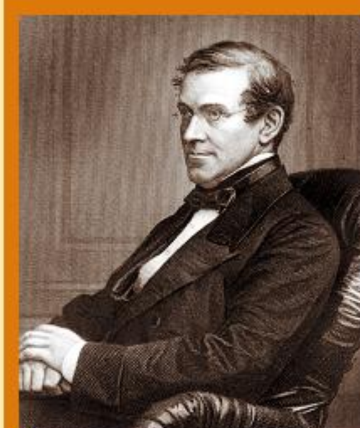
Methods of Analysis

OBJECTIVES

After studying this chapter you will be able to

- convert a voltage source into an equivalent current source,
- convert a current source into an equivalent voltage source,
- analyze circuits having two or more current sources in parallel,
- write and solve branch equations for a network,
- write and solve mesh equations for a network,
- write and solve nodal equations for a network,
- convert a resistive delta to an equivalent wye circuit or a wye to its equivalent delta circuit and solve the resulting simplified circuit,
- determine the voltage across or current through any portion of a bridge network,

PUTTING IT IN PERSPECTIVE



Sir Charles Wheatstone

CHARLES WHEATSTONE WAS BORN IN GLOUCESTER, England, on February 6, 1802. Wheatstone's original interest was in the study of acoustics and musical instruments. However, he gained fame and a knighthood as a result of inventing the telegraph and improving the electric generator.

Although he did not invent the bridge circuit, Wheatstone used one for measuring resistance very precisely. He found that when the currents in the Wheatstone bridge are exactly balanced, the unknown resistance can be compared to a known standard.

Sir Charles died in Paris, France, on October 19, 1875.

8.1 Constant-Current Sources

All the circuits presented so far have used voltage sources as the means of providing power. However, the analysis of certain circuits is easier if you work with current rather than with voltage. Unlike a voltage source, a **constant-current source** maintains the same current in its branch of the circuit regardless of how components are connected external to the source. The symbol for a constant-current source is shown in Figure 8–2.

The direction of the current source arrow indicates the direction of conventional current in the branch. In previous chapters you learned that the magnitude and the direction of current through a voltage source varies according to the size of the circuit resistances and how other voltage sources are connected in the circuit. For current sources, the voltage across the current source depends on how the other components are connected.

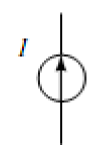


FIGURE 8–2 Ideal constant current source.

The constant-current source determines the current in its branch of the circuit.

The magnitude and polarity of voltage appearing across a constant-current source are dependent upon the network in which the source is connected.

8.2 Source Conversions

In the previous section you were introduced to the ideal constant-current source. This is a source which has no internal resistance included as part of the circuit. As you recall, voltage sources always have some series resistance, although in some cases this resistance is so small in comparison with other circuit resistance that it may effectively be ignored when determining the operation of the circuit. Similarly, a constant-current source will always have some shunt (or parallel) resistance. If this resistance is very large in comparison with the other circuit resistance, the internal resistance of the source may once again be ignored. **An ideal current source has an infinite shunt resistance.**

Figure 8–6 shows equivalent voltage and current sources.

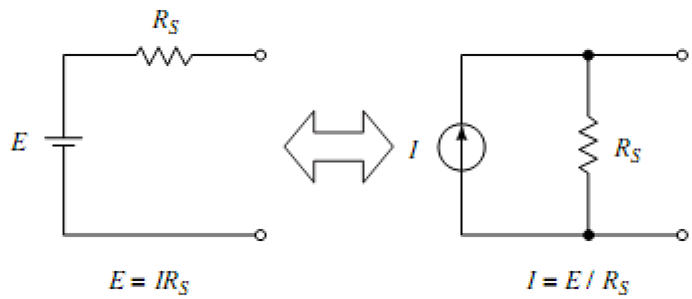


FIGURE 8–6

If the internal resistance of a source is considered, the source, whether it is a voltage source or a current source, is easily converted to the other type. The current source of Figure 8–6 is equivalent to the voltage source if

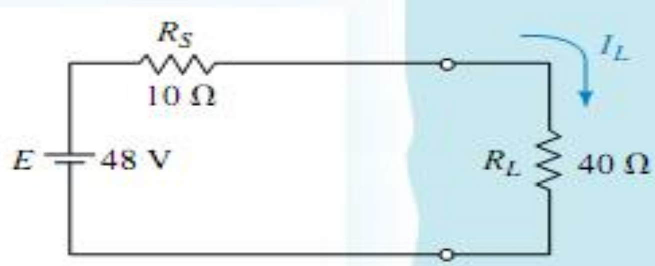
$$I = \frac{E}{R_S} \tag{8–1}$$

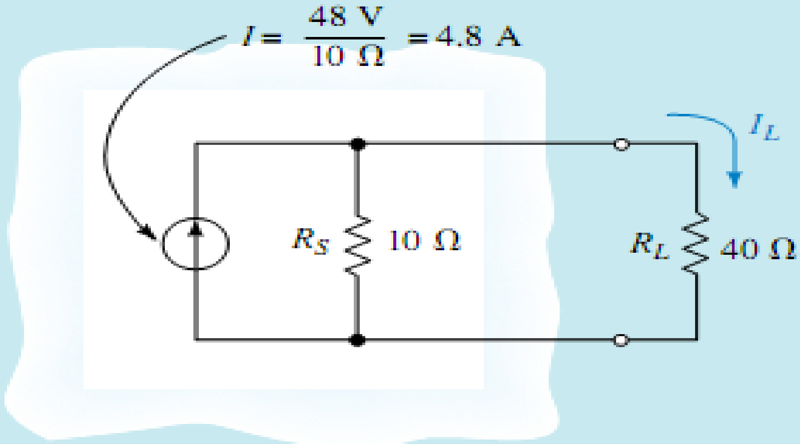
and the resistance in both sources is R_S.

Similarly, a current source may be converted to an equivalent voltage source by letting

$$E = IR_S \tag{8–2}$$

EXAMPLE 8–4 Convert the voltage source of Figure 8–9(a) into a current source and verify that the current, I_L, through the load is the same for each source.





(b)

FIGURE 8–9

Solution The equivalent current source will have a current magnitude given as

$$I = \frac{48\text{ V}}{10\ \Omega} = 4.8\text{ A}$$

The resulting circuit is shown in Figure 8–9(b).

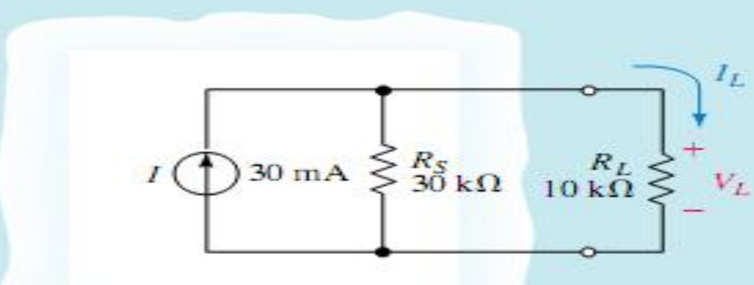
For the circuit of Figure 8–9(a), the current through the load is found as

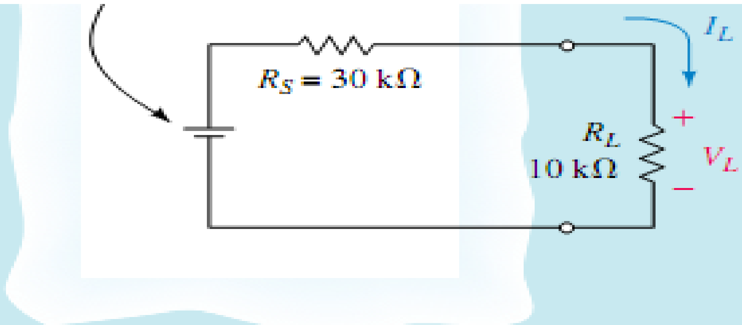
$$I_L = \frac{48\text{ V}}{10\ \Omega + 40\ \Omega} = 0.96\text{ A}$$

For the equivalent circuit of Figure 8–9(b), the current through the load is

$$I_L = \frac{(4.8\text{ A})(10\ \Omega)}{10\ \Omega + 40\ \Omega} = 0.96\text{ A}$$

EXAMPLE 8–5 Convert the current source of Figure 8–10(a) into a voltage source and verify that the voltage, V_L, across the load is the same for each source.





