5.2 Kirchhoff's Voltage Law

Next to Ohm's law, one of the most important laws of electricity is Kirchhoff's voltage law (KVL) which states the following:

The summation of voltage rises and voltage drops around a closed loop is equal to zero. Symbolically, this may be stated as follows:

$$\sum V = 0 \quad \text{for a closed loop} \qquad \textbf{(5–1)}$$

In the above symbolic representation, the uppercase Greek letter sigma (Σ) stands for summation and V stands for voltage rises and drops. A **closed loop** is defined as any path which originates at a point, travels around a circuit, and returns to the original point without retracing any segments.

An alternate way of stating Kirchhoff's voltage law is as follows:

The summation of voltage rises is equal to the summation of voltage drops around a closed loop.

$$\sum E_{\text{rises}} = \sum V_{\text{drops}} \quad \text{for a closed loop} \qquad \textbf{(5–2)}$$

If we consider the circuit of Figure 5–7, we may begin at point a in the lower left-hand corner. By arbitrarily following the direction of the current, I, we move through the voltage source, which represents a rise in potential from point a to point b. Next, in moving from point b to point c, we pass through resistor R_1, which presents a potential drop of V_1. Continuing through resistors R_2 and R_3, we have additional drops of V_2 and V_3 respectively. By applying Kirchhoff's voltage law around the closed loop, we arrive at the following mathematical statement for the given circuit:

$$E - V_1 - V_2 - V_3 = 0$$

Although we chose to follow the direction of current in writing Kirchhoff's voltage law equation, it would be just as correct to move around the circuit in the opposite direction. In this case the equation would appear as follows:

$$V_3 + V_2 + V_1 - E = 0$$

By simple manipulation, it is quite easy to show that the two equations are identical.

EXAMPLE 5–2 Verify Kirchhoff's voltage law for the circuit of Figure 5–8.

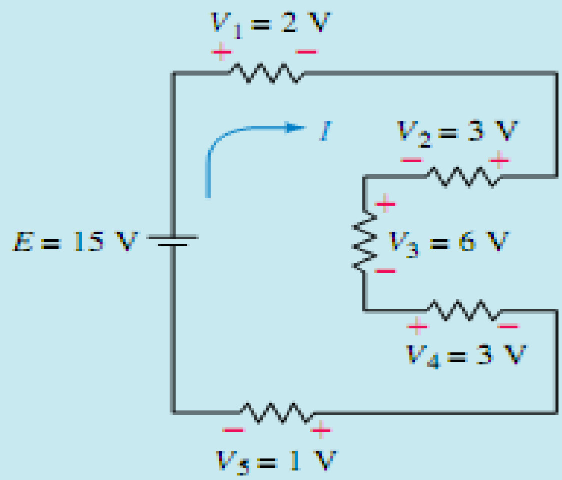


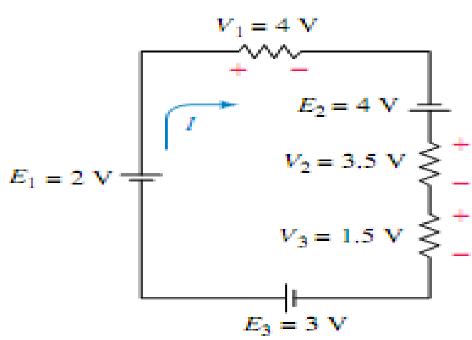
FIGURE 5–8

Solution If we follow the direction of the current, we write the loop equation as

$$15\text{ V} - 2\text{ V} - 3\text{V} - 6\text{ V} - 3\text{ V} - 1\text{ V} = 0$$

Verify Kirchhoff's voltage law for the circuit of Figure 5–9.

FIGURE 5–9



Answer: $2\text{ V} - 4\text{ V} + 4\text{ V} - 3.5\text{ V} - 1.5\text{ V} + 3\text{ V} = 0$

5.3 Resistors in Series

Almost all complicated circuits can be simplified. We will now examine how to simplify a circuit consisting of a voltage source in series with several resistors. Consider the circuit shown in Figure 5–10.

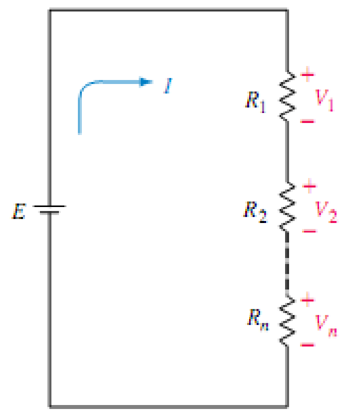


FIGURE 5–10

Since the circuit is a closed loop, the voltage source will cause a current I in the circuit. This current in turn produces a voltage drop across each resistor, where

$$V_x = IR_x$$

Applying Kirchhoff's voltage law to the closed loop gives

$$\begin{aligned} E &= V_1 + V_2 + \cdots + V_n \\ &= IR_1 + IR_2 + \cdots + IR_n \\ &= I(R_1 + R_2 + \cdots + R_n) \end{aligned}$$

If we were to replace all the resistors with an equivalent total resistance, R_T, then the circuit would appear as shown in Figure 5–11.

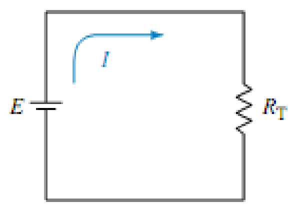


FIGURE 5–11

However, applying Ohm's law to the circuit of Figure 5–11 gives

$$E = IR_T \tag{5–3}$$

Since the circuit of Figure 5–11 is equivalent to the circuit of Figure 5–10, we conclude that this can only occur if the total resistance of the n series resistors is given as

$$R_T = R_1 + R_2 + \cdots + R_n \quad \text{[ohms, } \Omega\text{]} \tag{5–4}$$

If each of the n resistors has the same value, then the total resistance is determined as

$$R_T = nR \quad \text{[ohms, } \Omega\text{]} \tag{5–5}$$

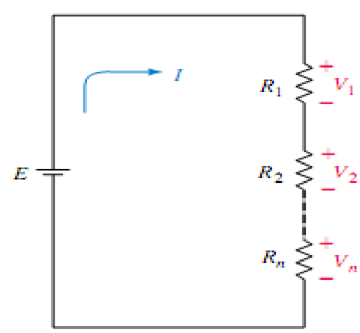


FIGURE 5–10

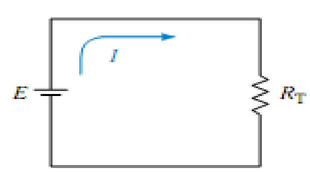


FIGURE 5–11

EXAMPLE 5–3 Determine the total resistance for each of the networks shown in Figure 5–12.

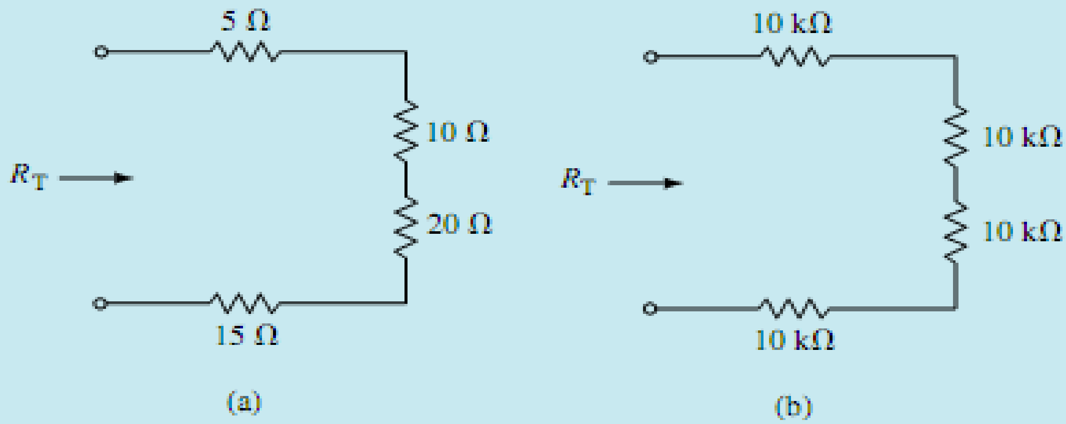


FIGURE 5–12

Any voltage source connected to the terminals of a network of series resistors will provide the same current as if a single resistance, having a value of R_T, were connected between the open terminals. From Ohm's law we get

$$I = \frac{E}{R_T} \quad \text{[amps, A]} \tag{5–6}$$

The power dissipated by each resistor is determined as

$$\begin{aligned} P_1 &= V_1 I = \frac{V_1^2}{R_1} = I^2 R_1 \quad \text{[watts, W]} \\ P_2 &= V_2 I = \frac{V_2^2}{R_2} = I^2 R_2 \quad \text{[watts, W]} \\ &\vdots \\ P_n &= V_n I = \frac{V_n^2}{R_n} = I^2 R_n \quad \text{[watts, W]} \end{aligned} \tag{5–7}$$

