, the unit pole will experience a force  $H_r$ .

Total work done round closed path

=Ampere turns linked.

$$H_r l = NI$$

$$2\pi r H_r = NI$$



$$H_r = \frac{NI}{2\pi r}$$

At the inner edge, where  $r=r_i$ ,

$$H_i = \frac{NI}{2\pi r_i}$$

At the outer edge, where  $r=r_o$ ,

$$H_o = \frac{NI}{2\pi r_o}$$

The magnetic field strength decreases as the radius increases. If the toroid has a mean radius much greater than its cross-sectional thickness ( $r_o-r_i$ ), then  $H_i$  is approximately equal to  $H_o$ . The magnetic field strength is assumed uniform and equal to  $H$ .

$$H = \frac{NI}{2\pi r_{av}} \quad \text{where } r_{av} = \text{the main radius.}$$

$$B = \mu_o \mu_r H = \mu_o \mu_r \frac{NI}{2\pi r_{av}}$$

$$\Phi = BA = \mu_o \mu_r A \frac{NI}{2\pi r_{av}}$$

$$L = \frac{N\phi}{I} = N^2 * \frac{\mu_o \mu_r A}{2\pi r_{av}} = \frac{N^2}{S}$$

Example: A wooden toroid of mean diameter 400 mm and cross-sectional area  $400 \text{ mm}^2$  is uniformly wound with a coil of 1000 turns, which carries a current of 2 A.

Determine the self inductance of the coil and the emf induced in it when the current is uniformly reduced to zero in 10 msec.

## 6.21 Mutual inductance :

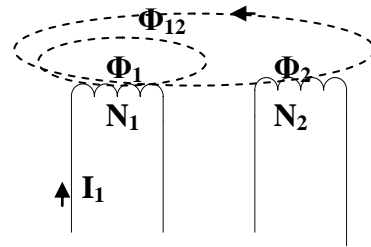
If two circuits are positioned in such a manner that the flux set up by one links the other circuit, then that flux which is common to both circuits is termed the mutual flux. If the current in the first circuit is varied, the mutual flux also varies and hence induces an emf in the second circuit. This effect is termed mutual

inductance.

$$e = M \frac{di}{dt}$$

where M= The mutual inductance.

The flux produced by  $I_1$  is  $\Phi_1$ . Part of this flux links  $N_2$  ( $k_{12} \Phi_1$ ).



$$L_1 = \frac{N_1 \phi_1}{I_1}$$

$$M_{12} = \frac{k_{12} N_2 \phi_1}{I_1}$$

$$F = \phi S = NI$$

$$\frac{\phi}{I} = \frac{N}{S}$$

$$\frac{k_{12}}{I_1} = \frac{N_1}{S}$$

$$M_{12} = \frac{N_1 N_2}{S} \quad , \quad \text{where } S \text{ is the reluctance of the common flux path.}$$

The flux produced by  $I_2$  in coil 2 is  $\Phi_2$ .

$$L_2 = \frac{N_2 \phi_2}{I_2}$$

$$M_{21} = \frac{k_{12} \phi_1 N_1}{I_2} = \frac{N_1 N_2}{S}$$

Therefore,  $M_{12} = M_{21} = M$

$$M^2 = M_{12} * M_{21}$$

$$= \frac{k_{12} N_2 \phi_1}{I_1} * \frac{k_{12} \phi_1 N_1}{I_2} = k_{12} k_{21} \frac{\phi_1 N_1}{I_1} \frac{\phi_2 N_2}{I_2}$$

$$= k_{12} k_{21} L_1 L_2 = k^2 L_1 L_2$$

$$M = k(L_1 L_2)^{1/2} \text{ (H)}$$

The coefficient of coupling k has a maximum value of unity when the entire self flux of each coil links the other.

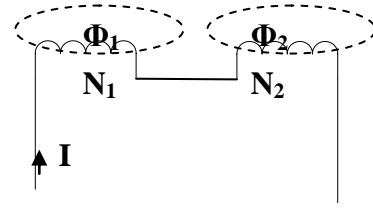
Example: A ferromagnetic ring of cross-sectional  $800 \text{ mm}^2$  and of mean radius  $170 \text{ mm}$  has two windings connected in series, one of 500 turns and one of 700 turns. If the relative permeability is 1200, calculate the self inductance of each coil and the mutual inductance of each assuming that there is no flux leakage.