(b)

FIGURE 8–10

Solution The equivalent voltage source will have a magnitude given as

$$E = (30\ \text{mA})(30\ \text{k}\Omega) = 900\ \text{V}$$

The resulting circuit is shown in Figure 8–10(b).

For the circuit of Figure 8–10(a), the voltage across the load is determined as

$$I_L = \frac{(30\ \text{k}\Omega)(30\ \text{mA})}{30\ \text{k}\Omega + 10\ \text{k}\Omega} = 22.5\ \text{mA}$$

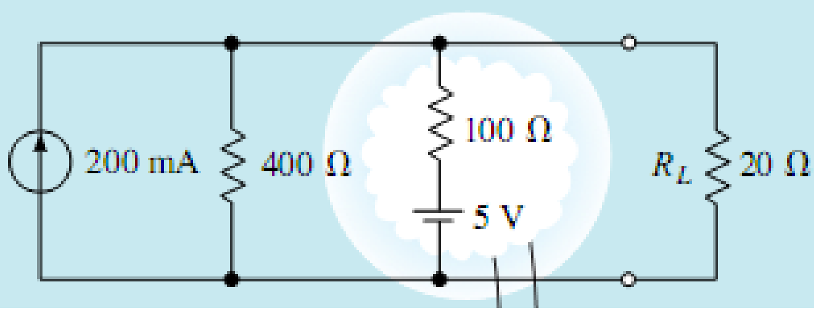
$$V_L = I_L R_L = (22.5\ \text{mA})(10\ \text{k}\Omega) = 225\ \text{V}$$

For the equivalent circuit of Figure 8–10(b), the voltage across the load is

$$V_L = \frac{10\ \text{k}\Omega}{10\ \text{k}\Omega + 30\ \text{k}\Omega}(900\ \text{V}) = 225\ \text{V}$$

Once again, we see that the circuits are equivalent.

EXAMPLE 8–7 Reduce the circuit of Figure 8–15 into a single current source and solve for the current through the resistor R_L.



Solution The voltage source in this circuit is converted to an equivalent current source as shown. The resulting circuit may then be simpified to a single current source where

$$I_S = 200 \text{ mA} + 50 \text{ mA} = 250 \text{ mA}$$

and

$$R_S = 400\ \Omega \| 100\ \Omega = 80\ \Omega$$

The simplified circuit is shown in Figure 8–16.

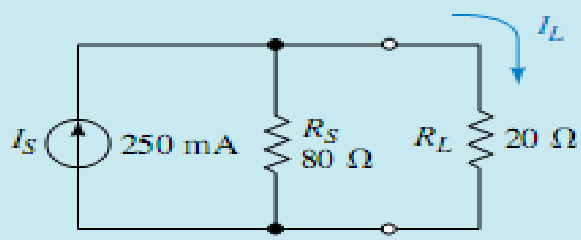


FIGURE 8–16

The current through R_L is now easily calculated as

$$I_L = \left(\frac{80\ \Omega}{80\ \Omega + 20\ \Omega}\right)(250 \text{ mA}) = 200 \text{ mA}$$

8.4 Branch-Current Analysis

In previous chapters we used Kirchhoff's circuit law and Kirchhoff's voltage law to solve equations for circuits having a single voltage source. In this section, you will use these powerful tools to analyze circuits having more than one source.

Branch-current analysis allows us to directly calculate the current in each branch of a circuit. Since the method involves the analysis of several simultaneous linear equations, you may find that a review of determinants is in order. Appendix B has been included to provide a review of the mechanics of solving simultaneous linear equations.

When applying branch-current analysis, you will find the technique listed below useful.

1. Arbitrarily assign current directions to each branch in the network. If a particular branch has a current source, then this step is not necessary since you already know the magnitude and direction of the current in this branch.
2. Using the assigned currents, label the polarities of the voltage drops across all resistors in the circuit.
3. Apply Kirchhoff's voltage law around each of the closed loops. Write just enough equations to include all branches in the loop equations. If a branch has only a current source and no series resistance, it is not necessary to include it in the KVL equations.
4. Apply Kirchhoff's current law at enough nodes to ensure that all branch currents have been included. In the event that a branch has only a current source, it will need to be included in this step.
5. Solve the resulting simultaneous linear equations.

EXAMPLE 8–8 Find the current in each branch in the circuit of Figure 8–18.

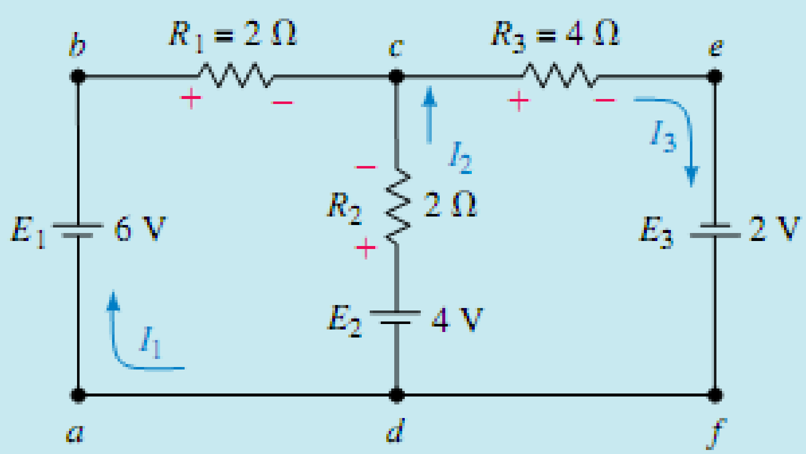


FIGURE 8–18

Solution

Step 1: Assign currents as shown in Figure 8–18.

Step 2: Indicate the polarities of the voltage drops on all resistors in the circuit, using the assumed current directions.

Step 3: Write the Kirchhoff voltage law equations.

Loop *abcda:* $6\text{ V} - (2\ \Omega)I_1 + (2\ \Omega)I_2 - 4\text{ V} = 0\text{ V}$

Notice that the circuit still has one branch which has not been included in the KVL equations, namely the branch *cefd.* This branch would be included if a loop equation for *cefdc* or for *abcefda* were written. There is no reason for choosing one loop over another, since the overall result will remain unchanged even though the intermediate steps will not give the same results.