In Chapter 4, we showed that the power delivered by a voltage source to a circuit is given as

$$P_T = EI \quad \text{[watts, W]} \tag{5–8}$$

Since energy must be conserved, the power delivered by the voltage source is equal to the total power dissipated by all the resistors. Hence

$$P_T = P_1 + P_2 + \cdots + P_n \quad \text{[watts, W]} \tag{5–9}$$

The Voltage Divider rule

Parallel Circuits

OBJECTIVES

After studying this chapter you will be able to

- recognize which elements and branches in a given circuit are connected in parallel and which are connected in series,
- calculate the total resistance and conductance of a network of parallel resistances,
- determine the current in any resistor in a parallel circuit,
- solve for the voltage across any parallel combinations of resistors,
- apply Kirchhoff's current law to solve for unknown currents in a circuit,
- explain why voltage sources of different magnitudes must never be connected in parallel,
- use the current divider rule to solve for the current through any resistor of a parallel combination,
- design a simple ammeter using a permanent-magnet, moving-coil meter movement,
- identify and calculate the loading effects of a voltmeter connected into a circuit,
- use Electronics Workbench to observe loading effects of a voltmeter,
- use PSpice to evaluate voltage and current in a parallel circuit.

6.1 Parallel Circuits

The illustration of Figure 6–1 shows that one terminal of each light bulb is connected to the positive terminal of the battery and that the other terminal of the light bulb is connected to the negative terminal of the battery. These points of connection are often referred to as **nodes.**

Elements or branches are said to be in a parallel connection when they have exactly two nodes in common.

Figure 6–2 shows several different ways of sketching parallel elements. The elements between the nodes may be any two-terminal devices such as voltage sources, resistors, light bulbs, and the like.

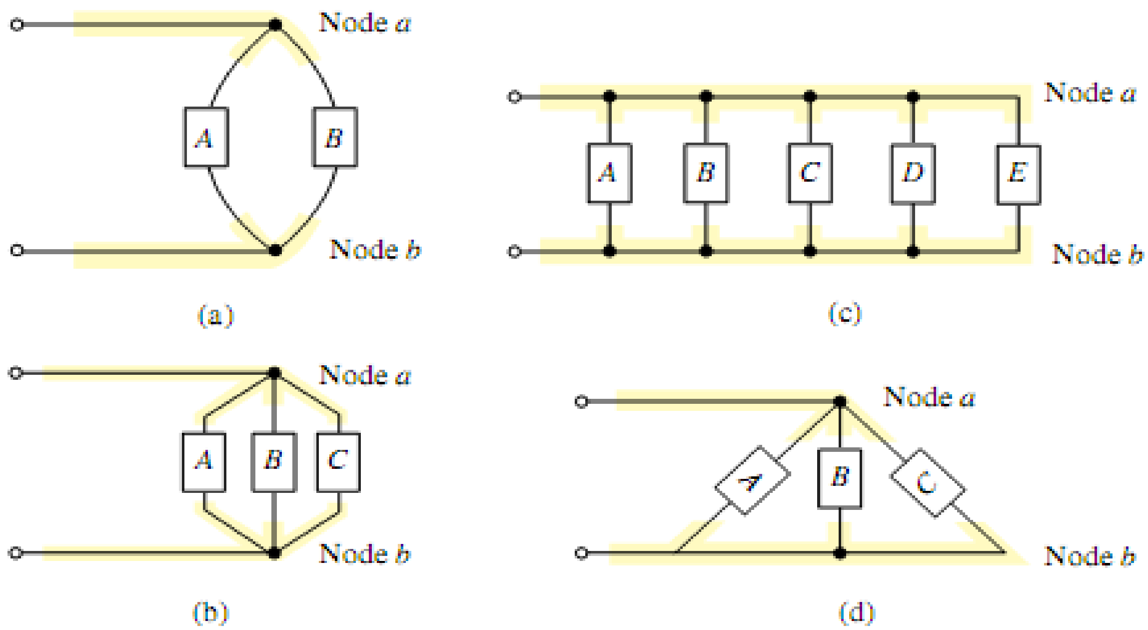
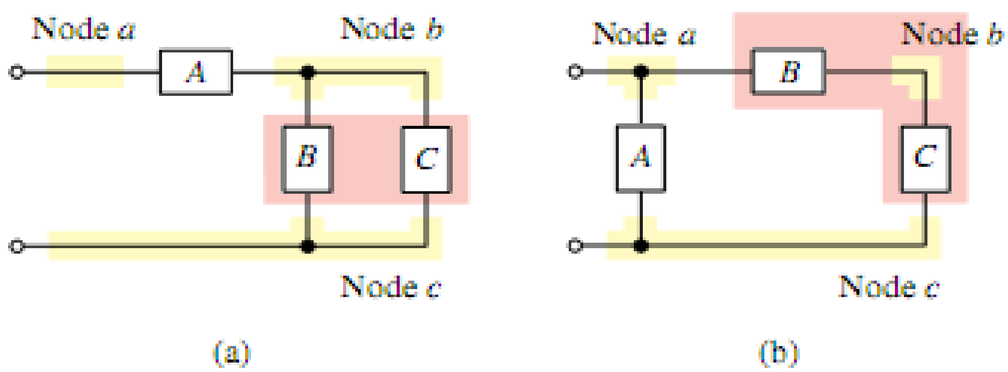


FIGURE 6–2 Parallel elements.



6.2 Kirchhoff's Current Law

Recall that Kirchhoff's voltage law was extremely useful in understanding the operation of the series circuit. In a similar manner, Kirchhoff's current law is the underlying principle which is used to explain the operation of a parallel circuit. Kirchhoff's current law states the following:

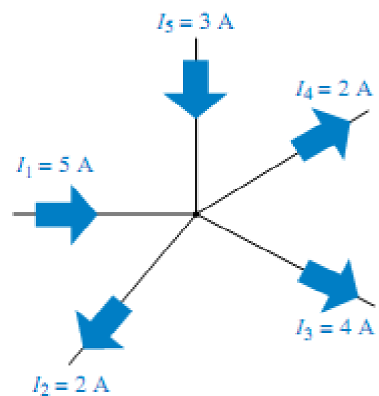
The summation of currents entering a node is equal to the summation of currents leaving the node.

An analogy which helps us understand the principle of Kirchhoff's current law is the flow of water. When water flows in a closed pipe, the amount of water entering a particular point in the pipe is exactly equal to the amount of water leaving, since there is no loss. In mathematical form, Kirchhoff's current law is stated as follows:

$$\sum I_{\text{entering node}} = \sum I_{\text{leaving node}} \qquad \textbf{(6–1)}$$

Figure 6–5 is an illustration of Kirchhoff's current law. Here we see that the node has two currents entering, $I_1 = 5$ A and $I_5 = 3$ A, and three currents leaving, $I_2 = 2$ A, $I_3 = 4$ A, and $I_4 = 2$ A. Now we can see that Equation 6–1 applies in the illustration, namely.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$
$$5\text{ A} + 3\text{ A} = 2\text{A} + 4\text{ A} + 8\text{ A}$$
$$8\text{ A} = 8\text{A} \quad \text{(checks!)}$$



EXAMPLE 6–1 Determine the magnitude and correct direction of the currents I_3 and I_5 for the network of Figure 6–7.