## **6.22 EMF in two series – connected coils :**

The flux  $\Phi_2$  in the second coil (see the Figure) is in the same direction of  $\Phi_1$ .

The total flux is  $\Phi_1 + \Phi_2$  (series adding).

The two coils can be represented by dot notation as follows:



If the current is changing at the ratio  $di/dt$ , then the total induced emf is:

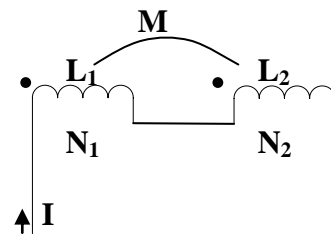
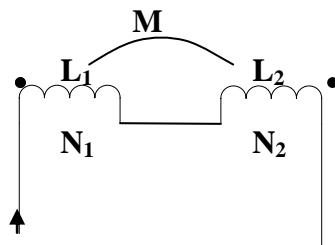
$$e = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + M \frac{di}{dt} + M \frac{di}{dt}$$

$$= (L_1 + L_2 + 2M) \frac{di}{dt} = L \frac{di}{dt}$$

$L = L_1 + L_2 + 2M$  = the effective inductance of the circuit.

If the flux  $\Phi_2$  is in opposite direction to  $\Phi_1$ , then the total flux is  $\Phi_1 - \Phi_2$

(series opposing).



$$e = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} - M \frac{di}{dt} - M \frac{di}{dt}$$

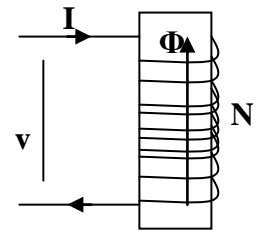
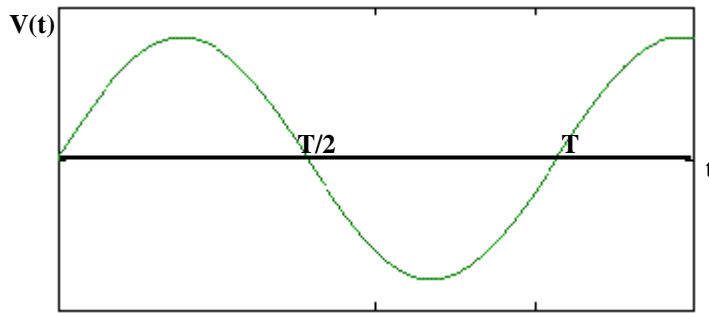
$$= (L_1 + L_2 - 2M) \frac{di}{dt} = L \frac{di}{dt}$$

$L = L_1 + L_2 - 2M$

Example: When two coils are connected in series, their effective inductance is found to be 10 H. However, when the connections to one

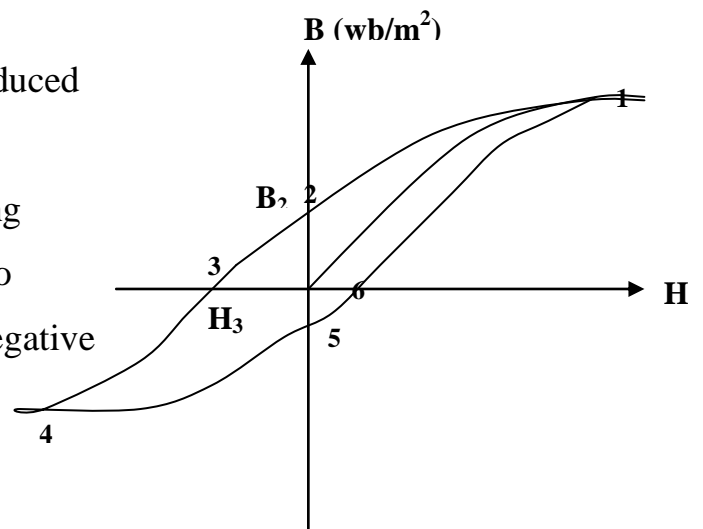
coil are reversed, the effective inductance is 6 H. If the coefficient of coupling is 0.6, calculate the self inductance of each coil and the mutual inductance.