Loop *cefdc:* $4\text{ V} - (2\ \Omega)I_2 - (4\ \Omega)I_3 + 2\text{ V} = 0\text{ V}$

Now that all branches have been included in the loop equations, there is no need to write any more. Although more loops exist, writing more loop equations would needlessly complicate the calculations.

Step 4: Write the Kirchhoff current law equation(s).

By applying KCL at node *c,* all branch currents in the network are included.

Node *c:* $I_3 = I_1 + I_2$

To simplify the solution of the simultaneous linear equations we write them as follows:

$$2I_1 - 2I_2 + 0I_3 = 2$$
$$0I_1 - 2I_2 - 4I_3 = -6$$
$$1I_1 + 1I_2 - 1I_3 = 0$$

$$D = \begin{vmatrix} 2 & -2 & 0 \\ 0 & -2 & -4 \\ 1 & 1 & -1 \end{vmatrix}$$
$$= 2\begin{vmatrix} -2 & -4 \\ 1 & -1 \end{vmatrix} - 0\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix}$$
$$= 2(2 + 4) - 0 + 1(8) = 20$$

Now, solving for the currents, we have the following:

$$I_1 = \frac{\begin{vmatrix} 2 & -2 & 0 \\ -6 & -2 & -4 \\ 0 & 1 & -1 \end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix} -2 & -4 \\ 1 & -1 \end{vmatrix} - (-6)\begin{vmatrix} -2 & 0 \\ 1 & -1 \end{vmatrix} + 0\begin{vmatrix} -2 & 0 \\ -2 & -4 \end{vmatrix}}{20}$$

$$= \frac{2(2 + 4) + 6(2) + 0}{20} = \frac{24}{20} = 1.200 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 0 \\ 0 & -6 & -4 \\ 1 & 0 & -1 \end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix} -6 & -4 \\ 0 & -1 \end{vmatrix} - 0\begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 0 \\ -6 & -4 \end{vmatrix}}{20}$$

$$= \frac{2(6) + 0 + 1(-8)}{20} = \frac{4}{20} = 0.200 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & -2 & 2 \\ 0 & -2 & -6 \\ 1 & 1 & 0 \end{vmatrix}}{D}$$

$$= \frac{2\begin{vmatrix} -2 & -6 \\ 1 & 0 \end{vmatrix} - 0\begin{vmatrix} -2 & 2 \\ 1 & 0 \end{vmatrix} + 1\begin{vmatrix} -2 & 2 \\ -2 & -6 \end{vmatrix}}{20}$$

$$= \frac{2(6) - 0 + 1(12 + 4)}{20} = \frac{28}{20} = 1.400 \text{ A}$$

EXAMPLE 8–9 Find the currents in each branch of the circuit shown in Figure 8–19. Solve for the voltage V_{ab}.

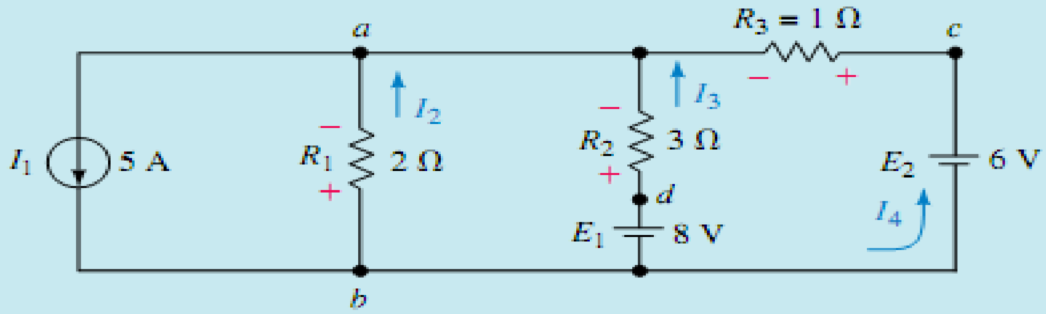


FIGURE 8–19

Solution Notice that although the above circuit has four currents, there are only three **unknown** currents: I_2, I_3, and I_4. The current I_1 is given by the value of the constant-current source. In order to solve this network we will need three linear equations. As before, the equations are determined by Kirchhoff's voltage and current laws.

Step 1: The currents are indicated in the given circuit.

Step 2: The polarities of the voltages across all resistors are shown.

Step 3: Kirchhoff's voltage law is applied at the indicated loops:

Loop *badb*: $-(2\ \Omega)(I_2) + (3\ \Omega)(I_3) - 8\ \text{V} = 0\ \text{V}$

Loop *bacb*: $-(2\ \Omega)(I_2) + (1\ \Omega)(I_4) - 6\ \text{V} = 0\ \text{V}$

Step 4: Kirchhoff's current law is applied as follows:

Node *a*: $I_2 + I_3 + I_4 = 5\ \text{A}$

Rewriting the linear equations,

$$-2I_2 + 3I_3 + 0I_4 = 8$$
$$-2I_2 + 0I_3 + 1I_4 = 6$$
$$1I_2 + 1I_3 + 1I_4 = 5$$

The determinant of the denominator is evaluated as

$$D = \begin{vmatrix} -2 & 3 & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 11$$

Now solving for the currents, we have

$$I_2 = \frac{\begin{vmatrix} 8 & 3 & 0 \\ 6 & 0 & 1 \\ 5 & 1 & 1 \end{vmatrix}}{D} = \frac{11}{11} = -1.00\text{ A}$$

$$I_3 = \frac{\begin{vmatrix} -2 & 8 & 0 \\ -2 & 6 & 1 \\ 1 & 5 & 1 \end{vmatrix}}{D} = \frac{22}{11} = 2.00\text{ A}$$

$$I_4 = \frac{\begin{vmatrix} -2 & 3 & 8 \\ -2 & 0 & 6 \\ 1 & 1 & 5 \end{vmatrix}}{D} = \frac{44}{11} = 4.00\text{ A}$$

The current I_2 is negative, which simply means that the actual direction of the current is opposite to the chosen direction.

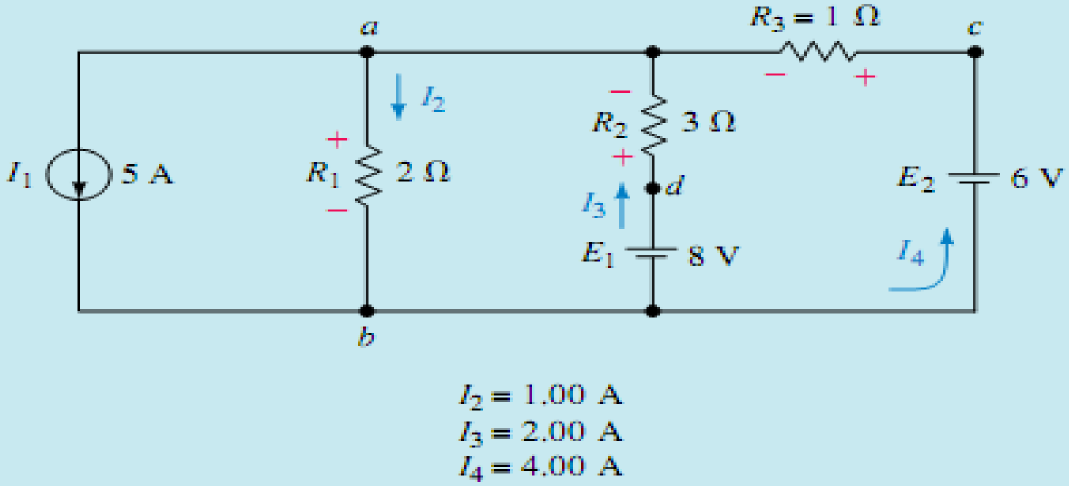


FIGURE 8–20

Using the actual direction for I_2,

$$V_{ab} = +(2\ \Omega)(1\text{ A}) = +2.00\text{ V}$$

Homework:

8.5 Mesh (Loop) Analysis

In the previous section you used Kirchhoff's laws to solve for the current in each branch of a given network. While the methods used were relatively simple, branch-current analysis is awkward to use because it generally involves solving several simultaneous linear equations. It is not difficult to see that the number of equations may be prohibitively large even for a relatively simple circuit.

