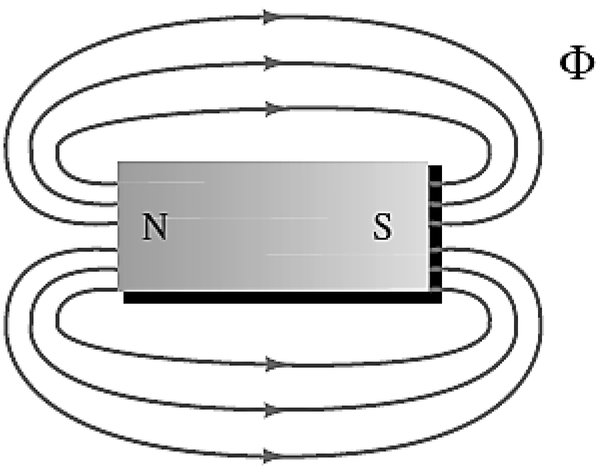
Magnetism and Magnetic Circuits

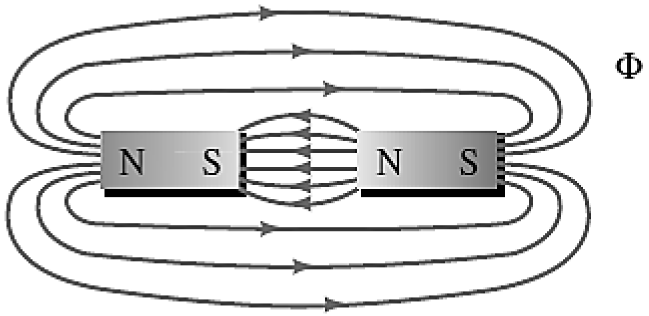
The Nature of a Magnetic Field:

- **Magnetism refers to the force that acts between magnets and magnetic materials. We know, for example, that magnets attract pieces of iron, deflect compass needles, attract or repel other magnets, and so on.**
- **The region where the force is felt is called the "field of the magnet" or simply, its magnetic field. Thus, a magnetic field is a force field.**
- **Using Faraday's representation, magnetic fields are shown as lines in space. These lines, called <u>flux lines</u> or <u>lines of force</u>, show the direction and intensity of the field at all points.**
- **The field is strongest at the poles of the magnet (where flux lines are most dense), its direction is from north (N) to south (S) external to the magnet, and flux lines never cross. The symbol for magnetic flux as shown is the Greek letter Φ (phi).**



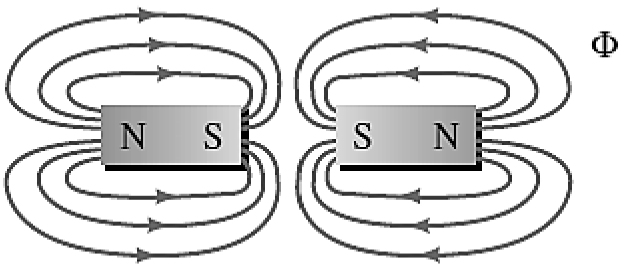
What happens when two magnets are brought close together?

- **If unlike poles attract, and flux lines pass from one magnet to the other.**



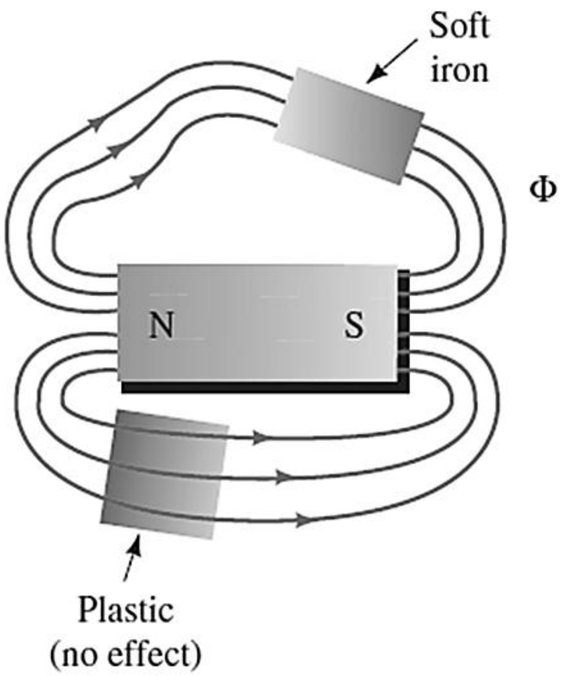
(a) Attraction

- If like poles repel, and the flux lines are pushed back as indicated by the flattening of the field between the two magnets.



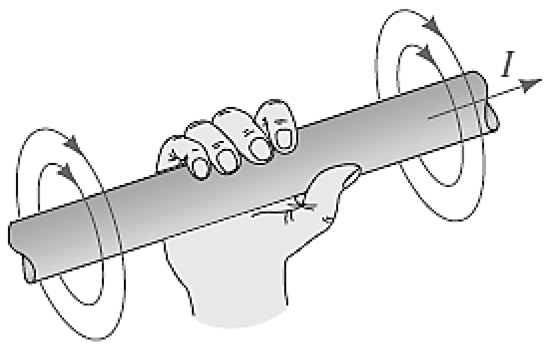
(b) Repulsion

- **<u>Ferromagnetic Materials</u> (magnetic materials that are attracted by magnets such as iron، nickel, cobalt, and their alloys) are called ferromagnetic materials. Ferromagnetic materials provide an easy path for magnetic flux.**
- **The flux lines take the longer (but easier) path through the soft iron, rather than the shorter path that they would normally take. Note, however, that nonmagnetic materials (plastic، wood, glass, and so on) have no effect on the field.**