### **6.23 Hysteresis loop :**



When coil shown supplied with alternating voltage, the B-H curve will be as follows:

When the magnetic field strength is reduced to zero, a flux will remain due to the reorientation of the domains. Reversing the field strength will reduce the flux to zero (point 3) and then to maximum negative (point 4). Each cycle of input current will give complete Hysteresis loop.



The area is a measure of the work that is done in taking the material through a cycle of magnetization. This work results in a loss of energy termed the hysteresis loss. This loss becomes apparent in the form of heat.

The hysteresis loss can be calculated as follows;

$$P_h = V * f * \text{area of loop}$$

Or

$$P_h = K_h f B_m^2 V$$

where:

$P_h$  = Hysteresis loss (w).

$V$  = Volume of the magnetic material.

$f$  = frequency.

$N$  = Steinmetz index (constant depending on the material used = 1.6 to 2.5)

$K_h$  = Hysteresis constant depending on the material used

= 100 to 200 for silicon steel.

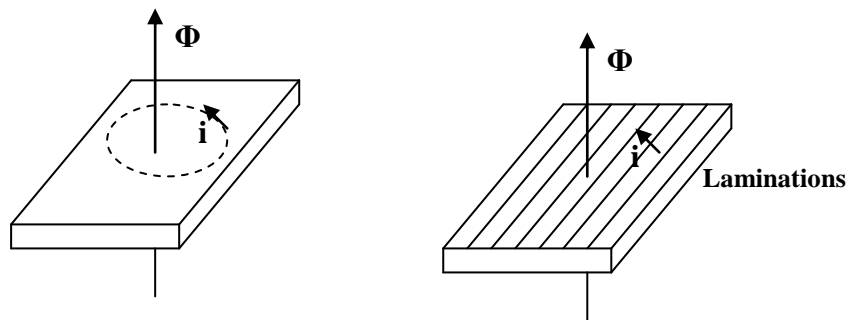
$B_m$  = maximum flux density.

Example: A sample of silicon steel has a hysteresis coefficient of 100 and a corresponding Steinmetz index of 1.6. Calculate the hysteresis power loss to  $10^6 \text{ mm}^3$  when the flux is alternating at 50 Hz, such that the maximum flux density is 2 T. Find also the area of the loop.

## **6.24 Eddy current loss :**



If a loop of conducting material is linked by a varying flux, an emf is induced in the loop and a circulating current will flow round the loop.



The eddy current give rise to a power loss in the resistance of the eddy current path. This loss is called the eddy current power loss.

$$\text{Let } \Phi = \Phi_m \sin \omega t = A B_m \sin \omega t$$

where A is the cross sectional area perpendicular to the direction of the field.

$$e \propto \frac{d\phi}{dt}$$

$$e \propto \frac{d(A B_m \sin \omega t)}{dt}$$

$$e \propto \omega A B_m \cos \omega t$$

$$E \propto \frac{\omega A B_m}{\sqrt{2} R} ;$$

$$I \propto \frac{\omega A B_m}{\sqrt{2} R} ; \quad I \propto \frac{f A B_m}{\sqrt{2} R}$$

where R= the effective resistance to the eddy current.

The eddy current power loss  $P_e$  is given by:

$$P_e = I^2 R$$

$$P_e \propto \frac{A^2 B_m^2 f^2}{R}$$

Since  $R = \rho l/A$  ;  $R \propto \rho$

$$P_e \propto \frac{A^2 B_m^2 f^2}{\rho} ; \quad P_e = \frac{K_e A^2 B_m^2 f^2}{R}$$

Therefore, to minimize the loss, only the cross-sectional area “A” and the resistivity can be varied. The net area of the core of a magnetic circuit can not be reduced since this is determined by the required flux and the maximum permissible

flux density, but the area can be divided into smaller sections. This is achieved by making the core of a number of thin sheets called laminations which are lightly insulated from one another. This reduces the area of each section and hence the induced emf. It also increases the resistance of the eddy current paths since the area through which the currents can pass is reduced. Further reduction of eddy current power loss can be obtained by using a material of high resistivity.

# Chapter Seven

## Electrostatics and Capacitance

**7.1: Properties of an electric field:** When a current flows at the rate of one ampere, the charge that passes through a cross sectional of the conductor during a period of one second is one coulomb. The coulomb is the unit of electric charge.