A better approach and one which is used extensively in analyzing linear bilateral networks is called **mesh** (or **loop**) **analysis.** While the technique is similar to branch-current analysis, the number of simultaneous linear equations tends to be less. The principal difference between mesh analysis and branch-current analysis is that we simply need to apply Kirchhoff's voltage law around closed loops without the need for applying Kirchhoff's current law.

1. Arbitrarily assign a clockwise current to each interior closed loop in the network. Although the assigned current may be in any direction, a clockwise direction is used to make later work simpler.
2. Using the assigned loop currents, indicate the voltage polarities across all resistors in the circuit. For a resistor which is common to two loops, the polarities of the voltage drop due to each loop current should be indicated on the appropriate side of the component.
3. Applying Kirchhoff's voltage law, write the loop equations for each loop in the network. Do not forget that resistors which are common to two loops will have two voltage drops, one due to each loop.
4. Solve the resultant simultaneous linear equations.
5. Branch currents are determined by algebraically combining the loop currents which are common to the branch.

EXAMPLE 8–10 Find the current in each branch for the circuit of Figure 8–22.

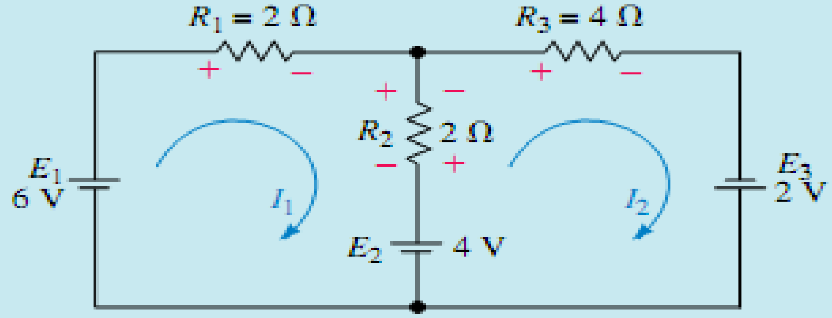


FIGURE 8–22

Solution

Step 1: Loop currents are assigned as shown in Figure 8–22. These currents are designated I_1 and I_2.

Step 2: Voltage polarities are assigned according to the loop currents. Notice that the resistor R_2 has two different voltage polarities due to the different loop currents.

Step 3: The loop equations are written by applying Kirchhoff's voltage law in each of the loops. The equations are as follows:

Loop 1: $6\text{ V} - (2\ \Omega)I_1 - (2\ \Omega)I_1 + (2\ \Omega)I_2 - 4\text{ V} = 0$

Loop 2: $4\text{ V} - (2\ \Omega)I_2 + (2\ \Omega)I_1 - (4\ \Omega)I_2 + 2\text{ V} = 0$

Note that the voltage across R_2 due to the currents I_1 and I_2 is indicated as two separated terms, where one term represents a voltage drop in the direction of I_1 and the other term represents a voltage rise in the same direction. The magnitude and polarity of the voltage across R_2 is determined by the actual size and directions of the loop currents. The above loop equations may be simplified as follows:

Loop 1: $(4\ \Omega)I_1 - (2\ \Omega)I_2 = 2\text{ V}$

Loop 2: $-(2\ \Omega)I_1 + (6\ \Omega)I_2 = 6\text{ V}$

Using determinants, the loop equations are easily solved as

$$I_1 = \frac{\begin{vmatrix} 2 & -2 \\ 6 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 6 \end{vmatrix}} = \frac{12 + 12}{24 - 4} = \frac{24}{20} = 1.20\text{ A}$$

and

$$I_2 = \frac{\begin{vmatrix} 4 & 2 \\ -2 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ -2 & 6 \end{vmatrix}} = \frac{24 + 4}{24 - 4} = \frac{28}{20} = 1.40\text{ A}$$

From the above results, we see that the currents through resistors R_1 and R_3 are I_1 and I_2 respectively.

The branch current for R_2 is found by combining the loop currents through this resistor:

$$I_{R_2} = 1.40\text{ A} - 1.20\text{ A} = 0.20\text{ A} \quad \text{(upward)}$$

The results obtained by using mesh analysis are exactly the same as those obtained by branch-current analysis. Whereas branch-current analysis required three equations, this approach requires the solution of only two simultaneous linear equations. Mesh analysis also requires that only Kirchhoff's voltage law be applied and clearly illustrates why mesh analysis is preferred to branch-current analysis.

EXAMPLE 8–12 Determine the current through R_1 for the circuit shown in Figure 8–25.

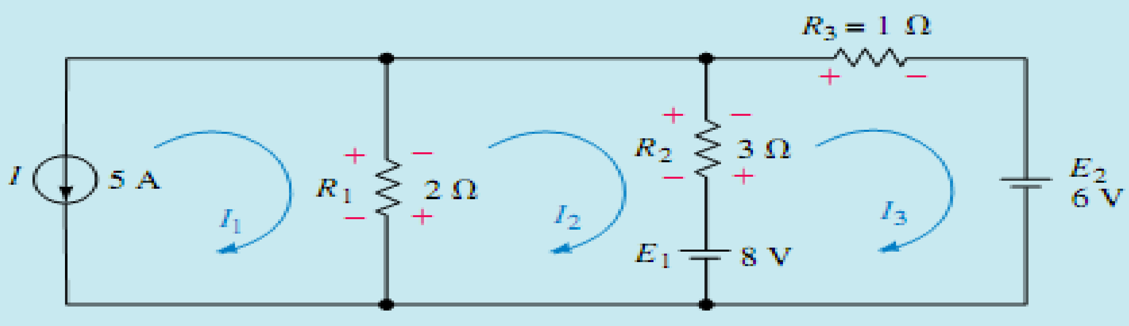


FIGURE 8–25

Solution By inspection, we see that the loop current $I_1 = -5\text{ A}$. The mesh equations for the other two loops are as follows:

Loop 2: $\quad -(2\ \Omega)I_2 + (2\ \Omega)I_1 - (3\ \Omega)I_2 + (3\ \Omega)I_3 - 8\text{ V} = 0$

Loop 3: $\quad 8\text{ V} - (3\ \Omega)I_3 + (3\ \Omega)I_2 - (1\ \Omega)I_3 - 6\text{ V} = 0$

Although it is possible to analyze the circuit by solving three linear equations, it is easier to substitute the known value $I_1 = -5V$ into the mesh equation for loop 2, which may now be written as

Loop 2: $\quad -(2\ \Omega)I_2 - 10\ \text{V} - (3\ \Omega)I_2 + (3\ \Omega)I_3 - 8\ \text{V} = 0$