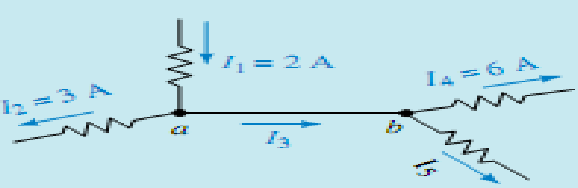


FIGURE 6–7

Solution Although points a and b are in fact the same node, we treat the points as two separate nodes with 0 Ω resistance between them.

Since Kirchhoff's current law must be valid at point a, we have the following expression for this node:

$$I_1 = I_2 + I_3$$

and so

$$\begin{aligned} I_3 &= I_1 - I_2 \\ &= 2\text{ A} - 3\text{ A} = -1\text{ A} \end{aligned}$$

Notice that the reference direction of current I_3 was taken to be from a to b, while the negative sign indicates that the current is in fact from b to a.

Similarly, using Kirchhoff's current law at point b gives

$$I_3 = I_4 + I_5$$

which gives current I_5 as

$$\begin{aligned} I_5 &= I_3 - I_4 \\ &= -1\text{ A} - 6\text{ A} = -7\text{ A} \end{aligned}$$

The negative sign indicates that the current I_5 is actually towards node b rather than away from the node. The actual directions and magnitudes of the currents are illustrated in Figure 6–8.

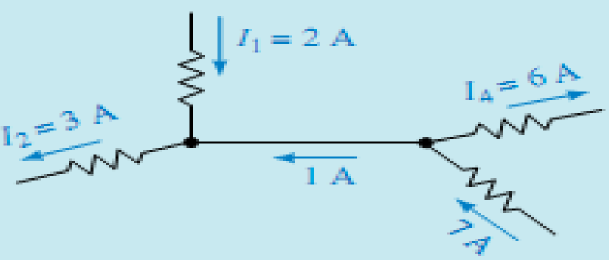


FIGURE 6–8

EXAMPLE 6–2 Find the magnitudes of the unknown currents for the circuit of Figure 6–9.

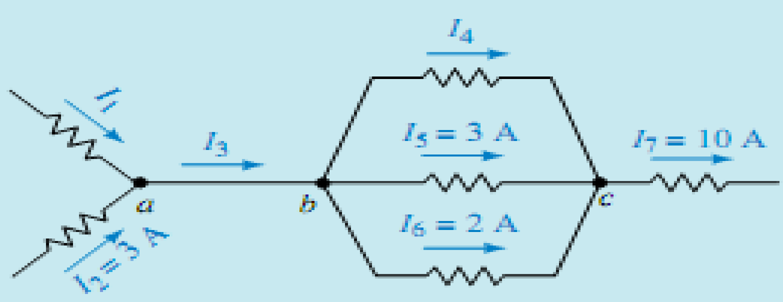


FIGURE 6–9

Solution If we consider point a, we see that there are two unknown currents, I_1 and I_3. Since there is no way to solve for these values, we examine the currents at point b, where we again have two unknown currents, I_3 and I_4. Finally we observe that at point c there is only one unknown, I_4. Using Kirchhoff's current law we solve for the unknown current as follows:

$$I_4 + 3\text{A} + 2\text{ A} = 10\text{ A}$$

Therefore,

$$I_4 = 10\text{ A} - 3\text{A} - 2\text{ A} = 5\text{ A}$$

Now we can see that at point b the current entering is

$$I_3 = 5\text{ A} + 3\text{ A} + 2\text{A} = 10\text{ A}$$

And finally, by applying Kirchhoff's current law at point a, we determine that the current I_1 is

$$I_1 = 10\text{ A} - 3\text{ A} = 7\text{ A}$$

EXAMPLE 6–3 Determine the unknown currents in the network of Figure 6–10.

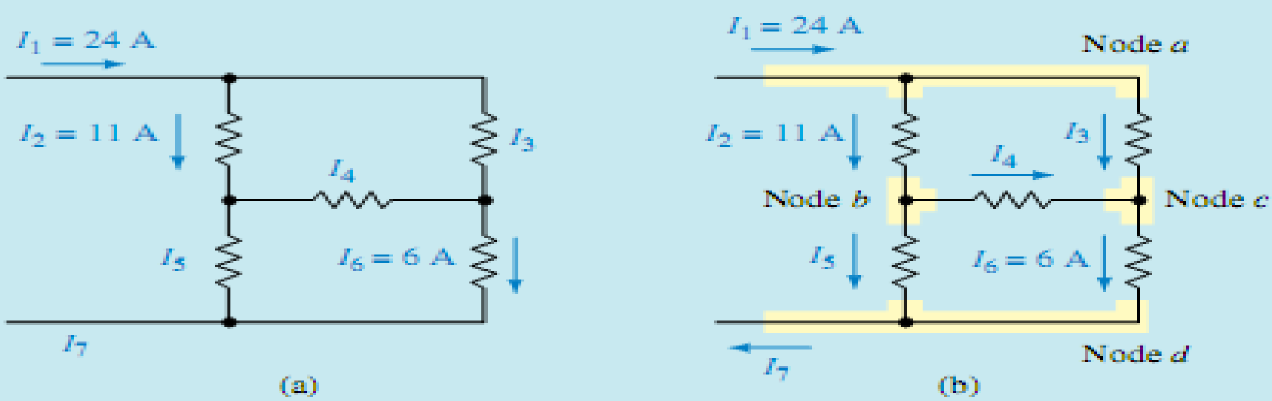


FIGURE 6–10

Solution We first assume reference directions for the unknown currents in the network.

Since we may use the analogy of water moving through conduits, we can easily assign directions for the currents I_3, I_5, and I_7. However, the direction for the current I_4 is not as easily determined, so we arbitrarily assume that its direction is to the right. Figure 6–10(b) shows the various nodes and the assumed current directions.

By examining the network, we see that there is only a single source of current $I_1 = 24$ A. Using the analogy of water pipes, we conclude that the current leaving the network is $I_7 = I_1 = 24$ A.

Now, applying Kirchhoff's current law to node *a*, we calculate the current I_3 as follows:

$$I_1 = I_2 + I_3$$

Therefore,

$$I_3 = I_1 - I_2 = 24\text{ A} - 11\text{ A} = 13\text{ A}$$

Similarly, at node *c*, we have

$$I_3 + I_4 = I_6$$

Therefore,

$$I_4 = I_6 - I_3 = 6\text{ A} - 13\text{ A} = -7\text{ A}$$

Although the current I_4 is opposite to the assumed reference direction, we do not change its direction for further calculations. We use the original direction together with the negative sign; otherwise the calculations would be needlessly complicated.

Applying Kirchhoff's current law at node *b*, we get

$$I_2 = I_4 + I_5$$

which gives

$$I_5 = I_2 - I_4 = 11\text{ A} - (-7\text{ A}) = 18\text{ A}$$

Finally, applying Kirchhoff's current law at node *d* gives

$$I_5 + I_6 = I_7$$

resulting in

$$I_7 = I_5 + I_6 = 18\text{ A} + 6\text{ A} = 24\text{ A}$$

6.3 Resistors in Parallel

A simple parallel circuit is constructed by combining a voltage source with several resistors as shown in Figure 6–12.