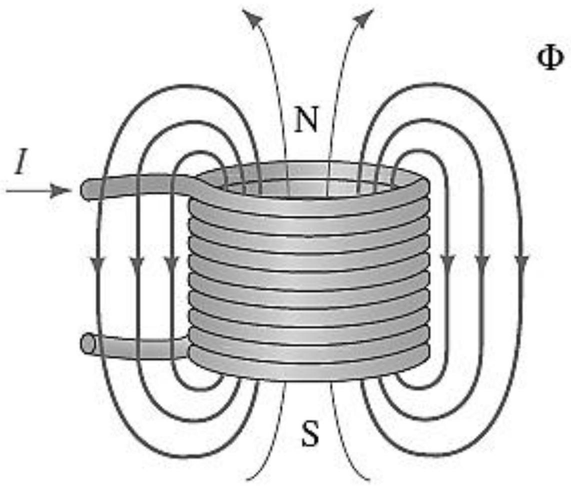
Electromagnetism:

Most applications of magnetism involve magnetic effects due to electric currents. The current, I, creates a magnetic field that is concentric about the conductor، uniform along its length, and whose strength is directly proportional to I. Note the direction of the field. It may be remembered with the aid of the right-hand rule, imagine placing your right hand around the conductor with your thumb pointing in the direction of current. Your fingers then point in the direction of the field. If you reverse the direction of the current, the direction of the field reverses.

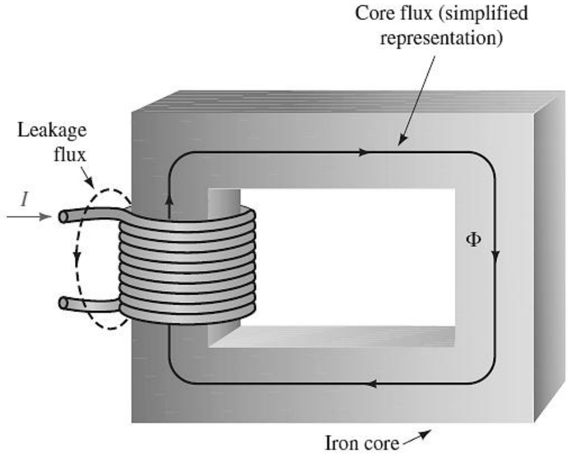


Right-hand rule

If the conductor is wound into a coil(Provided no ferromagnetic material is present) , the fields of its individual turns combine, producing a resultant field as in Figure. The direction of the coil flux can also be remembered by means of a simple rule: curl the fingers of your right hand around the coil in the direction of the current and your thumb will point in the direction of the field. If the direction of the current is reversed, the field also reverses. Provided no ferromagnetic material is present, the strength of the coil's field is directly proportional to its current.

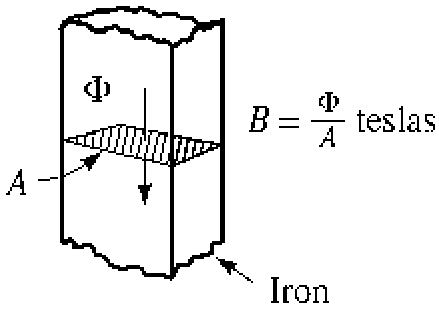


If the coil is wound on a ferromagnetic core as in Figure below (transformers are built this way), almost all flux is confined to the core, although a small amount (called stray or leakage flux) passes through the surrounding air. However, now that ferromagnetic material is present, the core flux is no longer proportional to current.



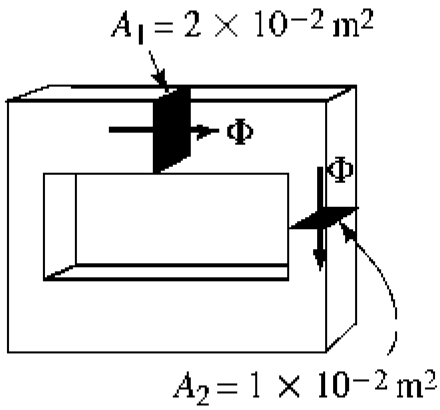
Flux and Flux Density:

The magnetic flux is represented by the symbol Φ. In the SI system, the unit of flux is the weber (Wb). However, we are often more interested in flux density B (i.e., flux per unit area) than in total flux Φ. Since flux Φ is measured in Wb and area A in m^2, flux density is measured as Wb/m^2, the unit of flux density is called the tesla (T) where 1 T = 1 Wb/m^2. Flux density is found by dividing the total flux passing perpendicularly through an area by the size of the area is,



$$\boldsymbol{B = \frac{\Phi}{A} \quad (Tesla, T)}$$

EXAMPLE : **For the magnetic core of Figure below, the flux density at cross section 1 is B1 = 0.4 T. Determine B2.**



Sol/ $\boldsymbol{\Phi = B_1 \times A_1 = 0.8 \times 10^{-2} wb}$

Since all flux is confined to the core,

the flux at cross section 2 is the

same as at cross section 1. Therefore,

$$\boldsymbol{B_2 = \frac{\Phi}{A_2} = 0.8\ T}$$

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PERMEABILITY:

If cores of different materials with the same physical dimensions are used in the electromagnet, the strength of the magnet will vary in accordance with the core used. This variation in strength is due to the greater or lesser number of flux lines passing through the core. Materials in which flux lines can readily be set up are said to be magnetic and to have high permeability. The permeability (μ) of a material It is similar in many respects to conductivity in electric circuits. The permeability of free space μ_0(vacuum) is.

$$\mu_o = 4\pi \times 10^{-7} \frac{Wb}{A.m}$$

The permeability of all nonmagnetic materials, such as copper, aluminum، wood, glass, and air, is the same as that for free space. Materials that have permeabilities slightly less than that of free space are said to be diamagnetic, and those with permeabilities slightly greater than that of free space are said to be paramagnetic. Materials with these very high permeabilities are referred to as ferromagnetic.

The ratio of the permeability of a material to that of free space is called its relative permeability; that is,

$$\mu_r = \frac{\mu}{\mu_o}$$

In general, for ferromagnetic materials, $\mu_r \geq 100$, and for nonmagnetic materials, $\mu_r = 1$.

RELUCTANCE:

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation,

$$R = \rho \frac{l}{A} \qquad (Ohms)$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\mathbb{R} = \frac{\ell}{\mu A} \quad (\frac{At}{Wb})$$

where $\mathbb{R}$ is the reluctance, ℓ is the length of the magnetic path, and A is the cross-sectional area. The units At/Wb is the number of turns of the applied winding. More is said about ampere-turns (At)

MMF: The Source of Magnetic Flux:

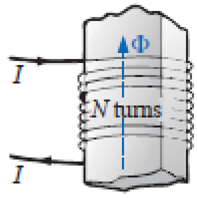
Current through a coil creates magnetic flux. The greater the current or the greater the number of turns, the greater will be the flux. This flux-producing ability of a coil is called its magnetomotive force (mmf). Magnetomotive force is given the symbol $\mathcal{F}$ and is defined as

$$\mathcal{F} = mmf = NI \qquad (amper - turn, At)$$

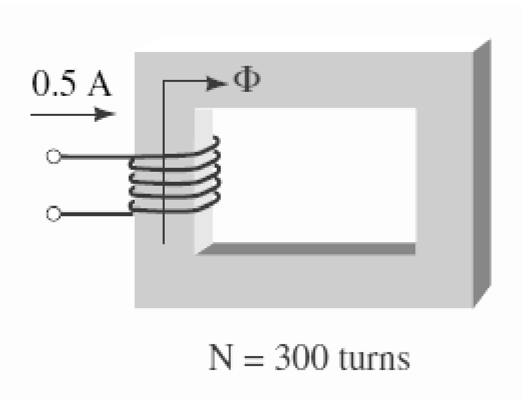
Ohm's Law for Magnetic Circuits:

The relationship between flux, mmf, and reluctance is

$$\Phi = \frac{\mathcal{F}}{\mathbb{R}} \quad (Wb)$$



EXAMPLE : For Figure shown below, if the reluctance of the magnetic circuit is $\mathbb{R} = 12 \times 10^4 \frac{At}{Wb}$, what is the flux in the circuit?



Sol/

$$\mathcal{F} = NI = 150\ At$$

$$\Phi = \frac{\mathcal{F}}{\mathbb{R}} = 12.5 \times 10^{-4}\ Wb$$

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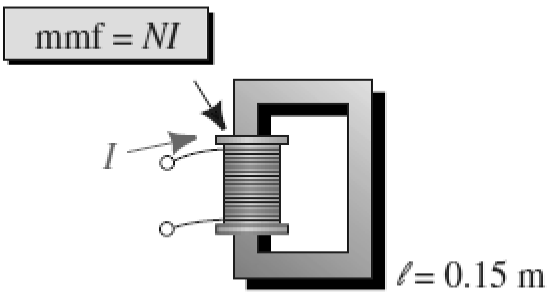
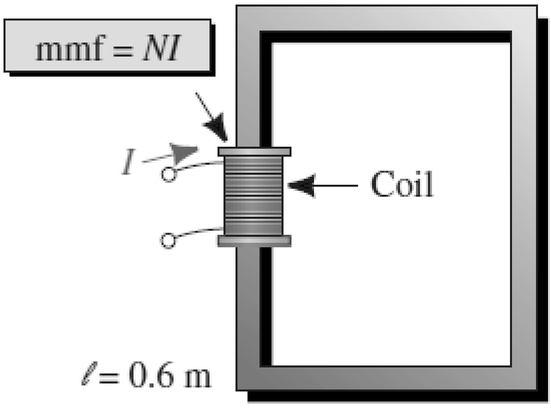
Magnetic Field Intensity and Magnetization Curves :

We now look at a more practical approach to analyzing magnetic circuits. First, we require a quantity called magnetic field intensity, H (also known as magnetizing force). It is a measure of the mmf per unit length of a circuit. We can define magnetic field intensity as the ratio of applied mmf to the length of path that it acts over. Thus,

$$H = \frac{\mathcal{F}}{\ell} = \frac{NI}{\ell} \quad \left(\frac{AT}{m}\right)$$



(b) A short path



(a) A long path

Rearranging Equation yields an important result:

$$NI = H\ell \quad \left(\frac{At}{m}\right)$$

In an analogy with electric circuits , the NI product is an mmf source, while the $H\ell$ product is an mmf drop.