The loop equations may now be simplified as

Loop 2: $\quad (5\ \Omega)I_2 - (3\ \Omega)I_3 = -18\ \text{V}$

Loop 3: $\quad -(3\ \Omega)I_2 + (4\ \Omega)I_3 = 2\ \text{V}$

The simultaneous linear equations are solved as follows:

$$I_2 = \frac{\begin{vmatrix} -18 & -3 \\ 2 & 4 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix}} = -\frac{66}{11} = -6.00\ \text{A}$$

$$I_3 = \frac{\begin{vmatrix} 5 & -18 \\ -3 & 2 \end{vmatrix}}{\begin{vmatrix} 5 & -3 \\ -3 & 4 \end{vmatrix}} = -\frac{44}{11} = -4.00\ \text{A}$$

The calculated values of the assumed reference currents allow us to determine the actual current through the various resistors as follows:

$$I_{R_1} = I_1 - I_2 = -5\ \text{A} - (-6\ \text{A}) = 1.00\ \text{A} \quad \text{downward}$$
$$I_{R_2} = I_3 - I_2 = -4\ \text{A} - (-6\ \text{A}) = 2.00\ \text{A} \quad \text{upward}$$
$$I_{R_3} = -I_3 = 4.00\ \text{A} \quad \text{left}$$

Format Approach for Mesh Analysis

The method used in applying the format approach of mesh analysis is as follows:

1. Convert current sources into equivalent voltage sources.
2. Assign clockwise currents to each independent closed loop in the network.
3. Write the simultaneous linear equations in the format outlined.
4. Solve the resulting simultaneous linear equations.

EXAMPLE 8–13 Solve for the currents through R_2 and R_3 in the circuit of Figure 8–26.

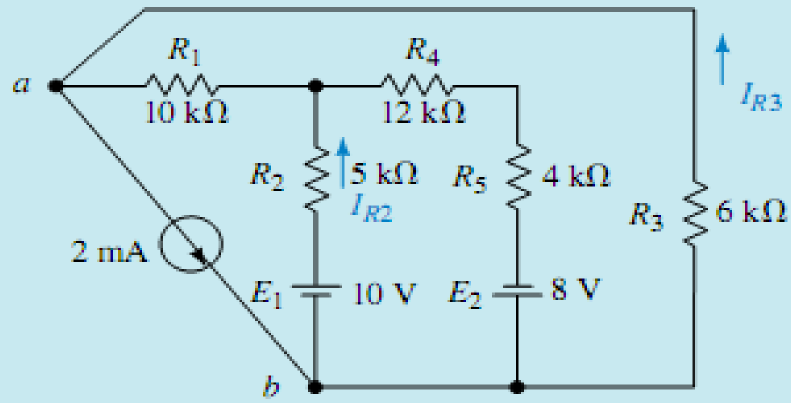


FIGURE 8–26

Solution

Step 1: Although we see that the circuit has a current source, it may not be immediately evident how the source can be converted into an equivalent voltage source. Redrawing the circuit into a more recognizable form, as shown in Figure 8–27, we see that the 2-mA current source is in parallel with a 6-kΩ resistor. The source conversion is also illustrated in Figure 8–27.

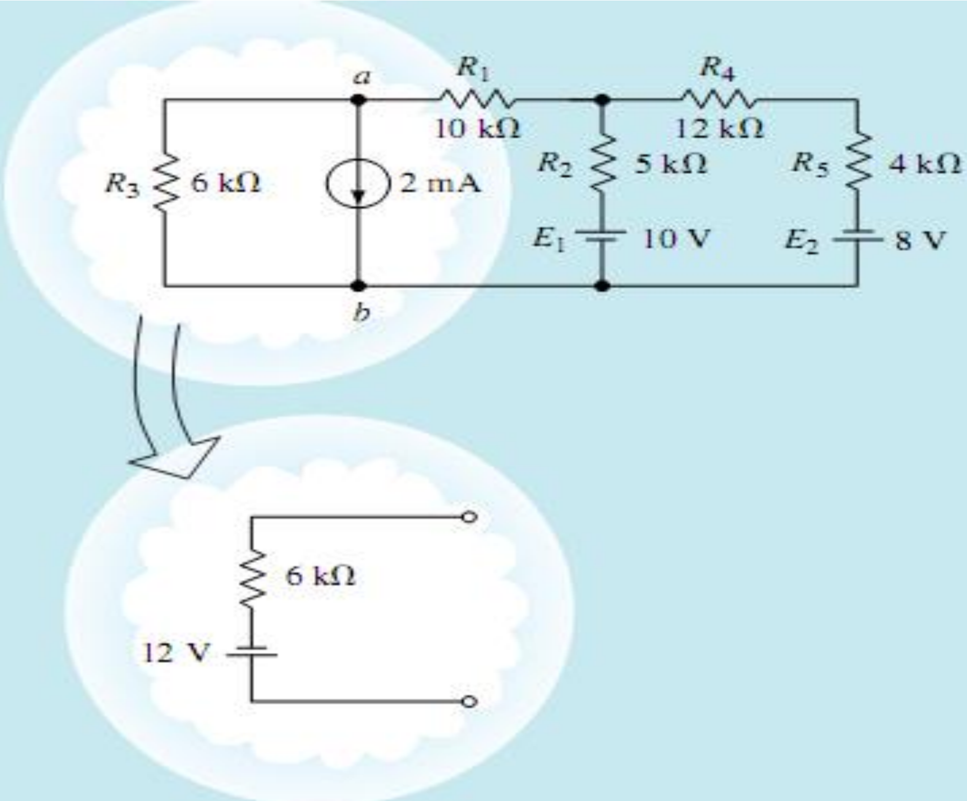


FIGURE 8–27

Step 2: Redrawing the circuit is further simplified by labelling some of the nodes, in this case *a* and *b*. After performing a source conversion, we have the two-loop circuit shown in Figure 8–28. The current directions for I_1 and I_2 are also illustrated.

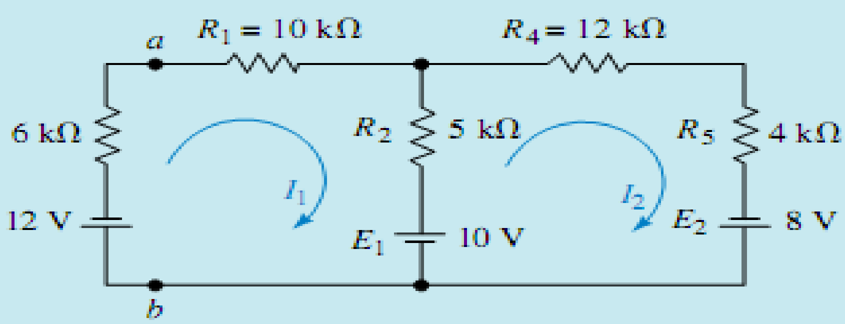


FIGURE 8–28

Step 3: The loop equations are

Loop 1: $(6\text{ k}\Omega + 10\text{ k}\Omega + 5\text{ k}\Omega)I_1 - (5\text{ k}\Omega)I_2 = -12\text{ V} - 10\text{ V}$

Loop 2: $-(5\text{ k}\Omega)I_1 + (5\text{ k}\Omega + 12\text{ k}\Omega + 4\text{ k}\Omega)I_2 = 10\text{ V} + 8\text{ V}$

In loop 1, both voltages are negative since they appear as voltage drops when following the direction of the loop current.