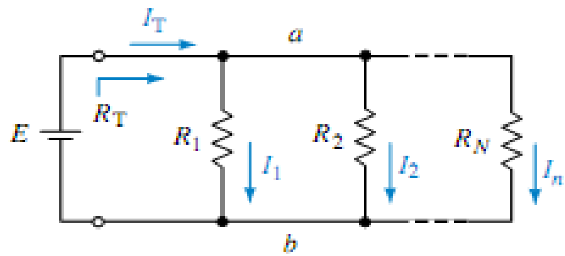


FIGURE 6–12

The voltage source will result in current from the positive terminal of the source toward node *a*. At this point the current will split between the various resistors and then recombine at node *b* before continuing to the negative terminal of the voltage source.

This circuit illustrates a very important concept of parallel circuits. If we were to apply Kirchhoff's voltage law around each closed loop in the parallel circuit of Figure 6–12, we would find that the voltage across all parallel resistors is exactly equal, namely $V_{R_1} = V_{R_2} = V_{R_3} = E$. Therefore, by applying Kirchhoff's voltage law, we make the following statement:

The voltage across all parallel elements in a circuit will be the same.

The above principle allows us to determine the equivalent resistance, R_T, of any number of resistors connected in parallel. The equivalent resistance, R_T, is the effective resistance "seen" by the source and determines the total current, I_T, provided to the circuit. Applying Kirchhoff's current law to the circuit of Figure 6–11, we have the following expression:

$$I_T = I_1 + I_2 + \cdots + I_n$$

However, since Kirchhoff's voltage law also applies to the parallel circuit, the voltage across each resistor must be equal to the supply voltage, *E*. The total current in the circuit, which is determined by the supply voltage and the equivalent resistance, may now be written as

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2} + \cdots + \frac{E}{R_n}$$

Simplifying the above expression gives us the general expression for total resistance of a parallel circuit as

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n} \quad \text{(siemens, S)} \qquad \textbf{(6–2)}$$

Since conductance was defined as the reciprocal of resistance, we may write the above equation in terms of conductance, namely,

$$G_T = G_1 + G_2 + \cdots + G_n \quad \text{(S)} \tag{6–3}$$

Whereas series resistors had a total resistance determined by the summation of the particular resistances, we see that any number of parallel resistors have a total conductance determined by the summation of the individual conductances.

The equivalent resistance of n parallel resistors may be determined in one step as follows:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots + \dfrac{1}{R_n}} \quad (\Omega) \tag{6–4}$$

An important effect of combining parallel resistors is that the resultant resistance will always be smaller than the smallest resistor in the combination.

Two Resistors in Parallel

Very often circuits have only two resistors in parallel. In such a case, the total resistance of the combination may be determined without the necessity of determining the conductance.

For two resistors, Equation 6–4 is written

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}}$$

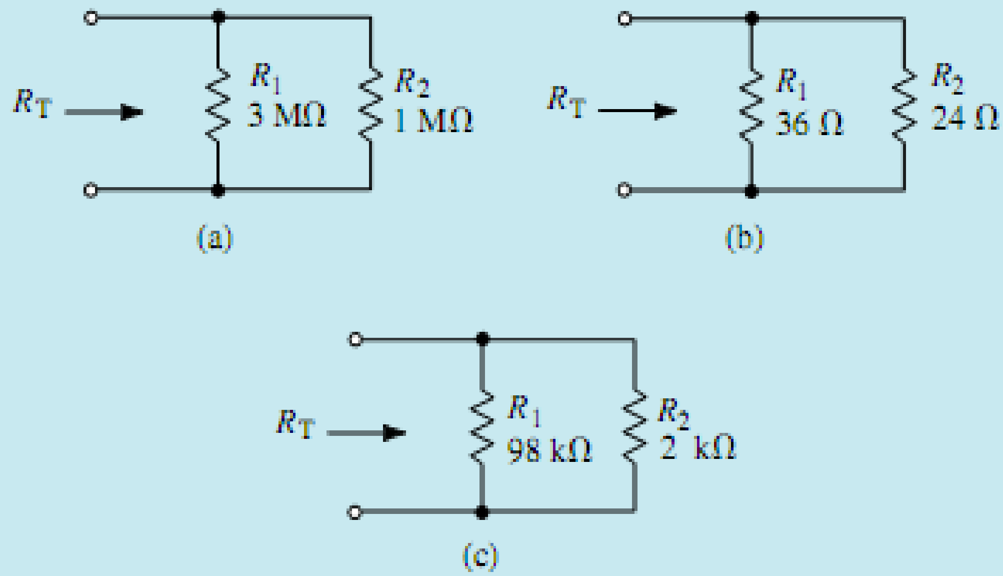
By cross multiplying the terms in the denominator, the expression becomes

$$R_T = \frac{1}{\dfrac{R_1 + R_2}{R_1 R_2}}$$

Thus, for two resistors in parallel we have the following expression:

$$R_T = \frac{R_1 R_2}{R_1 + R_2} \tag{6–6}$$

EXAMPLE 6–7 Determine the total resistance of the resistor combinations of Figure 6–17.



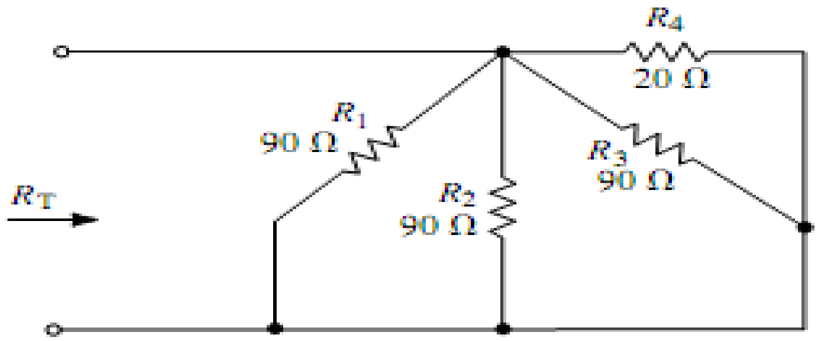
Solution

a. $R_T = \dfrac{(3\ \text{M}\Omega)(1\ \text{M}\Omega)}{3\ \text{M}\Omega + 1\ \text{M}\Omega} = 0.75\ \text{M}\Omega = 750\ \text{k}\Omega$

b. $R_T = \dfrac{(36\ \Omega)(24\ \Omega)}{36\ \Omega + 24\ \Omega} = 14.4\ \Omega$

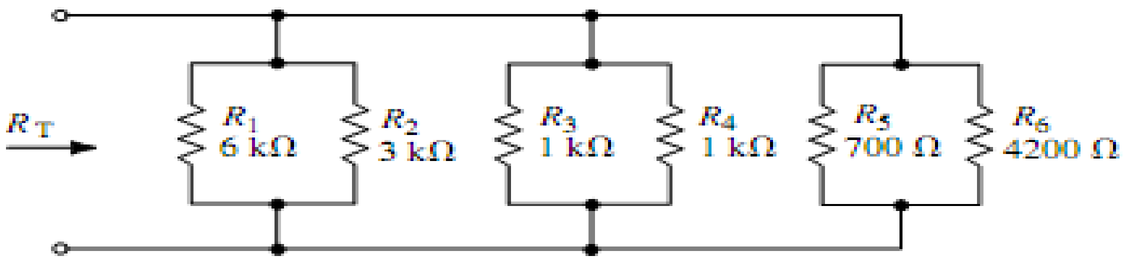
c. $R_T = \dfrac{(98\ \text{k}\Omega)(2\ \text{k}\Omega)}{98\ \text{k}\Omega + 2\ \text{k}\Omega} = 1.96\ \text{k}\Omega$

Example :



R_4
20 Ω
R_1
90 Ω
R_T
R_2
90 Ω
R_3
90 Ω

(a)



R_T
R_1
6 kΩ
R_2
3 kΩ
R_3
1 kΩ
R_4
1 kΩ
R_5
700 Ω
R_6
4200 Ω

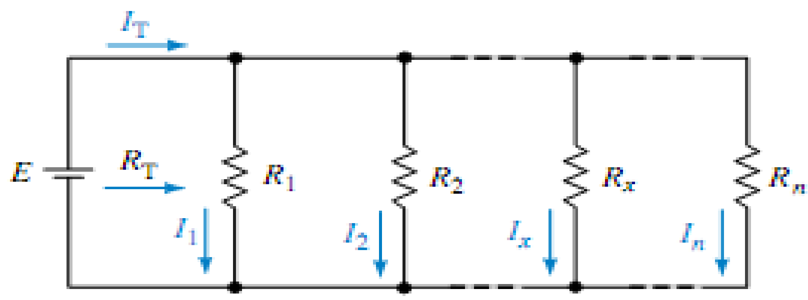
(b)

6.5 Current Divider Rule

When we examined series circuits we determined that the current in the series circuit was the same everywhere in the circuit, whereas the voltages across the series elements were typically different. The voltage divider rule (VDR) was used to determine the voltage across all resistors within a series network.