The Relationship between B and H:

The flux density and the magnetizing force are related by the following equation:

$$\mathcal{B} = \mu H$$

where μ is the permeability of the core, therefore, the larger the value of μ , the larger the flux density for a given magnetizing current.

For all practical purposes, the permeability of air and other nonmagnetic materials is the same as for a vacuum. Thus, in air gaps,

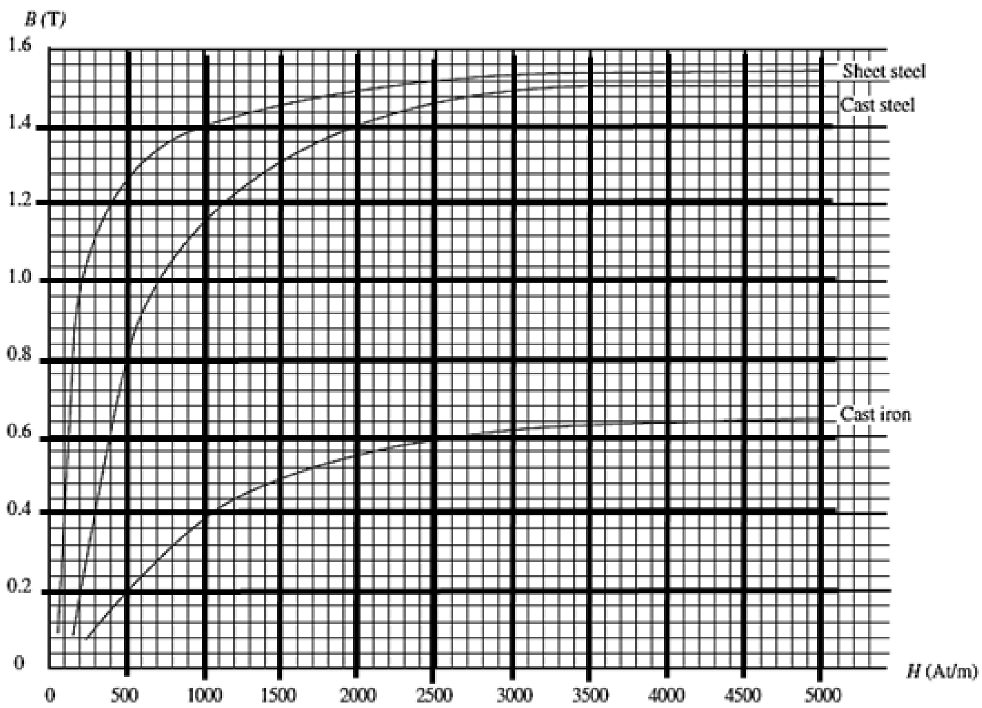
$$B_g = \mu_o H_g = 4\pi \times 10^{-7} \times H_g$$

$$H_g = \frac{B_g}{4\pi \times 10^{-7}} = 7.96 \times 10^5 B_g \qquad (\frac{At}{m})$$

B-H Curves:

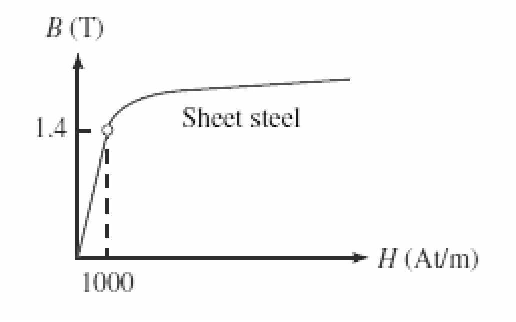
For ferromagnetic materials, μ is not constant but varies with flux density and there is no easy way to compute it. In reality, however, it isn't m that you are interested in: What you really want to know is, given B, what is H, and vice versa. A set of curves, called B-H or magnetization curves, provides this information. (These curves are obtained experimentally and are available in handbooks. A separate curve is required for each material.) Figure shows typical curves for cast iron, cast steel, and sheet steel.

B-H curve (magnetization curves)



<u>EXAMPLE :</u> If B =1.4 T for sheet steel, what is H?

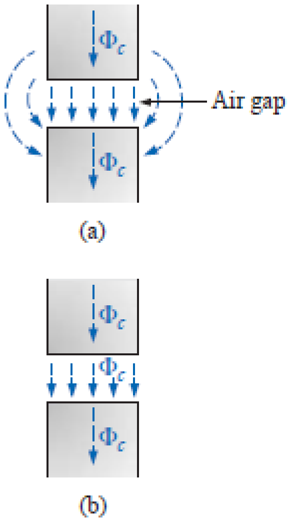
Sol/



For sheet steel, H = 1000 At/m, when B = 1.4 T.

Air Gaps, Fringing, and Laminated Cores:

The spreading of the flux lines outside the common area of the core for the air gap in Figure(a) is known as fringing. For our purposes, we shall neglect this effect and assume the flux distribution to be as in Figure (b) .For magnetic circuits with air gaps, fringing occurs, causing a decrease in flux density in the gap. For short gaps, fringing can usually be neglected. Alternatively, correction can be made by increasing each cross-sectional dimension of the gap by the size of the gap to approximate the decrease in flux density.



Air gaps: (a) with fringing; (b) ideal.

Ampere's Circuital Law:

If we apply the analogy to Kirchhoff's voltage law (for magnetic circuits).