These equations are rewritten as

$$(21\text{ k}\Omega)I_1 - (5\text{ k}\Omega)I_2 = -22\text{ V}$$
$$-(5\text{ k}\Omega)I_1 + (21\text{ k}\Omega)I_2 = 18\text{ V}$$

Step 4: In order to simplify the solution of the previous linear equations, we may eliminate the units (kΩ and V) from our calculations. By inspection, we see that the units for current must be in milliamps. Using determinants, we solve for the currents I_1 and I_2 as follows:

$$I_1 = \frac{\begin{vmatrix} -22 & -5 \\ 18 & 21 \end{vmatrix}}{\begin{vmatrix} 21 & -5 \\ -5 & 21 \end{vmatrix}} = \frac{-462 + 90}{441 - 25} = \frac{-372}{416} = -0.894\text{ mA}$$

$$I_2 = \frac{\begin{vmatrix} 21 & -22 \\ -5 & 18 \end{vmatrix}}{\begin{vmatrix} 21 & -5 \\ -5 & 21 \end{vmatrix}} = \frac{378 + 110}{441 - 25} = \frac{268}{416} = -0.644\text{ mA}$$

The current through resistor R_2 is easily determined to be

$$I_2 - I_1 = 0.644\text{ mA} - (-0.894\text{ mA}) = 1.54\text{ mA}$$

The current through R_3 is not found as easily. A common mistake is to say that the current in R_3 is the same as the current through the 6-kΩ resistor of the circuit in Figure 8–28. **This is not the case.** Since this resistor was part of the source conversion it is no longer in the same location as in the original circuit.

$$V_{ab} = -(6\text{ k}\Omega)I_1 - 12\text{ V} = -(6\text{ k}\Omega)(-0.894\text{ mA}) - 12\text{ V} = -6.64\text{ V}$$

The above calculation indicates that the current through R_3 is upward (since point a is negative with respect to point b). The current has a value of

$$I_{R_3} = \frac{6.64\text{ V}}{6\text{ k}\Omega} = 1.11\text{ mA}$$

8.6 Nodal Analysis

In the previous section we applied Kirchhoff's voltage law to arrive at loop currents in a network. In this section we will apply Kirchhoff's current law to determine the potential difference (voltage) at any node with respect to some arbitrary reference point in a network. Once the potentials of all nodes are known, it is a simple matter to determine other quantities such as current and power within the network.

The steps used in solving a circuit using **nodal analysis** are as follows:

1. Arbitrarily assign a reference node within the circuit and indicate this node as **ground**. The reference node is usually located at the bottom of the circuit, although it may be located anywhere.
2. Convert each voltage source in the network to its equivalent current source. This step, although not absolutely necessary, makes further calculations easier to understand.
3. Arbitrarily assign voltages (V_1, V_2, . . . , V_n) to the remaining nodes in the circuit. (Remember that you have already assigned a reference node, so these voltages will all be with respect to the chosen reference.)
4. Arbitrarily assign a current direction to each branch in which there is no current source. Using the assigned current directions, indicate the corresponding polarities of the voltage drops on all resistors.
5. With the exception of the reference node (ground), apply Kirchhoff's current law at each of the nodes. If a circuit has a total of $n + 1$ nodes (including the reference node), there will be n simultaneous linear equations.
6. Rewrite each of the arbitrarily assigned currents in terms of the potential difference across a known resistance.
7. Solve the resulting simultaneous linear equations for the voltages (V_1, V_2, . . . , V_n).

EXAMPLE 8–14 Given the circuit of Figure 8–30, use nodal analysis to solve for the voltage V_{ab}.

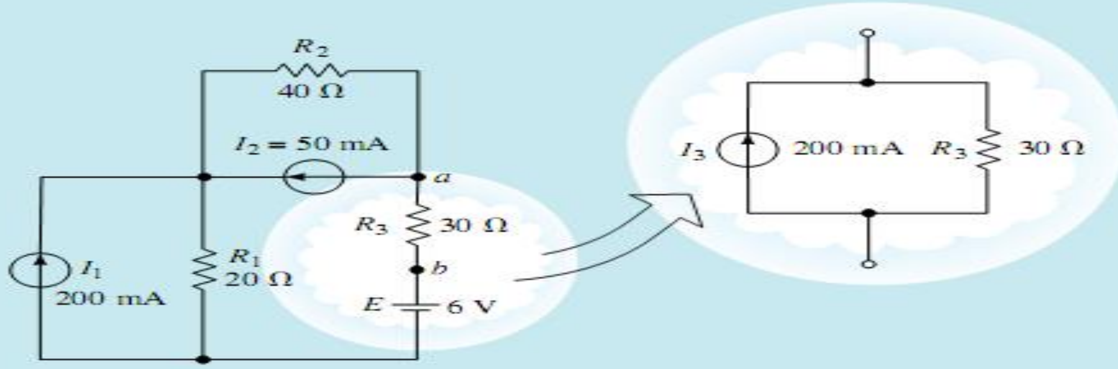


FIGURE 8–30

Solution

Step 1: Select a convenient reference node.

Step 2: Convert the voltage sources into equivalent current sources. The equivalent circuit is shown in Figure 8–31.

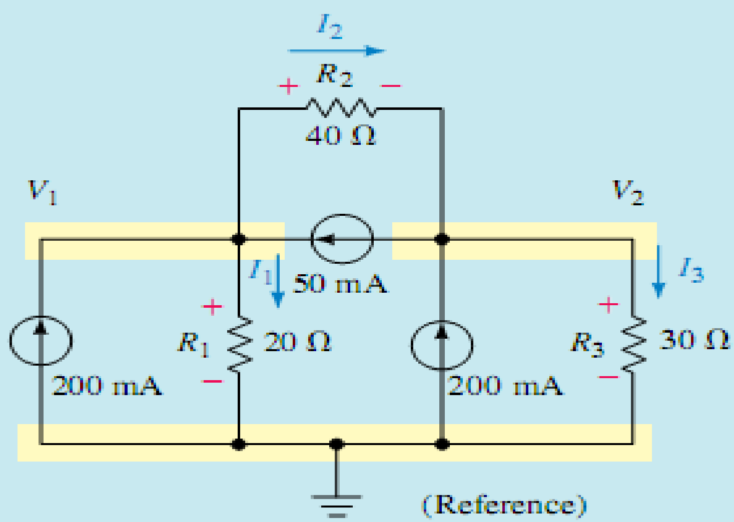


FIGURE 8–31

Steps 3 and 4: Arbitrarily assign node voltages and branch currents. Indicate the voltage polarities across all resistors according to the assumed current directions.