In parallel networks, the voltage across all parallel elements is the same. However, the currents through the various elements are typically different. The current divider rule (CDR) is used to determine how current entering a node is split between the various parallel resistors connected to the node.

Consider the network of parallel resistors shown in Figure 6–25.



$$I_x = \frac{R_T}{R_x} I_T$$

If this network of resistors is supplied by a voltage source, the total current in the circuit is

$$I_T = \frac{E}{R_T} \qquad (6\text{–}8)$$

Since each of the n parallel resistors has the same voltage, E, across its terminals, the current through any resistor in the network is given as

$$I_x = \frac{E}{R_x} \qquad (6\text{–}9)$$

By rewriting Equation 6–8 as $E = I_T R_T$ and then substituting this into Equation 6–9, we obtain the current divider rule as follows:

$$I_x = \frac{R_T}{R_x} I_T \qquad (6\text{–}10)$$

An alternate way of writing the current divider rule is to express it in terms of conductance. Equation 6–10 may be modified as follows:

$$I_x = \frac{G_x}{G_T} I_T \qquad (6\text{–}11)$$

The current divider rule allows us to calculate the current in any resistor of a parallel network if we know the total current entering the network. Notice the similarity between the voltage divider rule (for series components) and the current divider rule (for parallel components). The main difference is that the current divider rule of Equation 6–11 uses circuit conductance rather than resistance. While this equation is useful, it is generally easier to use resistance to calculate current.

If the network consists of only two parallel resistors, then the current through each resistor may be found in a slightly different way. Recall that for two resistors in parallel, the total parallel resistance is given as

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Now, by substituting this expression for total resistance into Equation 6–10, we obtain

$$I_1 = \frac{I_T R_T}{R_1} = \frac{I_T\left(\frac{R_1 R_2}{R_1 + R_2}\right)}{R_1}$$

which simplifies to

$$I_1 = \frac{R_2}{R_1 + R_2} I_T \quad \textbf{(6–12)}$$

Similarly,

$$I_2 = \frac{R_1}{R_1 + R_2} I_T \quad \textbf{(6–13)}$$

Several other important characteristics of parallel networks become evident.

If current enters a parallel network consisting of any number of equal resistors, then the current entering the network will split equally between all of the resistors.

If current enters a parallel network consisting of several values of resistance, then the smallest value of resistor in the network will have the largest amount of current. Inversely, the largest value of resistance will have the smallest amount of current.

This characteristic may be simplified by saying that *most of the current will follow the path of least resistance.*

EXAMPLE 6–10 For the network of Figure 6–26, determine the currents I_1, I_2, and I_3.

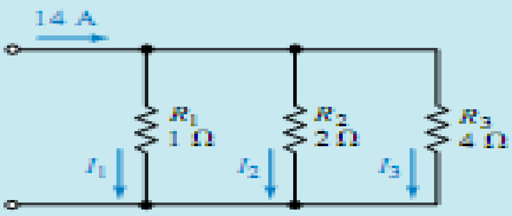


FIGURE 6–26

Solution First, we calculate the total conductance of the network.

$$G_T = \frac{1}{1\ \Omega} + \frac{1}{2\ \Omega} + \frac{1}{4\ \Omega} = 1.75\ \text{S}$$

Now the currents may be evaluated as follows:

$$I_1 = \frac{G_1}{G_T} I_T = \left(\frac{1\ \text{S}}{1.75\ \text{S}}\right) 14\ \text{A} = 8.00\ \text{A}$$

$$I_2 = \frac{G_2}{G_T} I_T = \left(\frac{0.5\ \text{S}}{1.75\ \text{S}}\right) 14\ \text{A} = 4.00\ \text{A}$$

$$I_3 = \frac{G_3}{G_T} I_T = \left(\frac{0.25\ \text{S}}{1.75\ \text{S}}\right) 14\ \text{A} = 2.00\ \text{A}$$

An alternate approach is to use circuit resistance, rather than conductance.

$$R_T = \frac{1}{G_T} = \frac{1}{1.75\ \text{S}} = 0.571\ \Omega$$

$$I_1 = \frac{R_T}{R_1} I_T = \left(\frac{0.571\ \Omega}{1\ \Omega}\right) 14\ \text{A} = 8.00\ \text{A}$$

$$I_2 = \frac{R_T}{R_2} I_T = \left(\frac{0.571\ \Omega}{2\ \Omega}\right) 14\ \text{A} = 4.00\ \text{A}$$

$$I_3 = \frac{R_T}{R_3} I_T = \left(\frac{0.571\ \Omega}{5\ \Omega}\right) 14\ \text{A} = 2.00\ \text{A}$$

EXAMPLE 6–11 For the network of Figure 6–27, determine the currents I_1, I_2, and I_3.

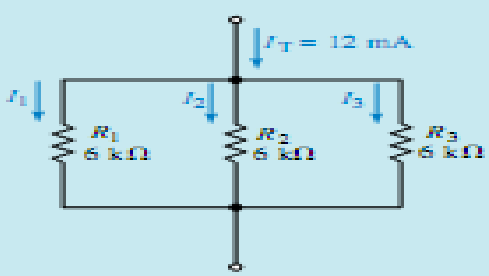


FIGURE 6–27

Solution Since all the resistors have the same value, the incoming current will split equally between the resistances. Therefore,

$$I_1 = I_2 = I_3 = \frac{12\ \text{mA}}{3} = 4.00\ \text{mA}$$

EXAMPLE 6–14

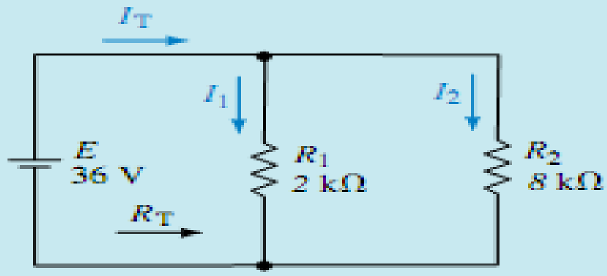


FIGURE 6–31

For the circuit of Figure 6–31, determine the following quantities:

a. R_T
b. I_T
c. Power delivered by the voltage source
d. I_1 and I_2 using the current divider rule
e. Power dissipated by the resistors

Solution

a. $R_T = \dfrac{R_1R_2}{R_1 + R_2} = \dfrac{(2\ \text{k}\Omega)(8\ \text{k}\Omega)}{2\ \text{k}\Omega + 8\ \text{k}\Omega} = 1.6\ \text{k}\Omega$

b. $I_T = \dfrac{E}{R_T} = \dfrac{36\ \text{V}}{1.6\ \text{k}\Omega} = 22.5\ \text{mA}$

c. $P_T = EI_T = (36\ \text{V})(22.5\ \text{mA}) = 810\ \text{mW}$

d. $I_2 = \dfrac{R_1}{R_1 + R_2} I_T = \left(\dfrac{2\ \text{k}\Omega}{2\ \text{k}\Omega + 8\ \text{k}\Omega}\right)(22.5\ \text{mA}) = 4.5\ \text{mA}$