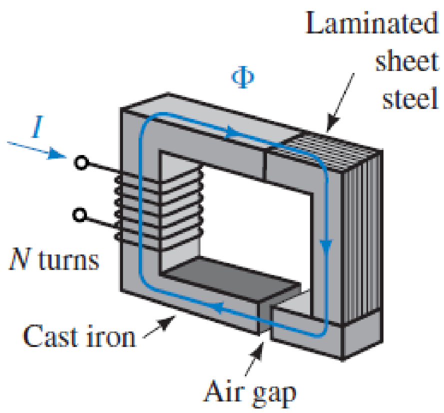
$$\sum_{\circlearrowleft} \mathcal{F} = 0$$

States that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop. Equation is referred to as Ampère's circuital law. When it is applied to magnetic circuits, sources of mmf are expressed by the equation. The summation is algebraic and terms are additive or subtractive, depending on the direction of flux and how the coils are wound.

$$\mathcal{F} = NI \qquad (At)$$

$$\sum_{\circlearrowleft} NI = \sum_{\circlearrowleft} H\ell$$

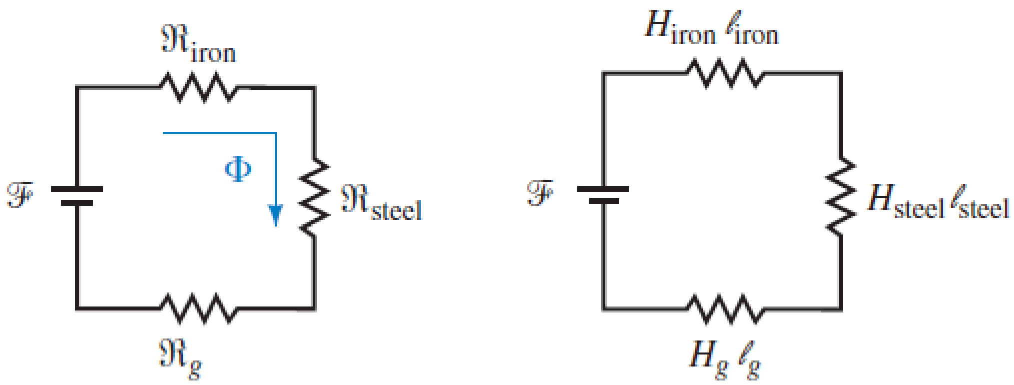
Consider the magnetic circuit appearing in Figure constructed of three different ferromagnetic materials. Applying Ampère's circuital law, we have



$$NI - H_{iron}\ell_{iron} - H_{steel}\ell_{steel} - H_g\ell_g = 0$$

Which states that the applied mmf NI is equal to the sum of the drops $H\ell$ around the loop. The path to use for the $H\ell$ terms is the mean (average) path.

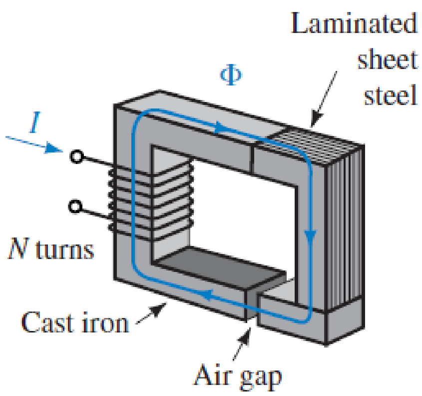
You now have two magnetic circuit models (Figure below). While the reluctance model (a) is not very useful for solving problems, it helps relate magnetic circuit problems to familiar electrical circuit concepts. The Ampere's law model, on the other hand, permits us to solve practical problems.



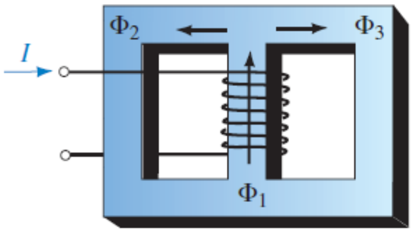
(a) Reluctance model (b) Ampere's circuital law model

<u>Series Elements and Parallel Elements:</u>

Magnetic circuits may have sections of different materials. For example, the circuit of Figure has sections of cast iron, sheet steel, and an air gap. For this circuit, flux is the same in all sections. Such a circuit is called a series magnetic circuit. Although the flux is the same in all sections, the flux density in each section may vary, depending on its effective cross-sectional area as you saw earlier.



A circuit may also have elements in parallel (Figure). At each junction, the sum of fluxes entering is equal to the sum leaving. This is the counterpart of Kirchhoff's current law. Thus



$$\boldsymbol{\Phi_1 = \Phi_2 + \Phi_3}$$

<u>Series Magnetic Circuits: Given Φ, Find NI</u>

You now have the tools needed to solve basic magnetic circuit problems. This type can be solved using four basic steps:

1) **Compute *B* for each section using $B = \frac{\Phi}{A}$.**
2) **Determine *H* for each magnetic section from the *B-H* curves. Use $H_g = 7.96 \times 10^5 B_g$ for air gaps.**
3) **Compute *NI* using Ampere's circuital law.**
4) **Use the computed *NI* to determine coil current or turns as required**

PRACTICAL NOTES...