Step 5: We now apply Kirchhoff's current law at the nodes labelled as V_1 and V_2:

Node V_1:

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}}$$
$$200 \text{ mA} + 50 \text{ mA} = I_1 + I_2$$

Node V_2:

$$\sum I_{\text{entering}} = \sum I_{\text{leaving}}$$
$$200 \text{ mA} + I_2 = 50 \text{ mA} + I_3$$

Step 6: The currents are rewritten in terms of the voltages across the resistors as follows:

$$I_1 = \frac{V_1}{20\ \Omega}$$
$$I_2 = \frac{V_1 - V_2}{40\ \Omega}$$
$$I_3 = \frac{V_2}{30\ \Omega}$$

The nodal equations become

$$200 \text{ mA} + 50 \text{ mA} = \frac{V_1}{20\ \Omega} + \frac{V_1 - V_2}{40\ \Omega}$$
$$200 \text{ mA} + \frac{V_1 - V_2}{40\ \Omega} = 50 \text{ mA} + \frac{V_2}{30\ \Omega}$$

Substituting the voltage expressions into the original nodal equations, we have the following simultaneous linear equations:

$$\left(\frac{1}{20\ \Omega} + \frac{1}{40\ \Omega}\right)V_1 - \left(\frac{1}{40\ \Omega}\right)V_2 = 250 \text{ mA}$$
$$-\left(\frac{1}{40\ \Omega}\right)V_1 + \left(\frac{1}{30\ \Omega} + \frac{1}{40\ \Omega}\right)V_2 = 150 \text{ mA}$$

These may be further simplified as

$$(0.075 \text{ S})V_1 - (0.025 \text{ S})V_2 = 250 \text{ mA}$$
$$-(0.025 \text{ S})V_1 + (0.058\overline{3})V_2 = 150 \text{ mA}$$

Step 7: Use determinants to solve for the nodal voltages as

$$V_1 = \frac{\begin{vmatrix} 0.250 & -0.025 \\ 0.150 & 0.058\overline{3} \end{vmatrix}}{\begin{vmatrix} 0.075 & -0.025 \\ 0.025 & 0.058\overline{3} \end{vmatrix}}$$
$$= \frac{(0.250)(0.058\overline{3}) - (0.150)(-0.025)}{(0.075)(0.058\overline{3}) - (-0.025)(-0.025)}$$
$$= \frac{0.018\overline{3}}{0.00375} = 4.89 \text{ V}$$

and

$$V_2 = \frac{\begin{vmatrix} 0.075 & 0.250 \\ -0.025 & 0.150 \end{vmatrix}}{\begin{vmatrix} 0.075 & 0.025 \\ -0.025 & 0.058\overline{3} \end{vmatrix}}$$

$$= \frac{(0.075)(0.150) - (-0.025)(0.250)}{0.00375}$$

$$= \frac{0.0175}{0.00375} = 4.67\ \text{V}$$

If we go back to the original circuit of Figure 8–30, we see that the voltage V_2 is the same as the voltage V_a, namely

$$V_a = 4.67\ \text{V} = 6.0\ \text{V} + V_{ab}$$

Therefore, the voltage V_{ab} is simply found as

$$V_{ab} = 4.67\ \text{V} - 6.0\ \text{V} = -1.33\ \text{V}$$

EXAMPLE 8–15 Determine the nodal voltages for the circuit shown in Figure 8–32.

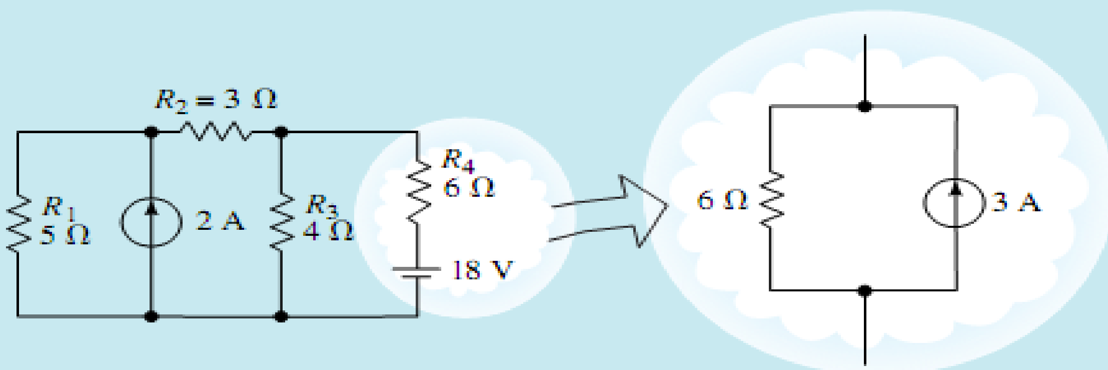
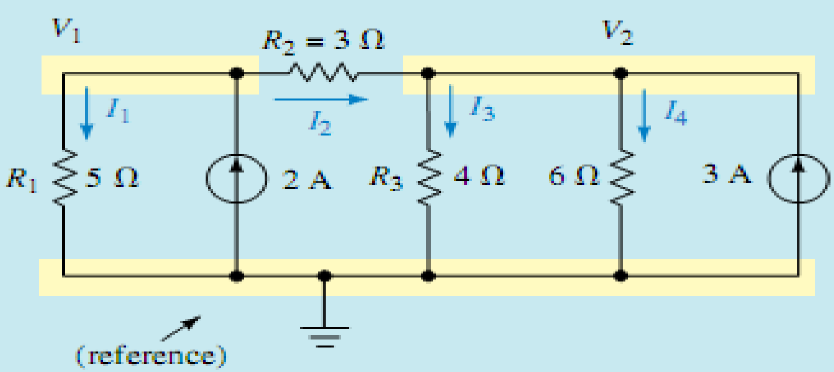


FIGURE 8–32

Solution By following the steps outlined, the circuit may be redrawn as shown in Figure 8–33.



Applying Kirchhoff's current law to the nodes corresponding to V_1 and V_2, the following nodal equations are obtained:

$$\sum I_{\text{leaving}} = \sum I_{\text{entering}}$$

Node V_1: $\quad I_1 + I_2 = 2\text{ A}$

Node V_2: $\quad I_3 + I_4 = I_2 + 3\text{ A}$

The currents may once again be written in terms of the voltages across the resistors:

$$I_1 = \frac{V_1}{5\ \Omega}$$

$$I_2 = \frac{V_1 - V_2}{3\ \Omega}$$

$$I_3 = \frac{V_2}{4\ \Omega}$$

$$I_4 = \frac{V_2}{6\ \Omega}$$

The nodal equations become

Node V_1: $$\frac{V_1}{5\ \Omega} + \frac{(V_1 - V_2)}{3\ \Omega} = 2\text{ A}$$

Node V_2: $$\frac{V_2}{4\ \Omega} + \frac{V_2}{6\ \Omega} = \frac{(V_1 - V_2)}{3\ \Omega} + 3\text{ A}$$

These equations may now be simplified as

Node V_1: $$\left(\frac{1}{5\ \Omega} + \frac{1}{3\ \Omega}\right)V_1 - \left(\frac{1}{3\ \Omega}\right)V_2 = 2\text{ A}$$

Node V_2: $$-\left(\frac{1}{3\ \Omega}\right)V_1 + \left(\frac{1}{4\ \Omega} + \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega}\right)V_2 = 3\text{ A}$$

The solutions for V_1 and V_2 are found using determinants:

$$V_1 = \frac{\begin{vmatrix} 2 & -0.333 \\ 3 & 0.750 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.333 \\ -0.333 & 0.750 \end{vmatrix}} = \frac{2.500}{0.289} = 8.65\text{ V}$$

$$V_2 = \frac{\begin{vmatrix} 0.533 & 2 \\ -0.333 & 3 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.333 \\ -0.333 & 0.750 \end{vmatrix}} = \frac{2.267}{0.289} = 7.85\text{ V}$$

Format Approach

The method used in applying the format approach of nodal analysis is as follows:

1. Convert voltage sources into equivalent current sources.
2. Label the reference node as ⏚. Label the remaining nodes as $V_1, V_2, \ldots, V_n$.
3. Write the linear equation for each node using the format outlined.
4. Solve the resulting simultaneous linear equations for $V_1, V_2, \ldots, V_n$.