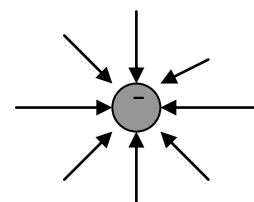
The space surrounding a charge can be investigated using a small charge body. This investigation is similar to that applied to the magnetic field surrounding a current carrying conductor.

As in the magnetic case, the lines of force can be traced out. These lines are again given certain properties:

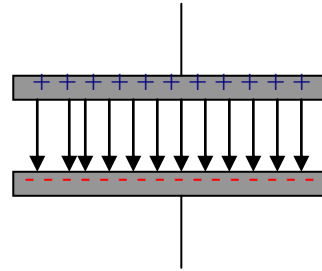
- 1- In an electric field, each line of force will emanate from or terminate in a charge. Usually it will lead from a positive charge to a negative charge.
- 2- The direction of the line is that experienced by a positive charge placed at a point in the electric field. It is assumed that the search charge has no effect on the field direction.
- 3- The lines of force never intersect since the resultant force at any point in an electric field can have only one direction.

It should be noticed that whilst it is possible to observe the electric force acting on a small charged body in principle, it is extremely difficult to obtain experimental verification of the field distribution and indirect methods have to be used.

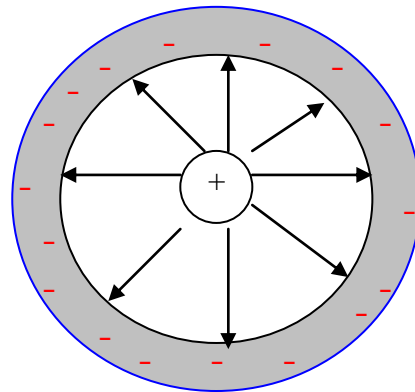
Electric field about an isolated spherical charge.



Parallel Plates



Concentric cylinders (cables)



**7.2 Electric flux and flux density:** The total electric effect of a system as described by the lines of electric force is termed the electric flux linking the system. The unit of electric flux is the coulomb.

Electric Flux  $Q$  (C).

Electric flux density  $D$  ( $C/m^2$ )

$$D = \frac{Q}{A}$$

**7.3 Permittivity:** The flux density may be considered to result from the electric field strength. For any given value of electric flux strength  $E$ , the value of the resulting flux density  $D$  depends on the medium in which the flux is produced.

The ratio of  $D$  to  $E$  is termed the absolute permittivity of the medium.

Absolute permittivity  $\epsilon$  (Farad/m)

$$\frac{D}{E} = \epsilon$$

Permittivity of the free space =  $\epsilon_0$  (Farads per meter) (F/m)

$$\epsilon_0 = 8.854 \times 10^{-12} \text{ (F/m)}$$

$$\epsilon_r = \epsilon / \epsilon_0$$

$$\epsilon = \epsilon_0 \epsilon_r$$

$$D = \epsilon_0 \epsilon_r E$$

#### **7.4 Simple parallel plate capacitor:**

Let the plates be given a charge of Q coulombs hence giving rise to an electric flux of Q coulombs and a potential difference of V volts between the plates.

$$D = Q/A$$

$$Q = D * A$$

$$Ed = V$$

$$C = Q/V$$

$$= D * A / (E d)$$

$$= D/E * A/d$$

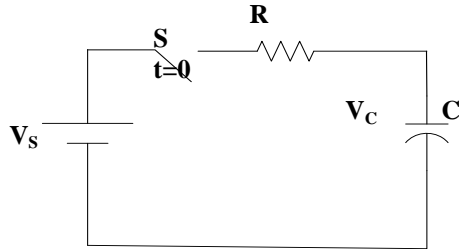
$$D = \epsilon_0 \epsilon_r E$$

$$D/E = \epsilon_0 \epsilon_r$$

$$C = \epsilon_0 \epsilon_r A/d$$

$$\epsilon = Cd/A$$

## 7.1 Charging of capacitor:



1- At instant  $t=0$  (switching instant)  
the potential difference across the  
capacitor =  $V_C=0$

Potential difference across the resistance =  $V_s$

$$I_m = \frac{V_s}{R} \quad \dots (1)$$

where,  $I_m$  is the initial charging current which is the maximum value.