$I_1 = \dfrac{R_2}{R_1 + R_2} I_T = \left(\dfrac{8\ \text{k}\Omega}{2\ \text{k}\Omega + 8\ \text{k}\Omega}\right)(22.5\ \text{mA}) = 18.0\ \text{mA}$

e. Since we know the voltage across each of the parallel resistors must be 36 V, we use this voltage to determine the power dissipated by each resistor. It would be equally correct to use the current through each resistor to calculate the power. However, it is generally best to use given information rather than calculated values to perform further calculations since it is then less likely that an error is carried through.

$$P_1 = \frac{E^2}{R_1} = \frac{(36\ \text{V})^2}{2\ \text{k}\Omega} = 648\ \text{mW}$$

$$P_2 = \frac{E^2}{R_2} = \frac{(36\ \text{V})^2}{8\ \text{k}\Omega} = 162\ \text{mW}$$

Notice that the power delivered by the voltage source is exactly equal to the total power dissipated by the resistors, namely $P_T = P_1 + P_2$.

EXAMPLE 6–15 Refer to the circuit of Figure 6–32:

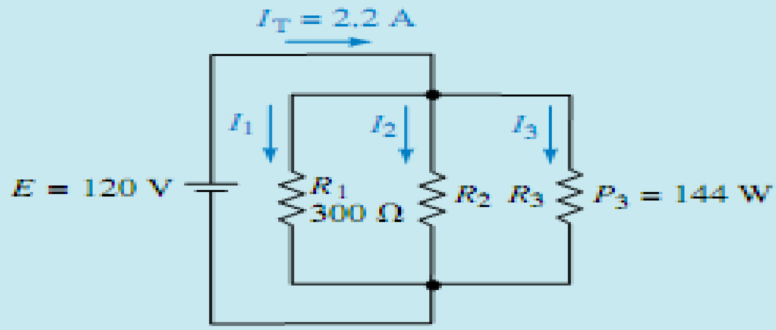


FIGURE 6–32

a. Solve for the total power delivered by the voltage source.
b. Find the currents I_1, I_2, and I_3.
c. Determine the values of the unknown resistors R_2 and R_3.
d. Calculate the power dissipated by each resistor.
e. Verify that the power dissipated is equal to the power delivered by the voltage source.

Solution

a. $P_T = EI_T = (120\text{ V})(2.2\text{ A}) = 264\text{ W}$

b. Since the three resistors of the circuit are in parallel, we know that the voltage across all resistors must be equal to $E = 120$ V.

$$I_1 = \frac{V_1}{R_1} = \frac{120\text{ V}}{300\ \Omega} = 0.4\text{ A}$$

$$I_3 = \frac{P_3}{V_3} = \frac{144\text{ W}}{120\text{ V}} = 1.2\text{ A}$$

Because KCL must be maintained at each node, we determine the current I_2 as

$$I_2 = I_T - I_1 - I_3$$
$$= 2.2\text{ A} - 0.4\text{ A} - 1.2\text{ A} = 0.6\text{ A}$$

c. $R_2 = \frac{V_2}{I_2} = \frac{120\text{ V}}{0.6\text{ A}} = 200\ \Omega$

Although we could use the calculated current I_3 to determine the resistance, it is best to use the given data in calculations rather than calculated values.

$$R_3 = \frac{V_3^2}{P_3} = \frac{(120\text{ V})^2}{144\text{ W}} = 100\ \Omega$$

d. $P_1 = \frac{V_1^2}{R_1} = \frac{(120\text{ V})^2}{300\ \Omega} = 48\text{ W}$

$P_2 = I_2 E_2 = (0.6\text{ A})(120\text{ V}) = 72\text{ W}$

e. $P_{in} = P_{out}$

$264\text{ W} = P_1 + P_2 + P_3$

$264\text{ W} = 48\text{ W} + 72\text{ W} + 144\text{ W}$

$264\text{ W} = 264\text{ W}$ (checks!)

EXAMPLE 7–2 Consider the circuit of Figure 7–4.

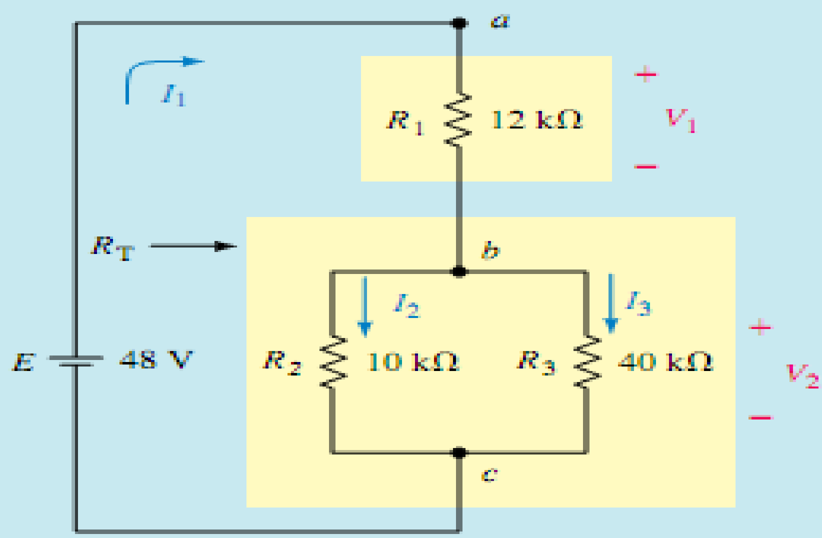


FIGURE 7–4

a. Find R_T.
b. Calculate I_1, I_2, and I_3.
c. Determine the voltages V_1 and V_2.

Solution By examining the circuit of Figure 7–4, we see that resistors R_2 and R_3 are in parallel. This parallel combination is in series with the resistor R_1.

The combination of resistors may be represented by a simple series network shown in Figure 7–5. Notice that the nodes have been labelled using the same notation.

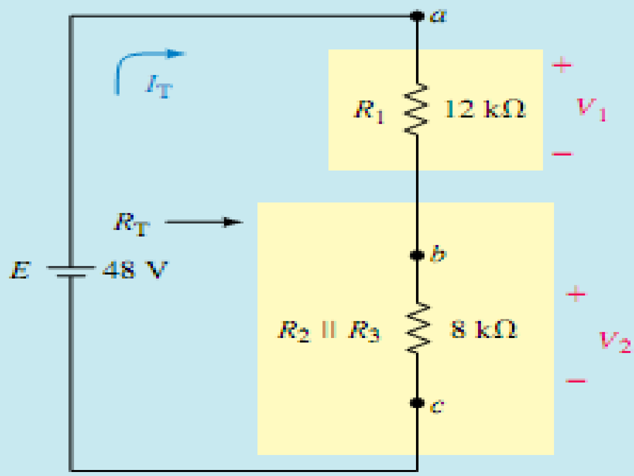


FIGURE 7–5

a. The total resistance of the circuit may be determined from the combination

$$R_T = R_1 + R_2 \| R_3$$

$$R_T = 12\ \text{k}\Omega + \frac{(10\ \text{k}\Omega)(40\ \text{k}\Omega)}{10\ \text{k}\Omega + 40\ \text{k}\Omega}$$

$$= 12\ \text{k}\Omega + 8\ \text{k}\Omega = 20\ \text{k}\Omega$$

b. From Ohm's law, the total current is

$$I_T = I_1 = \frac{48\ \text{V}}{20\ \text{k}\Omega} = 2.4\ \text{mA}$$

The current I_1 will enter node b and then split between the two resistors R_2 and R_3. This current divider may be simplified as shown in the partial circuit of Figure 7–6.