The next examples illustrate how the format approach is used to solve circuit problems.

EXAMPLE 8–16 Determine the nodal voltages for the circuit shown in Figure 8–34.

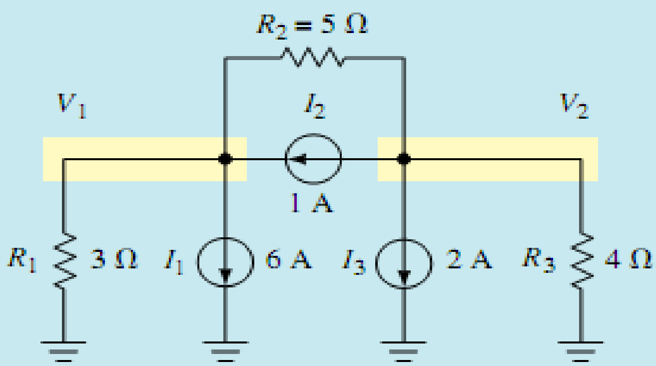


FIGURE 8–34

Solution The circuit has a total of three nodes: the reference node (at a potential of zero volts) and two other nodes, V_1 and V_2.

By applying the format approach for writing the nodal equations, we get two equations:

Node V_1:
$$\left(\frac{1}{3\ \Omega} + \frac{1}{5\ \Omega}\right)V_1 - \left(\frac{1}{5\ \Omega}\right)V_2 = -6\ \text{A} + 1\ \text{A}$$

Node V_2:
$$-\left(\frac{1}{5\ \Omega}\right)V_1 + \left(\frac{1}{5\ \Omega} + \frac{1}{4\ \Omega}\right)V_2 = -1\ \text{A} - 2\ \text{A}$$

On the right-hand sides of the above, those currents that are leaving the nodes are given a negative sign.

These equations may be rewritten as

Node V_1: $(0.533\ \mathrm{S})V_1 - (0.200\ \mathrm{S})V_2 = -5\ \mathrm{A}$

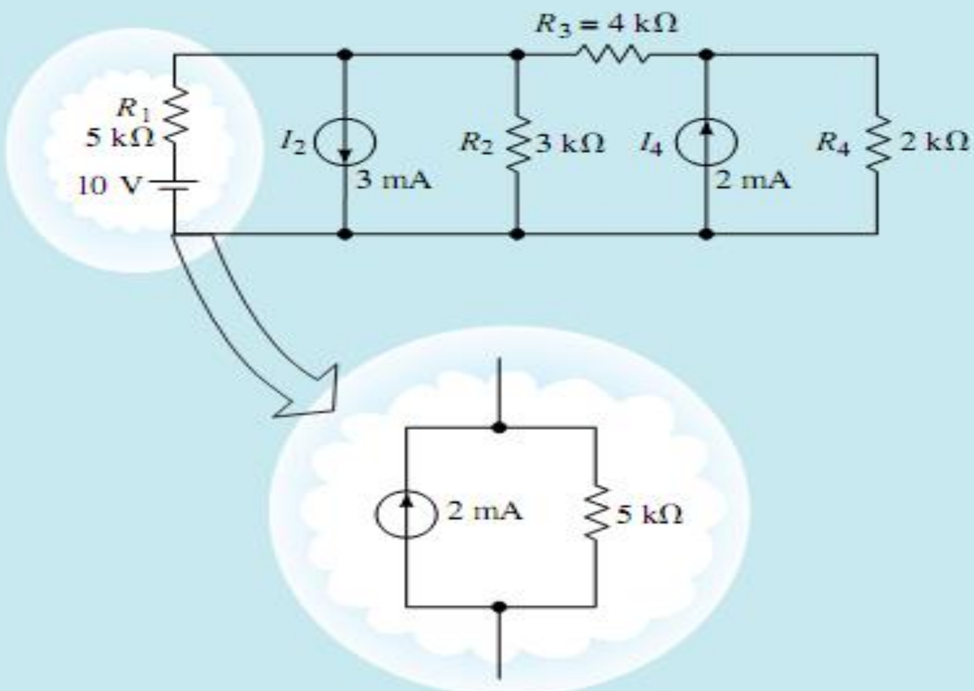
Node V_2: $-(0.200\ \mathrm{S})V_1 + (0.450\ \mathrm{S})V_2 = -3\ \mathrm{A}$

Using determinants to solve these equations, we have

$$V_1 = \frac{\begin{vmatrix} -5 & -0.200 \\ -3 & 0.450 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.200 \\ -0.200 & 0.450 \end{vmatrix}} = \frac{-2.85}{0.200} = -14.3\ \mathrm{V}$$

$$V_2 = \frac{\begin{vmatrix} 0.533 & -5 \\ -0.200 & -3 \end{vmatrix}}{\begin{vmatrix} 0.533 & -0.200 \\ -0.200 & 0.450 \end{vmatrix}} = \frac{-2.60}{0.200} = -13.0\ \mathrm{V}$$

EXAMPLE 8–17 Use nodal analysis to find the nodal voltages for the circuit of Figure 8–35. Use the answers to solve for the current through R_1.



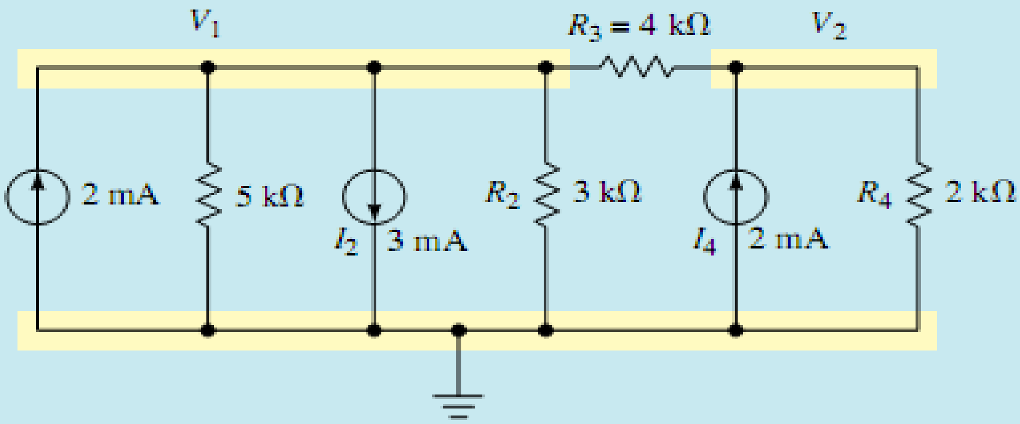


FIGURE 8–36

Labelling the nodes and writing the nodal equations, we obtain the following:

Node V_1: $$\left(\frac{1}{5\ \text{k}\Omega} + \frac{1}{3\ \text{k}\Omega} + \frac{1}{4\ \text{k}\Omega}\right)V_1 - \left(\frac{1}{4\ \text{k}\Omega}\right)V_2 = 2\ \text{mA} - 3\ \text{mA}$$

Node V_2: $$-\left(\frac{1}{4\ \text{k}\Omega}\right)V_1 + \left(\frac{1}{4\ \text{k}\Omega} + \frac{1}{2\ \text{k}\Omega}\right)V_2 = 2\ \text{mA}$$