2- At any instant:

$$\sum V = 0$$

$$V_s = v_C + iR \quad \dots (2)$$

where  $i$  = charging current at any instant.

$V_C$  = potential difference across the capacitor at any instant.

$$I_m = C \frac{dv_C}{dt} \quad \dots (3)$$

From (3) and (2):

$$V_s = v_C + RC \frac{dv_C}{dt}$$

$$V_s - v_C = RC \frac{dv_C}{dt}$$

$$\int \frac{dV_C}{V_S - V_C} = \int \frac{dt}{RC} + K$$

$$-\ln(V_S - V_C) = \frac{t}{RC} + K \quad \dots (4)$$

Where K is a constant which can be determine from the initial conditions.

Substituting  $t=0$  and  $V_C=0$  in (4):

$K = -\ln V_S$  in equation (4):

$$-\ln(V_S - V_C) = \frac{t}{RC} - \ln V_S$$

$$-\ln(V_S - V_C) + \ln V_S = \frac{t}{RC}$$

$$\ln\left(\frac{V_S}{V_S - V_C}\right) = \frac{t}{RC}$$

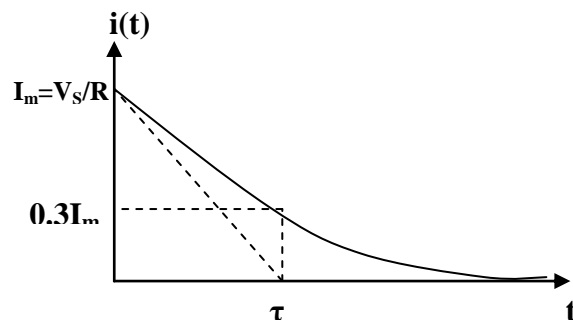
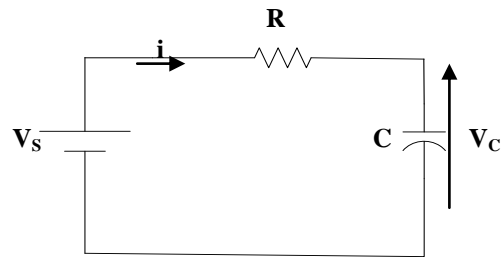
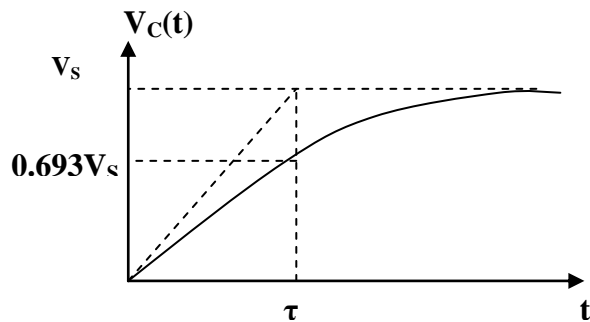
$$\frac{V_S}{V_S - V_C} = e^{t/RC}$$

$$V_C = V_S(1 - e^{-t/RC}) \quad \dots (5)$$

$$i = \frac{V_S - V_C}{R} = \frac{V_S - V_S(1 - e^{-t/RC})}{R}$$

$$i = \frac{V_S}{R} e^{-t/RC}$$

$$i = I_m e^{-t/RC} \quad \dots (6)$$



$\tau = RC =$  Time constant.

## 7.2 Discharging of capacitor:

$$0 = V_R + V_C$$

$$0 = i R + V_C$$

$$\int \frac{dV_C}{V_C} = \int -\frac{dt}{RC}$$

$$\ln V_C = \frac{t}{RC} + K \quad \text{at } t=0 \quad V_C = V_S$$

$$K = \ln V_S$$

$$\ln V_C = -\frac{t}{RC} + \ln V_S$$

$$\ln \frac{V_C}{V_S} = -\frac{t}{RC}$$

$$\frac{V_C}{V_S} = e^{-t/RC}$$

$$V_C = V_S e^{-t/RC} \quad \dots (7)$$

$$i = C \frac{dv_C}{dt} = -\frac{V_S}{R} e^{-t/RC} \quad \dots (8)$$

$$i = I_m e^{-t/RC}$$