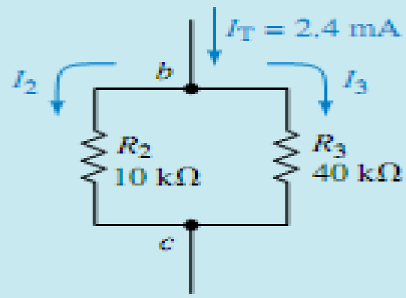


FIGURE 7–6

Applying the current divider rule to these two resistors gives

$$I_2 = \frac{(40\ \text{k}\Omega)(2.4\ \text{mA})}{10\ \text{k}\Omega + 40\ \text{k}\Omega} = 1.92\ \text{mA}$$

$$I_3 = \frac{(10\ \text{k}\Omega)(2.4\ \text{mA})}{10\ \text{k}\Omega + 40\ \text{k}\Omega} = 0.48\ \text{A}$$

c. Using the above currents and Ohm's law, we determine the voltages:

$$V_1 = (2.4\ \text{mA})(12\ \text{k}\Omega) = 28.8\ \text{V}$$
$$V_3 = (0.48\ \text{mA})(40\ \text{k}\Omega) = 19.2\ \text{V} = V_2$$

In order to check the answers, we may simply apply Kirchhoff's voltage law around any closed loop which includes the voltage source:

$$\begin{aligned}\sum V &= E - V_1 - V_3 \\ &= 48\ \text{V} - 28.8\ \text{V} - 19.2\ \text{V} \\ &= 0\ \text{V (checks!)}\end{aligned}$$

The solution may be verified by ensuring that the power delivered by the voltage source is equal to the summation of powers dissipated by the resistors.

EXAMPLE 7–3 Find the voltage V_{ab} for the circuit of Figure 7–7.

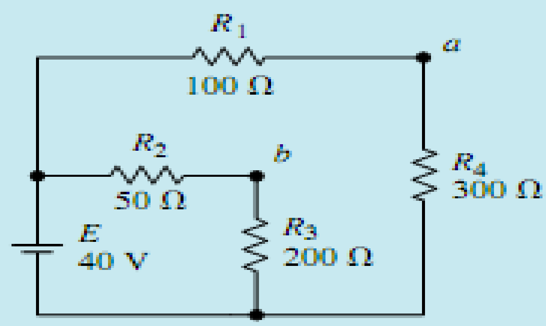


FIGURE 7–7

Solution We begin by redrawing the circuit in a more simple representation as shown in Figure 7–8.

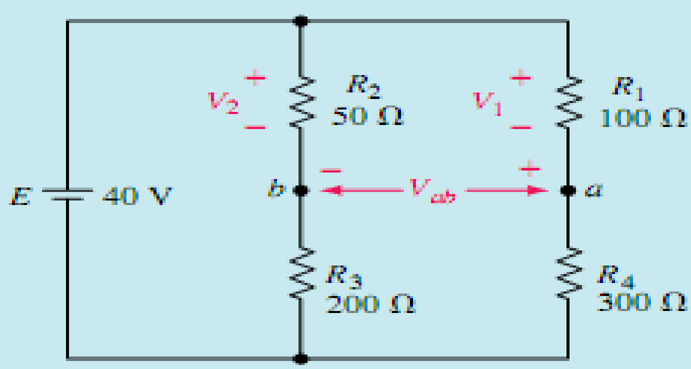


FIGURE 7–8

From Figure 7–8, we see that the original circuit consists of two parallel branches, where each branch is a series combination of two resistors.

If we take a moment to examine the circuit, we see that the voltage V_{ab} may be determined from the combination of voltages across R_1 and R_2. Alternatively, the voltage may be found from the combination of voltages across R_3 and R_4.

As usual, several methods of analysis are possible. Because the two branches are in parallel, the voltage across each branch must be 40 V. Using the voltage divider rules allows us to quickly calculate the voltage across each resistor. Although equally correct, other methods of calculating the voltages would be more lengthy.

$$V_2 = \frac{R_2}{R_2 + R_3}E$$

$$= \left(\frac{50\ \Omega}{50\ \Omega + 200\ \Omega}\right)(40\ \text{V}) = 8.0\ \text{V}$$

$$V_1 = \frac{R_1}{R_1 + R_4}E$$

$$= \left(\frac{100\ \Omega}{100\ \Omega + 300\ \Omega}\right)(40\ \text{V}) = 10.0\ \text{V}$$

As shown in Figure 7–9, we apply Kirchhoff's voltage law to determine the voltage between terminals *a* and *b*.

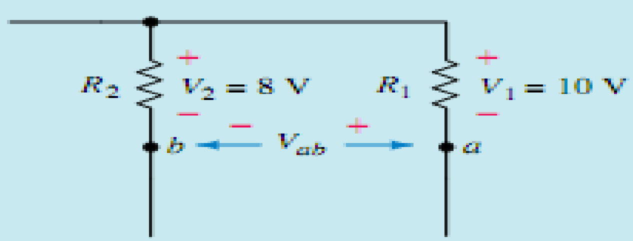
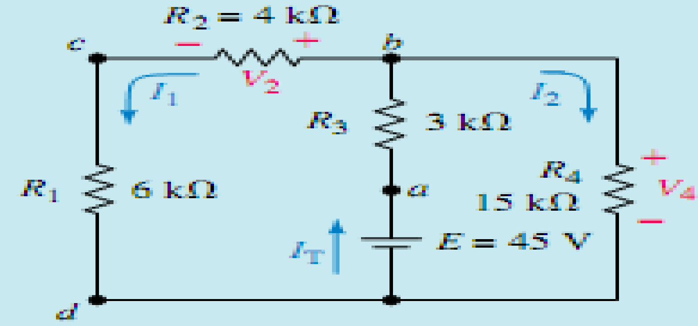


FIGURE 7–9

$$V_{ab} = -10.0\ \text{V} + 8.0\ \text{V} = -2.0\ \text{V}$$

EXAMPLE 7–4 Consider the circuit of Figure 7–10:



EWB

FIGURE 7–10

a. Find the total resistance R_T "seen" by the source E.

b. Calculate I_T, I_1, and I_2.

c. Determine the voltages V_2 and V_4.

Solution We begin the analysis by redrawing the circuit. Since we generally like to see the source on the left-hand side, one possible way of redrawing the resultant circuit is shown in Figure 7–11. Notice that the polarities of voltages across all resistors have been shown.

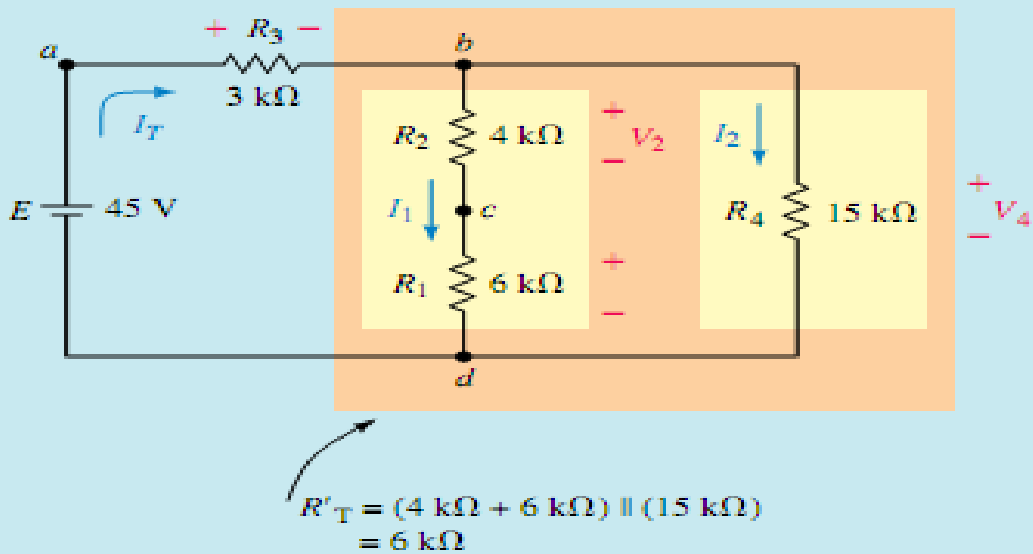


FIGURE 7–11

a. From the redrawn circuit, the total resistance of the circuit is

$$R_T = R_3 + [(R_1 + R_2) \| R_4]$$
$$= 3\text{ k}\Omega + \frac{(4\text{ k}\Omega + 6\text{ k}\Omega)(15\text{ k}\Omega)}{(4\text{ k}\Omega + 6\text{ k}\Omega) + 15\text{ k}\Omega}$$
$$= 3\text{ k}\Omega + 6\text{ k}\Omega = 9.00\text{ k}\Omega$$

b. The current supplied by the voltage source is

$$I_T = \frac{E}{R_T} = \frac{45\text{ V}}{9\text{ k}\Omega} = 5.00\text{ mA}$$

We see that the supply current divides between the parallel branches as shown in Figure 7–12.