Magnetic circuit analysis is not as precise as electric circuit analysis because

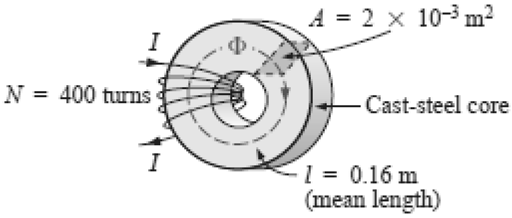
- ✓ **The assumption of uniform flux density breaks down at sharp corners.**
- ✓ **The B-H curve is a mean curve and has considerable uncertainty.**

Although the answers are approximate, they are adequate for most purposes.

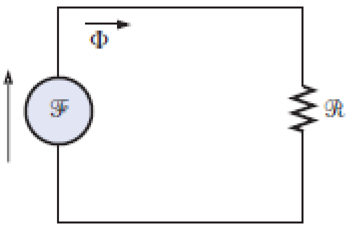
EXAMPLE : For the series magnetic circuit of Figure

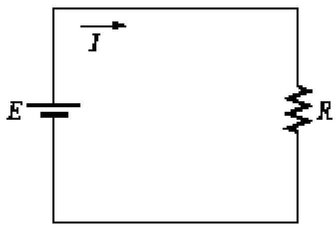
a) **Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4} Wb$.**

b) **Determine μ and μ_r for the material under these conditions.**



Sol/ $\Phi = 4 \times 10^{-4} Wb$, $A = 2 \times 10^{-3} m^2$,
$N = 400\ turns$, $\ell = 0.16\ m$





The flux density B is

$$B = \frac{\Phi}{A} = 0.2\ T$$

Using the B-H curves, we can determine the magnetizing force H:

$$H(Cast\ steel) = 170\ At/m$$

Applying Ampère's circuital law yields:

$$NI = H\ell, \qquad I = \frac{H\ell}{N} = 68\ mA$$

b. The permeability of the material can be found,

$$\mu = \frac{B}{H} = 1.17 \times 10^{-3} \frac{Wb}{A * m}$$

$$\mu_r = \frac{\mu}{\mu_o} = 935.83$$

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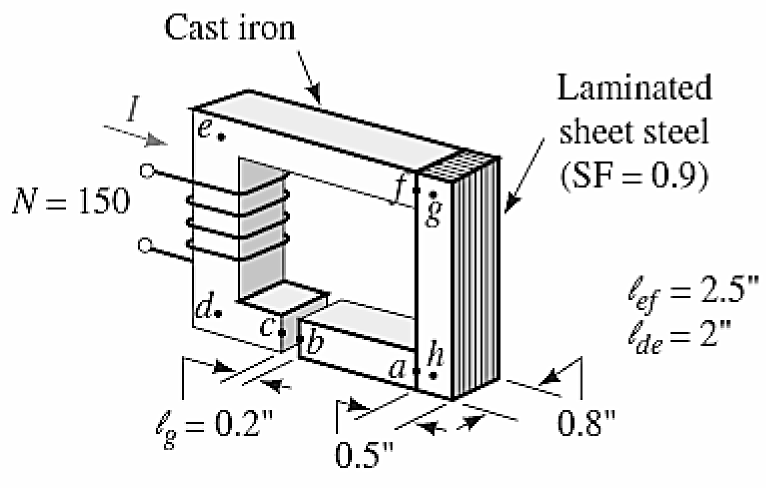
Example: **The laminated sheet steel section of Figure has a stacking factor of 0.9. Compute the current required to establish a flux of $\Phi = 1.4 \times 10^{-4}\ Wb$. Neglect fringing.**



Cross section = 0.5" × 0.8" (all members)
$\Phi = 1.4 \times 10^{-4}$ Wb

Sol/ Convert all dimensions to metric.

Cast iron:

$$\ell_{iron} = \ell_{ab} + \ell_{cdef} = 2.5 + 2 + 2.5 - 0.2 = 6.8\ in$$

$$\ell_{iron} = \frac{6.8}{39.37} = 0.173\ m$$

$$A_{iron} = (0.5 \times 0.8) = 0.4\ in^2 = 0.258 \times 10^{-3}\ m^2$$

$$B_{iron} = \frac{\Phi}{A_{iron}} = 0.54\ T$$

From B-H curve (Cast iron)

$$H_{iron} = 1850\ At/m$$

Sheet steel:

$$\ell_{steel} = \ell_{fg} + \ell_{gh} + \ell_{ha} = 0.25 + 2 + 0.25 = 2.5\ in = 6.35 \times 10^{-2} m$$

$$A_{steel} = (0.9)(0.258 \times 10^{-3}) = 0.232 \times 10^{-3} m^2$$

$$B_{steel} = \frac{\Phi}{A_{steel}} = 0.6\ T$$

From B-H curve (Sheet steel)

$$H_{steel} = 125\ \frac{At}{m}$$

Air gap:

$$\ell_g = 0.2\ in = 5.08 \times 10^{-3} m$$

$$B_g = B_{iron} = 0.54T$$

$$H_g = (7.96 \times 10^5)(0.54) = 4.3 \times 10^5\ At/m$$

Ampere's law

$$NI = H_{iron}\ell_{iron} + H_{steel}\ell_{steel} + H_g\ell_g = 2512\ At$$

$$I = \frac{2512}{N} = 16.7\ Amps.$$

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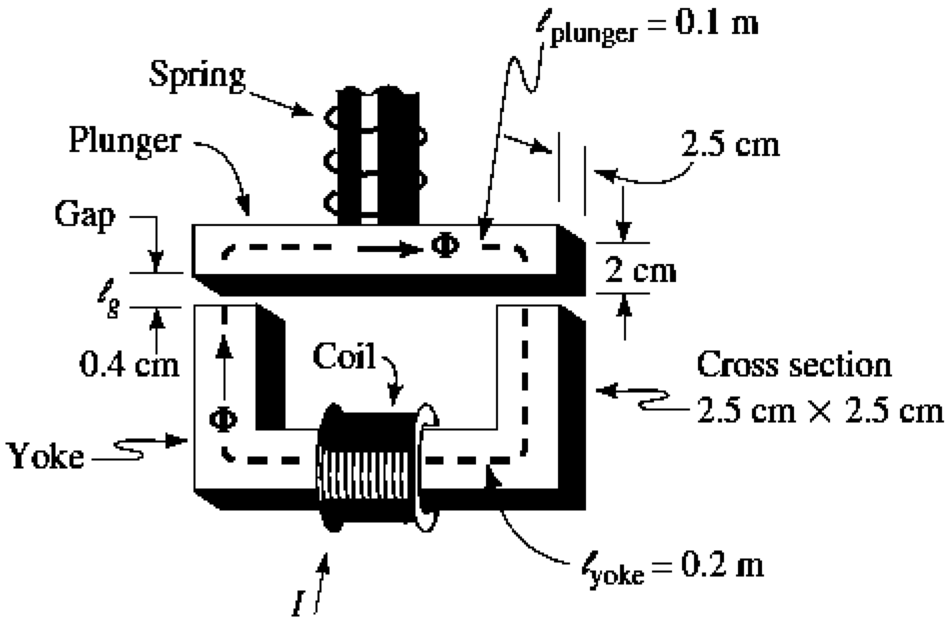
Example: Figure shows below a portion of a solenoid. Flux $\Phi = 4 \times 10^{-4}\ Wb$ when *I* = 2.5 amps. Find the number of turns on the coil.(Solenoid. All parts are cast steel)

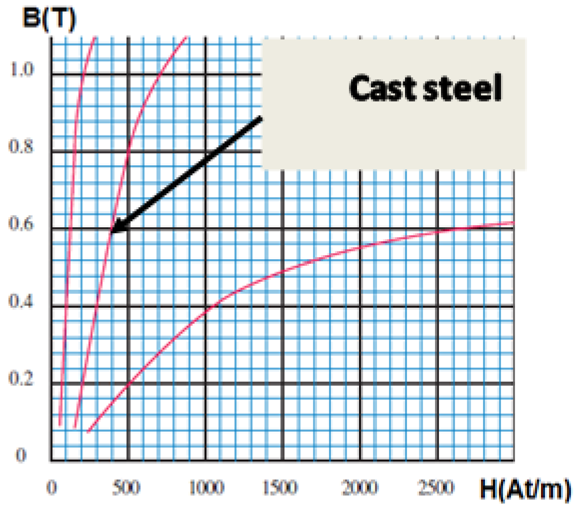


Sol

yoke:

$$A = 6.25\ cm^2 = 6.25 \times 10^{-4}\ m^2$$

$B = \frac{\Phi}{A} = 0.64\ T$



From B-H curve (cast steel)

$H = 410\ At\ /m$

Plunger:

$A = 2 \times 2.5 = 5\ cm^2 = 5 \times 10^{-4}\ m^2$

$B = \frac{\Phi}{A} = 0.8\ T$

From B-H curve (cast steel)

$H = 500\ At\ /m$

Air gap:

$B_g = B_{yoke} = 0.64T$

$H_g = (7.96 \times 10^5)(0.64) = 5.09 \times 10^5\ At \backslash m$

Ampere's law

$NI = H_{yoke}\ell_{yoke} + H_{plunger}\ell_{plunger} + 2H_g\ell_g = 4204\ At$

$\mathrm{N} = \frac{4204}{2.5} = 1682$ **turns**

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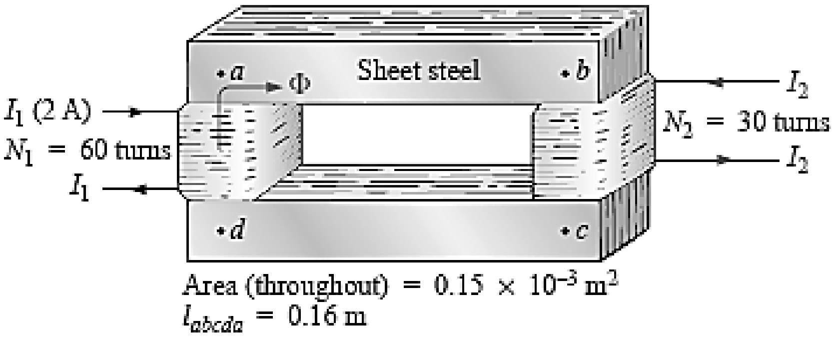
EXAMPLE: **Determine the secondary current I_2 for the transformer of Figure shown, if the resultant clockwise flux in the core is $\Phi = 1.5 \times 10^{-5}\ Wb$.**

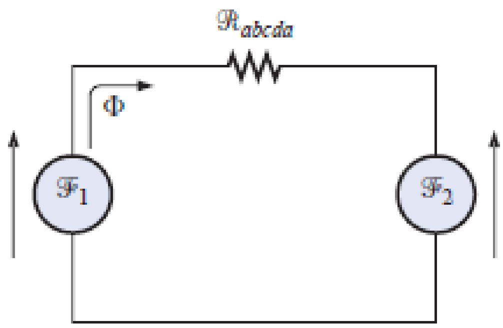


Sol/ you will note that the resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