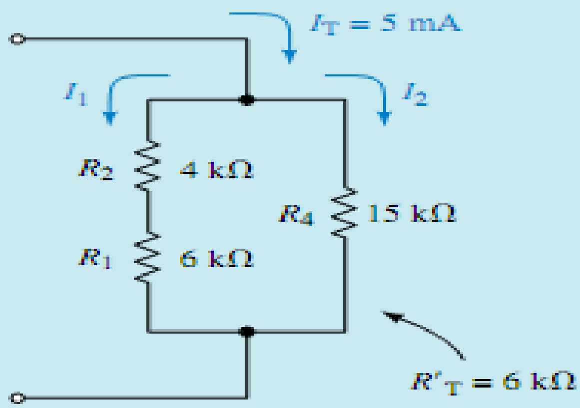


FIGURE 7–12

Applying the current divider rule, we calculate the branch currents as

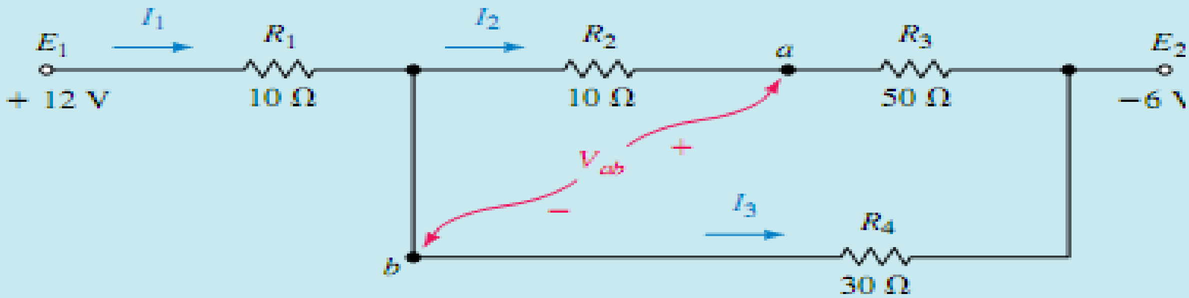
$$I_1 = I_T \frac{R'_T}{(R_1 + R_2)} = \frac{(5\text{ mA})(6\text{ k}\Omega)}{4\text{ k}\Omega + 6\text{ k}\Omega} = 3.00\text{ mA}$$
$$I_2 = I_T \frac{R'_T}{R_4} = \frac{(5\text{ mA})(6\text{ k}\Omega)}{15\text{ k}\Omega} = 2.00\text{ mA}$$

Notice: When determining the branch currents, the resistance R'_T is used in the calculations rather the the total circuit resistance. This is because the current I_T = 5 mA splits between the two branches of R'_T and the split is not affected by the value of R_3.

c. The voltages V_2 and V_4 are now easily calculated by using Ohm's law:

$$V_2 = I_1 R_2 = (3\text{ mA})(4\text{ k}\Omega) = 12.0\text{ V}$$
$$V_4 = I_2 R_4 = (2\text{ mA})(15\text{ k}\Omega) = 30.0\text{ V}$$

EXAMPLE 7–5 For the circuit of Figure 7–13, find the indicated currents and voltages.



EWB **FIGURE 7–13**

Solution Because the above circuit contains voltage point sources, it is easier to analyze if we redraw the circuit to help visualize the operation.

The point sources are voltages with respect to ground, and so we begin by drawing a circuit with the reference point as shown in Figure 7–14.

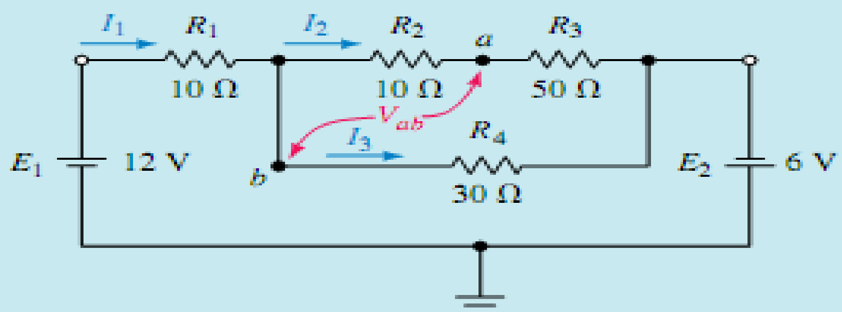
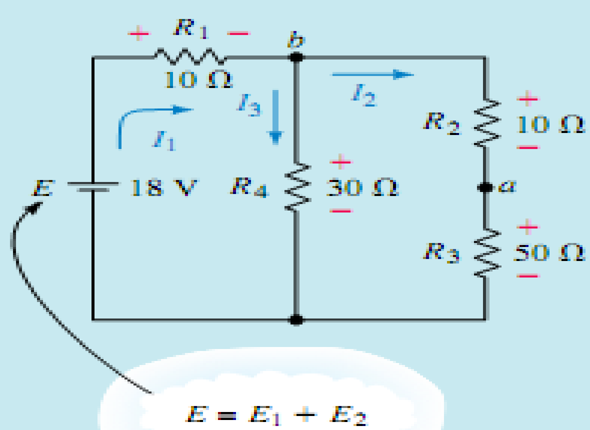


FIGURE 7–14

Now, we can see that the circuit may be further simplified by combining the voltage sources ($E = E_1 + E_2$) and by showing the resistors in a more suitable location. The simplified circuit is shown in Figure 7–15.



The total resistance "seen" by the equivalent voltage source is

$$R_T = R_1 + [R_4 \| (R_2 + R_3)]$$
$$= 10\ \Omega + \frac{(30\ \Omega)(10\ \Omega + 50\ \Omega)}{30\ \Omega + (10\ \Omega + 50\ \Omega)} = 30.0\ \Omega$$

And so the total current provided into the circuit is

$$I_1 = \frac{E}{R_T} = \frac{18\ \text{V}}{30\ \Omega} = 0.600\ \text{A}$$

At node b this current divides between the two branches as follows:

$$I_3 = \frac{(R_2 + R_3)I_1}{R_4 + R_2 + R_3} = \frac{(60\ \Omega)(0.600\ \text{A})}{30\ \Omega + 10\ \Omega + 50\ \Omega} = 0.400\ \text{A}$$
$$I_2 = \frac{R_4 I_1}{R_4 + R_2 + R_3} = \frac{(30\ \Omega)(0.600\ \text{A})}{30\ \Omega + 10\ \Omega + 50\ \Omega} = 0.200\ \text{A}$$

The voltage V_{ab} has the same magnitude as the voltage across the resistor R_2, but with a negative polarity (since b is at a higher potential than a):

$$V_{ab} = -I_2 R_2 = -(0.200\ \text{A})(10\ \Omega) = -2.0\ \text{V}$$