$\Phi = 1.5 \times 10^{-5}\ Wb$, **$I_1$= 2A, N_2= 30 turns, N_1=60 turns**

$A = 0.15 \times 10^{-3}\ m^2$, $\ell_{abcda} = 0.16\ m$, $I_2 = ?$

$B = \frac{\Phi}{A} = 0.1T$



From B-H curve (sheet steel)

$H = 20\ At\ /m$

Applying Ampère's circuital law,

$N_1 I_1 - N_2 I_2 = H_{abcda}\ell_{abcda}$

$120\ At - (30\ t)I_2 = (20\ At\ /m)(0.16\ m)$

$I_2 = 3.89\ A$

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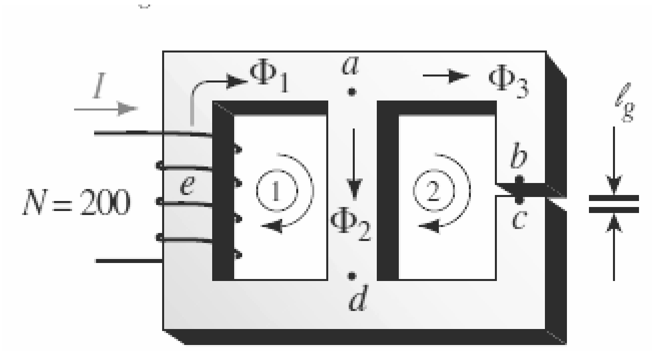
Series-Parallel Magnetic Circuits:

Series-parallel magnetic circuits are handled using the sum of fluxes principle and Ampere's law.

Example : **The core of Figure below is cast steel. Determine the current to establish an air-gap flux $\Phi_g = 6 \times 10^{-3} Wb$. Neglect fringing.**



Cast steel

$$\ell_{ab} = \ell_{cd} = 0.25\ m$$

$$\ell_g = \ell_{bc} = 0.25 \times 10^{-3} m$$

$$\ell_{da} = 0.2\ m$$

$$\ell_{dea} = 0.35\ m$$

$$A = 2 \times 10^{-2} m^2$$

$$\Phi_g = \Phi_3$$

Sol/ Air Gap

$$B_g = \frac{\Phi_g}{A_g} = 0.3T$$

$$H_g = (7.96 \times 10^5)(0.3) = 2.388 \times 10^5\ At/m$$

Sections ab and cd

$$B_{ab} = B_{cd} = B_g = 0.3T$$

From B-H curve

$$H_{ab} = H_{cd} = 250\ At/m$$

Ampere's Law (Loop 2), $\sum_{\circlearrowright} NI = \sum_{\circlearrowright} H\ell$

$$0 = H_{ab}\ell_{ab} + H_g\ell_g + H_{cd}\ell_{cd} - H_{da}\ell_{da}$$

$$0 = 62.5 + 59.7 + 62.5 - 0.2\ H_{da}$$

$$H_{da} = 925\ At \backslash m$$

From B-H curve $B_{da} \cong 1.12T$

$$\Phi_2 = B_{ad}.A = 2.24 \times 10^{-2} Wb$$

$$\Phi_1 = \Phi_2 + \Phi_3 = 2.84 \times 10^{-2} Wb$$

$$B_{dea} = \frac{\Phi_1}{A} = 1.42T$$

From B-H curve

$$H_{dea} = 2125\ At/m$$

Ampere's Law (Loop 1)

$$NI = H_{dea}\ell_{dea} + H_{ad}\ell_{ad}$$

$$NI = 929\ At$$

$$I = 4.65\ A$$

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Series Magnetic Circuits: Given NI, Find Φ :

For the special case of a core of one material and constant cross section

Example: **For the circuit of Figure , *NI* =250 At. Determine Φ**

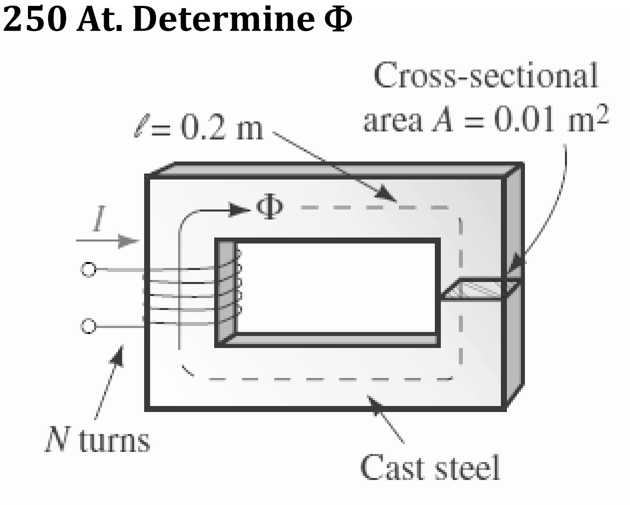
sol/

$$NI = H\ell \ \rightarrow H = \frac{NI}{\ell} = 1250\ At/m$$

From B-H curve(cast steel)

B= 1.2T , therefore

$$\Phi = BA = 1.24 \times 10^{-4} Wb$$



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For circuits with two or more sections, the process is not so simple.